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Numerical Solutions of Unsteady Laminar Free Convection from a Vertical Cone with Non-Uniform Surface Heat Flux

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ABSTRACT

Numerical solutions of, unsteady laminar free convection from an incompressible viscous fluid past a vertical cone with non-uniform surface heat flux $q_w(x)=a\,x^m$ varying as a power function of the distance from the apex of the cone ($x=0$) is presented. Here m is the exponent in power law variation of the surface heat flux. The dimensionless governing equations of the flow that are unsteady, coupled and non-linear partial differential equations are solved by an efficient, accurate and unconditionally stable finite difference scheme of Crank-Nicolson type. The velocity and temperature fields have been studied for various parameters viz. Prandtl number $\Pr$, semi vertical angle ϕ and the exponent m. The local as well as average skin-friction and Nusselt number are also presented and analyzed graphically. The present results are compared with available results in literature and are found to be in good agreement

Keywords: Vertical cone, Non-uniform surface heat flux, Free convection

NOMENCLATURE

a	Constant	v	Velocity component in $y-$ direction
$f''(0)$	Local skin-friction	X	Dimensionless spatial co-ordinate along cone generator
$f'(\eta)$	Dimensionless velocity in $X-$ direction	x	Spatial co-ordinate along cone
$F_0''(0)$	Local skin friction	Y	Dimensionless spatial co-ordinate along the normal to the cone generator
Gr_L	Grashof number	y	Spatial co-ordinate along the normal to the cone generator
Gr_L^*	Modified Grashof number (= $Gr_L \cos\phi$)	α	Thermal diffusivity
g	Acceleration due to gravity	β	Volumetric thermal expansion
k	Thermal conductivity	η	Dimensionless independent variable
L	Reference length	ΔT	Dimensionless time-step
Nu_X	Non-dimensional local Nusselt number	ΔX	Dimensionless finite difference grid size in $X-$ direction
$\overline{Nu}$	Non-dimensional average Nusselt number	ΔY	Dimensionless finite difference grid size in $Y-$ direction
$\Pr$	Prandtl number	ϕ	Semi vertical angle of the cone
q_w	Rate of heat transfer per unit area	$1/\Phi_0(0)$	Local Nusselt number

R	Dimensionless local radius of the cone	r	Local radius of the cone
T'	Temperature	V	Dimensionless velocity in $Y-$ direction
T	Dimensionless temperature	υ	Kinematic viscosity
t'	Time	τ_X	Dimensionless local skin-friction
t	Dimensionless time	$\bar{\tau}$	Dimensionless average skin-friction
U	Dimensionless velocity in $X-$ direction	$-\theta(0)$	Temperature
u	Velocity component in $x-$ direction	∞	Free stream condition

1. INTRODUCTION

Natural convection flows under the influence of gravitational force have been investigated most extensively because they occur frequently in nature as well as in science and engineering applications. When a heated surface is in contact with the fluid, the result of temperature difference causes buoyancy force, which induces the natural convection. Recently heat flux applications are widely used in industries, engineering and science fields. Heat flux sensors can be used in industrial measurement and control systems. Examples of few applications are detection fouling (Boiler Fouling Sensor), monitoring of furnaces (Blast Furnace Monitoring/General Furnace Monitoring) and flare monitoring. Use of heat flux sensors can lead to improvements in efficiency, system safety and modeling.

From a technological point of view, the study of convection heat transfer from a cone is of special interest and has wide range of practical applications. Mainly, these types of heat transfer problems deal with the design of spacecrafts, nuclear reactor, solar power collectors, power transformers, steam generators etc. Since 1953 many investigations (Merk and Prins 1953, 1954, Hering and Grosh 1962, Hering 1965, Roy 1974, Gorla *et al.* 1986, Alamgir 1989, Pop and Takhar 1991, Ramanaiah and Kumaran 1992, Hossain and Paul 2001, Pop *et al.* 2003, Takhar *et al*. 2004, Alam *et al.* 2007) are carried out by developing similarity/non-similarity solutions for axi-symmetrical problems for natural convection flows over a vertical cone in steady state. Recently, Bapuji and Ekambavanan (2006) have numerically studied the solutions of unsteady flows past plane/axi-symmetrical shape bodies. Bapuji *et al.* (2007, 2008) have numerically studied the problem of transient natural convection from a vertical cone with isothermal, non-isothermal surface temperature using an implicit finite-difference method.

Recently theoretical studies on laminar free convection flow of axi-symmetric bodies have received wide attention especially in case of uniform and non-uniform surface heat flux. Similarity solutions for the laminar free convection from a right circular cone with prescribed uniform heat flux conditions for various values of Prandtl number (i.e. $\Pr = 0.72, 1, 2, 4, 6, 8, 10, 100$) and expressions for both wall skin friction and wall temperature distributions at $\Pr \to \infty$ were presented by Lin (1976). Further, Pop and Watanabe (1992) focused the theoretical study on the effects of suction or injection on steady free convection from a vertical cone with uniform surface heat flux condition. Kumari and Pop (1998) studied free convection from vertical rotating cone with uniform wall heat flux. Hasan and Mujumdar (1984) analyzed double diffusion effects in free convection under flux condition along a vertical cone. Hossain and Paul (2001), Hossain *et al.* (2002) studied non-similarity solutions for the free convection from a vertical permeable cone with non-uniform surface heat flux and the problem of laminar natural convective flow and heat transfer from a vertical circular cone immersed in a thermally stratified medium with either a uniform surface temperature or a uniform surface heat flux. Using a finite difference method, a series solution method and asymptotic solution method, the solutions have been obtained for the non-similarity boundary layer equations.

Na and Chiou (1979, 1980) studied the non-similar solutions for transverse curvature effects of the natural convection flow over a slender frustum of a cone. Na and Chiou (1979a) studied without transverse curvature effects on the laminar natural convection flow over a frustum of a cone. In above investigations the constant wall temperature as well as the constant wall heat flux was considered. Gorla *et al.* (1994) presented numerical solution for laminar free convection from a vertical frustum of a cone without transverse curvature effect (i.e. large cone angles when the boundary layer thickness is small compared with the local radius of the cone) to power-law fluids. Pop and Na (1999) studied the effects of amplitude of the wavy surfaces associated with natural convection over a vertical frustum of a cone with constant wall temperature or constant wall heat flux.

The present investigation, namely, unsteady laminar free convection from a vertical cone with uniform surface heat flux has not received any attention. Hence, the present work is considered to deal with transient free convection from a vertical cone with non-uniform surface heat flux. The governing boundary layer equations are solved by an implicit finite difference scheme of Crank-Nicolson type with various parameters $\Pr$, m and ϕ. In order to check the accuracy of the numerical results, the present results are compared with the available results of Lin (1976), Pop and Watanabe (1992), Na and Chiou (1979a), Hossain and Paul (2001) and are found to be in excellent agreement.

2. MATHEMATICAL ANALYSIS

An axi-symmetric unsteady laminar free convection of a viscous incompressible flow past vertical cone with non-uniform surface heat flux is considered. It is assumed that the viscous dissipation effects and pressure gradient along the boundary layer are negligible. Also it is assumed that the cone surface and the surrounding fluid, i.e. is at rest are with the same temperature T'_∞. Then at time $t' > 0$, it is assumed that heat is supplied from cone surface to the fluid at the rate $q_w(x) = a\, x^m$ and it is maintained at this value with m being a constant. The co-ordinate system is chosen (as shown in Fig.1) such that x measures the distance along surface of the cone from the apex $(x = 0)$, and y measures the distance normally outward. Here, ϕ is the semi vertical angle of the cone and r is the local radius of the cone. The fluid properties are assumed constant except for density variations, which induce buoyancy force and it plays main role in free convection. The governing boundary layer equations of continuity, momentum and energy under Boussinesq approximation are as follows:

Continuity equation

$$\frac{\partial}{\partial x}(r u) + \frac{\partial}{\partial y}(r v) = 0 \tag{1}$$

Momentum equation

$$\frac{\partial u}{\partial t'} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = g\beta(T' - T'_\infty)\cos\phi + \upsilon\frac{\partial^2 u}{\partial y^2} \tag{2}$$

Energy equation

$$\frac{\partial T'}{\partial t'} + u\frac{\partial T'}{\partial x} + v\frac{\partial T'}{\partial y} = \alpha\frac{\partial^2 T'}{\partial y^2} \tag{3}$$

The initial and boundary conditions are

$$\begin{aligned}
& t' \le 0: u = 0,\ v = 0,\ T' = T'_\infty \quad \text{for all } x \text{ and } y, \\
& t' > 0: u = 0,\ v = 0,\ \frac{\partial T'}{\partial y} = \frac{-q_w(x)}{k} \quad \text{at } y = 0, \\
& \qquad u = 0,\ T' = T'_\infty \quad \text{at } x = 0, \\
& \qquad u \to 0,\ T' \to T'_\infty \quad \text{as } y \to \infty.
\end{aligned} \tag{4}$$

where u and v are the velocity components along $x-$ and $y-$ axes, T' is the fluid temperature and other physical quantities are mentioned in the Nomenclature.

Further, we introduce the following non-dimensional variables:

$$\begin{aligned}
& X = \frac{x}{L},\ Y = \frac{y}{L}\ Gr_L^{1/5},\ t = \left(\frac{\upsilon}{L^2}\ Gr_L^{2/5}\right)t', \\
& R = \frac{r}{L},\ U = \left(\frac{L}{\upsilon}\ Gr_L^{-2/5}\right)u,\ V = \left(\frac{L}{\upsilon}\ Gr_L^{-1/5}\right)v, \\
& T = \frac{(T' - T'_\infty)}{L\left[q_w(L)/k\right]}\ Gr_L^{1/5},
\end{aligned} \tag{5}$$

Where $Gr_L = g\beta\left[q_w(L)\right]L^4/\upsilon^2 k$ is the Grashof number based on L, $\Pr = \upsilon/\alpha$ is the Prandtl number and $r = x\sin\phi$. Equations. (1), (2) and (3) can then be written in the following non-dimensional form:

$$\frac{\partial}{\partial X}(RU) + \frac{\partial}{\partial Y}(RV) = 0 \tag{6}$$

$$\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = T\cos\phi + \frac{\partial^2 U}{\partial Y^2} \tag{7}$$

$$\frac{\partial T}{\partial t} + U\frac{\partial T}{\partial X} + V\frac{\partial T}{\partial Y} = \frac{1}{\Pr}\frac{\partial^2 T}{\partial Y^2} \tag{8}$$

The corresponding non-dimensional initial and boundary conditions are

$$\begin{aligned}
& t \le 0: \quad U = 0,\quad V = 0,\quad T = 0 \quad \text{for all } X \text{ and } Y, \\
& t > 0: \quad U = 0,\quad V = 0,\quad \frac{\partial T}{\partial Y} = -X^m \quad \text{at} \quad Y = 0, \\
& \qquad U = 0,\ T = 0 \quad \text{at} \quad X = 0, \\
& \qquad U \to 0,\ T \to 0 \quad \text{as} \quad Y \to \infty.
\end{aligned} \tag{9}$$

3. SOLUTION PROCEDURE

The governing partial differential Eqs. (6)–(8) are coupled and non-linear with initial and derivative boundary conditions, Eq. (9). They are solved numerically by an implicit finite-difference method of Crank-Nicolson type which is discussed by many authors, namely, Soundalgekar and Ganesan (1981), Ganesan and Rani (1999), Muthucumaraswamy and Ganesan (1999, 2000), Ganesan and Palani (2004). Recently the heat transfer problem which deals with, unsteady free convection flow past a vertical cone are solved numerically by an implicit finite-difference method of Crank-Nicolson type as described in detail by Bapuji *et al.* (2007, 2008). The finite difference scheme of dimensionless governing equations is reduced to tri-diagonal system of equations and is solved by Thomas algorithm as discussed in Carnahan *et al*. (1969).

The region of integration is considered as a rectangle with $X_{\max}(=1)$ and $Y_{\max}(=26)$ where $Y_{\max}$ corresponds to $Y \to \infty$ which lies very well out side both the momentum and thermal boundary layers. The maximum of Y was chosen as 26, after some preliminary investigation so that the last two boundary conditions of (9) are satisfied within the tolerance limit of 10^{-5}. The mesh sizes have been fixed as

$\Delta X = 0.05, \Delta Y = 0.05$ with time step $\Delta t = 0.01$. The computations are carried out first by reducing the spatial mesh sizes by 50 % in one direction, and later in both directions by 50 %. The results are compared. It is observed in all cases, that the results differ only in the fifth decimal place. Hence, the choice of the mesh sizes seems to be appropriate. The scheme is unconditionally stable and is described by Bapuji *et al.* (2008). The local truncation error is $O\left(\Delta t^2 + \Delta Y^2 + \Delta X\right)$ and it tends to zero as $\Delta t, \Delta Y$ and ΔX tend to zero. Hence, the scheme is compatible. Stability and compatibility ensure the convergence.

4. Results And Discussion

In order to prove the accuracy of our numerical results, the present results in steady state at $X = 1.0$ is obtained and considering the modified Grashof number $Gr_L^* = Gr_L \cos\phi$, (i.e. the numerical solutions obtained from the Eqs. (6)-(8) are independent of semi vertical angle of the cone ϕ) are compared with available similarity solutions in literature. The velocity and temperature profiles of the cone for $\Pr = 0.72$ are displayed in Fig. 2 and the numerical values of local skin-friction τ_X, temperature T, for different values of Prandtl number are shown in Table 1 and that are compared with similarity solutions of Lin (1976) in steady state using suitable transformation [i.e. $Y = \left(20/9\right)^{1/5}\eta, T = \left(20/9\right)^{1/5}\left[-\theta(0)\right]$, $U = \left(20/9\right)^{3/5} f'(\eta), \tau_X = \left(20/9\right)^{2/5} f''(0)$]. In addition, the local skin-friction τ_X and the local Nusselt number Nu_X for different values of Prandtl number, when heat flux gradient $m = 0.5$ at $X = 1.0$ in steady state, are compared with the non-similarity results of Hossain and Paul (2001) in Table. 2. It is observed that the results are in good agreement with each other. It is also noticed that the present results agree well with those of Pop and Watanabe (1992), Na and Chiou (1979) (as pointed out in Table. 1).

Figures 3 through 8 shows the transient velocity and temperature profiles at $X = 1.0$, with various parameters $\Pr$, m and ϕ. The value of t with star (*) symbol denotes the time taken to reach steady state. In Fig. 3, transient velocity profiles are plotted for various values of ϕ, with $\Pr = 0.71$ and $m = 0.25$. When ϕ increases, near the cone apex, it leads to decrease the impulsive force along the cone surface. Hence, the difference between temporal maximum velocity values and steady state values decreases with increasing the values of semi vertical angle of the cone ϕ. The tangential component of buoyancy force reduces as the semi vertical angle increases. This causes the velocity to reduce as angle ϕ increases. The momentum boundary layer becomes thick, and the time taken to reach steady state increases for increasing ϕ.

Figure 4 shows the transient temperature profiles for different values of ϕ with $\Pr = 0.71$ and $m = 0.25$. It is observed that the temperature value and thermal boundary layer thickness increase on increasing ϕ. The difference between temporal maximum temperature values and steady state values decrease on increasing ϕ. In Figs. 5 and 6, transient velocity and temperature profiles are plotted for various values of $\Pr$ with $\phi = 15^0$ and $m = 0.25$. It may be noted that Prandtl number $\Pr$ increases when viscosity increases and thermal diffusivity decreases. This causes a reduction in the velocity and temperature as expected. It is observed from the figures that the difference between temporal maximum values and steady state values are reduced when $\Pr$ increases. It is also noticed that the time taken to reach steady state increases and thermal boundary layer thickness reduces with increasing $\Pr$.

In Figs. 7 and 8 transient velocity and temperature profiles are plotted for various values of m with $\phi = 15^0$ and $\Pr = 0.71$. Impulsive forces are reduced along the surface of the cone near the apex for increasing values of m (i.e. the gradient of heat flux along the cone near the apex reduces with the increasing values of m). Due to this, the difference between temporal maximum values and steady state values reduces with increasing m. It is also observed that the velocity as well as temperature reduces on increasing m and takes more time to reach steady state.

Once velocity and temperature profiles are known, it is interesting to study the local as well as the average skin-friction, and the rate of heat transfer in steady state and transient levels.

The local non-dimensional skin friction τ_X and the local Nusselt number Nu_X are given by

$$\tau_X = Gr_L^{3/5}\left(\frac{\partial U}{\partial Y}\right)_{Y=0} \quad \& \qquad Nu_X = \frac{X\,Gr_L^{1/5}}{T_{Y=0}}\left(-\frac{\partial T}{\partial Y}\right)_{Y=0} \tag{10}$$

Also, the non-dimensional average skin-friction $\bar{\tau}$ and the average Nusselt number $\overline{Nu}$ can be written as

$$\bar{\tau} = 2Gr_L^{3/5}\int_0^1 X\left(\frac{\partial U}{\partial Y}\right)_{Y=0} dX \quad \& \qquad \overline{Nu} = 2Gr_L^{1/5}\int_0^1 \frac{X}{(T)_{Y=0}}\left(-\frac{\partial T}{\partial Y}\right)_{Y=0} dX \tag{11}$$

The derivatives involved in Eqs. (10) and (11) are obtained using five-point approximation formula and then the integrals are evaluated using Newton-Cotes closed integration formula. The variations of local skin-friction τ_X and local Nusselt number Nu_X for different values of ϕ, at various positions on the surface of the cone ($X = 0.25$ and 1.0) in the transient period are shown in Figs. 9 and 10 respectively.

It is observed from the Fig. 9 that local skin-friction τ_X decreases with increasing ϕ, due to the fact that velocity decreases with increasing angle ϕ as shown in Fig. 3 and the influence of ϕ on skin friction τ_X increases as ϕ increases in the transient period along the surface of the cone compare to nearer apex. Figure 10 reveals that local Nusselt number Nu_X values decrease with increasing angle ϕ as temperature distribution increases with ϕ which is shown in Fig. 4. It is observed that this effect is less near the cone apex. The variation of the local skin-friction τ_X and the local Nusselt number Nu_X in the transient regime is displayed in Figs. 11 and 12 for different values of $\Pr$ and at various positions on the surface of the cone ($X = 0.25$ and 1.0). The local wall shear stress decreases as $\Pr$ increases because velocity decreases with an increasing value of $\Pr$ as shown in Fig. 5. Local Nusselt number Nu_X increases with increasing $\Pr$ and it is clear from the Fig. 12, that decreasing rate of Nu_X increases when the distance increases from the cone vertex along the surface of the cone.

The variation of the local skin-friction τ_X and the local Nusselt number Nu_X in the transient period at various positions on the surface of the cone ($X = 0.25$ and 1.0) and for different values of m, are shown in Figs. 13 and 14. It is observed from Fig. 13 that the local skin-friction decreases with increasing m and the effect of m over the local skin-friction τ_X is more near the apex of the cone and reduces gradually with increasing the distance along the surface of the cone from the apex. From Fig. 14, it is noticed that near the apex, local Nusselt number Nu_X reduces with increasing m, but that trend is slowly changed and reversed as distance increases along the surface from apex.

The influence of ϕ, Pr and m on average skin-friction $\bar{\tau}$ in transient period is shown in Fig. 15 and it is more for smaller values of angles ϕ, m and lower values of $\Pr$. Fig. 16 displays the influence on average Nusselt number $\overline{Nu}$ in transient period for various values of $\Pr$, ϕ and m. It is clear that $\overline{Nu}$ is more for smaller values of ϕ and larger values of $\Pr$. Finally, noticed from Fig. 16, there is no significant influence of m over the average Nusselt number.

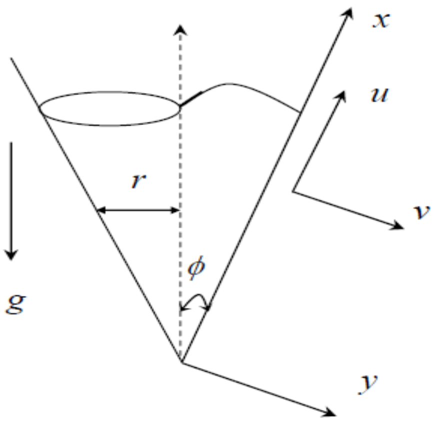

Fig. 1. Physical model and coordinate system

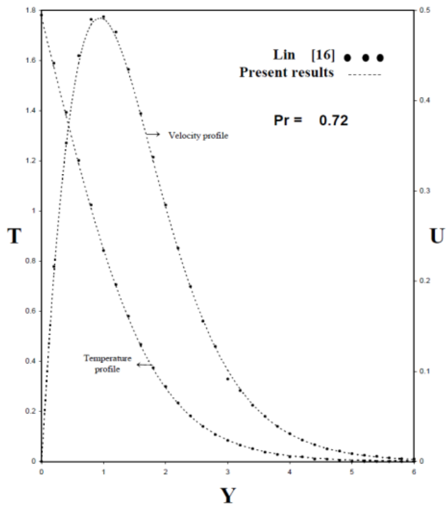

Fig. 2. Comparison of steady state temperature and velocity profiles at X=1.0

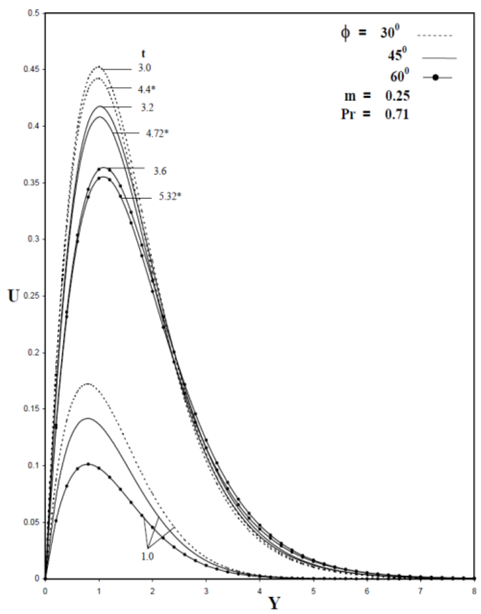

Fig. 3. Transient velocity profiles at X=1.0 for different values of ϕ

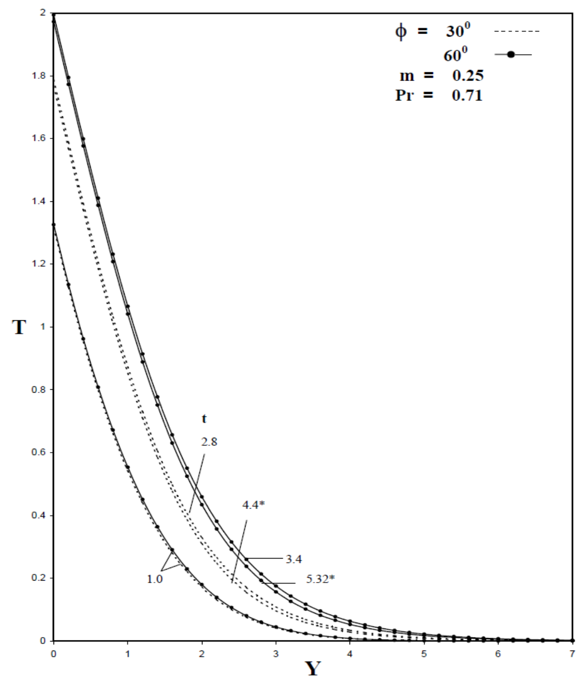

Fig. 4. Transient temperature profiles at X=1.0 for different values of ϕ

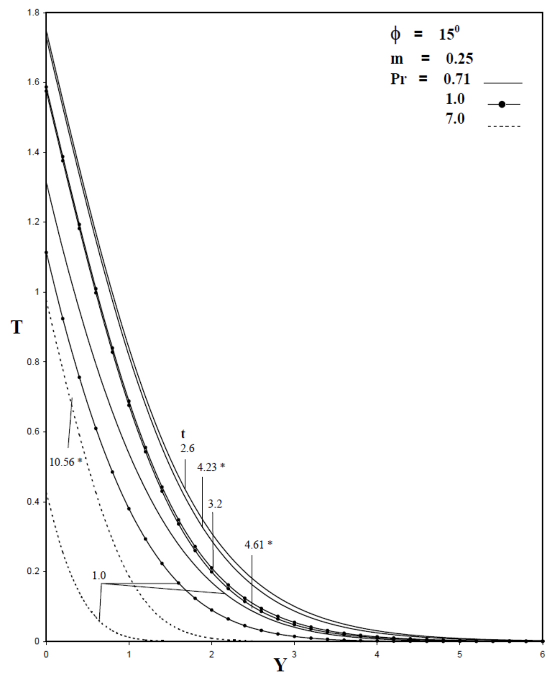

Fig. 6. Transient temperature profiles at X=1.0 for different values of Pr

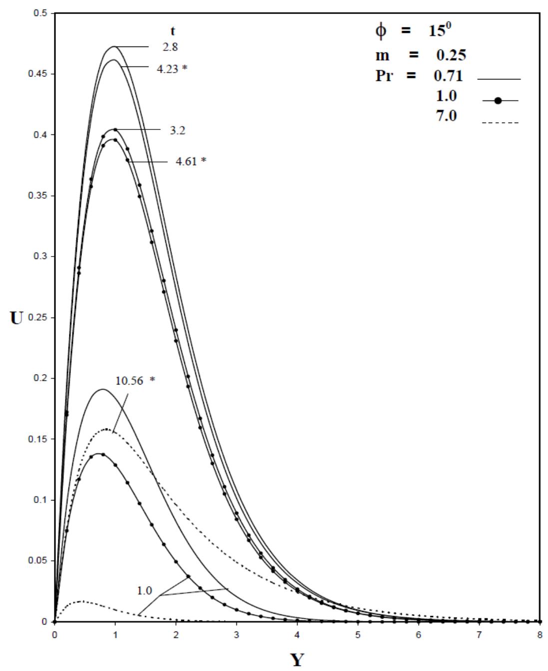

Fig. 5. Transient velocity profiles at X=1.0 for different values of Pr

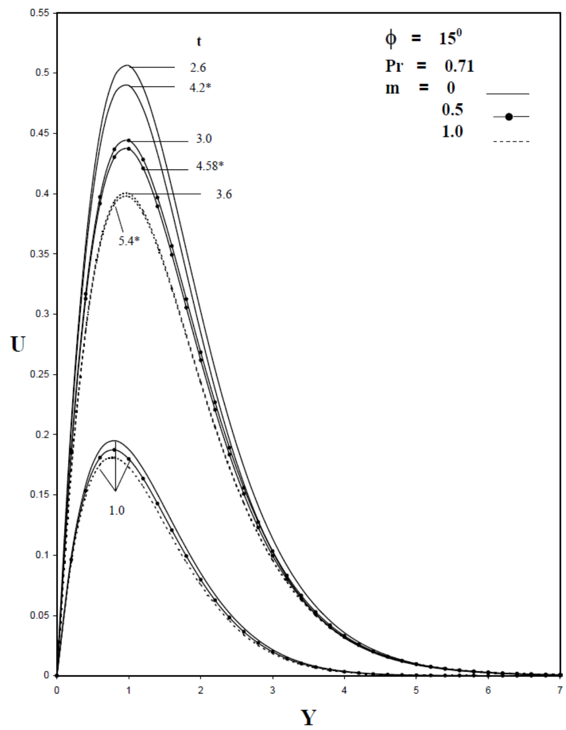

Fig. 7. Transient velocity profiles at X=1.0 for different values of m

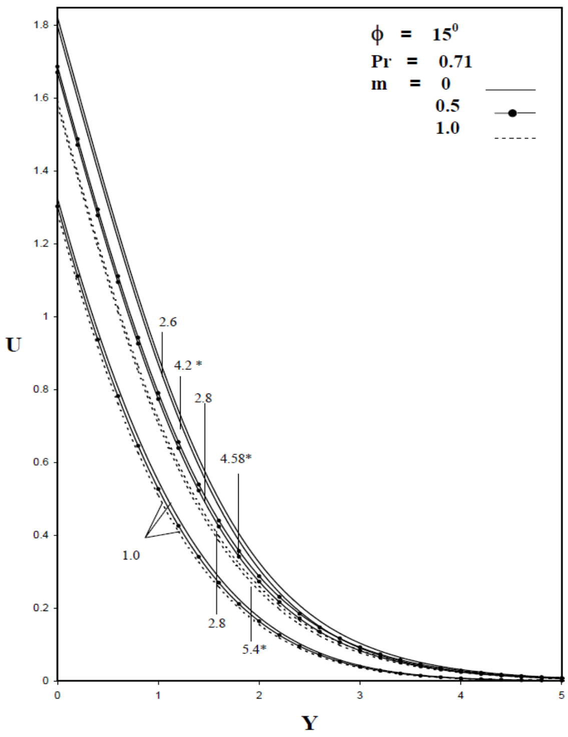

Fig. 8. Transient temperature profiles at X=1.0 for different values of m

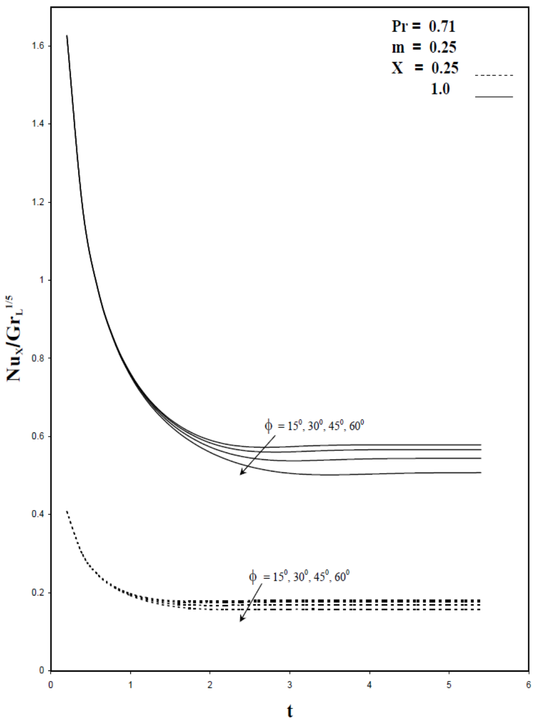

Fig. 10. Local Nusselt number at X=0.25 and 1.0 for different values of ϕ in transient state

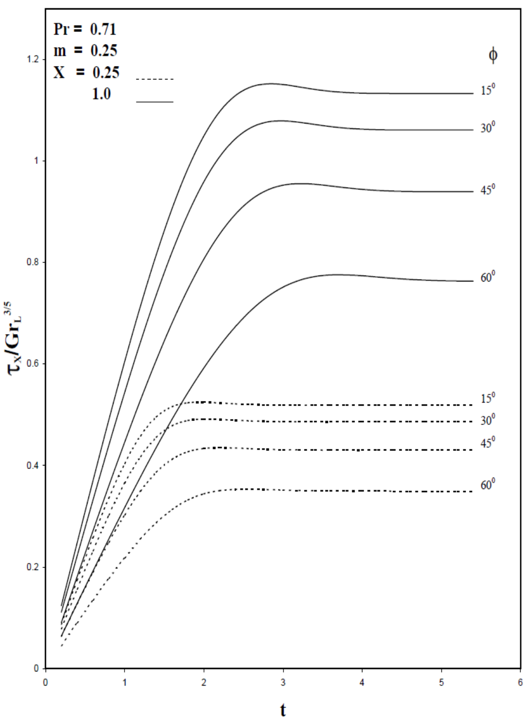

Fig. 9. Local skin Fraction at X=0.25 for different values of ϕ in transient state

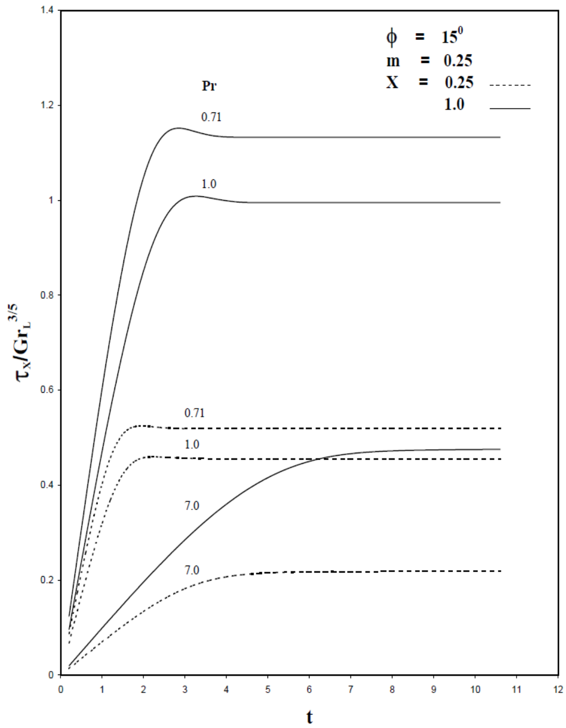

Fig. 11. Local skin fraction at X=0.25 and 1.0 for different values of Pr in transient state

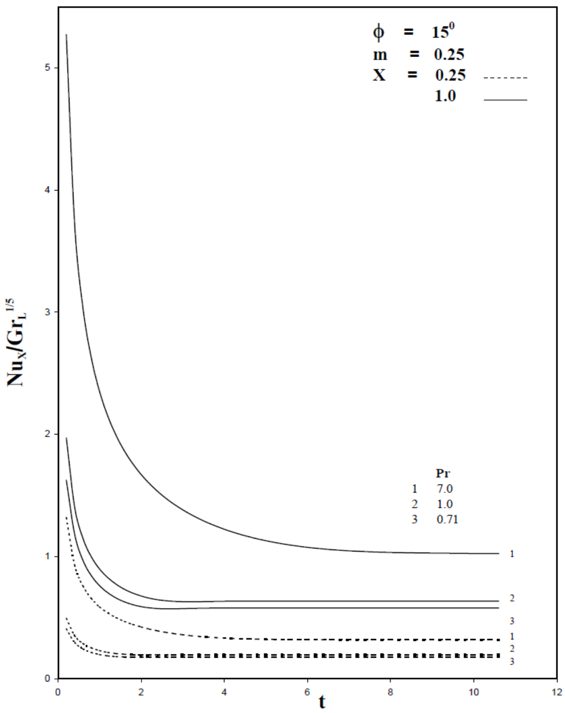

Fig. 12. Local Nusselt number at X=0.25 and 1.0 for different values of Pr in transient state

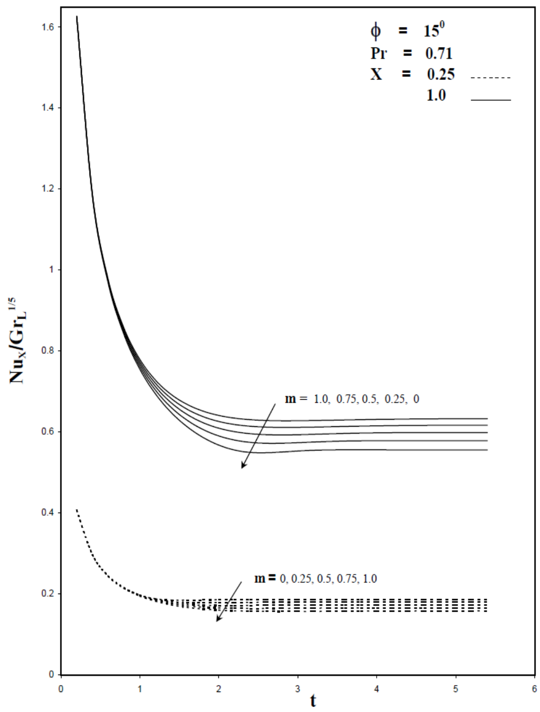

Fig. 14. Local Nusselt number at X=0.25 and 1.0 for different values of m in transient state

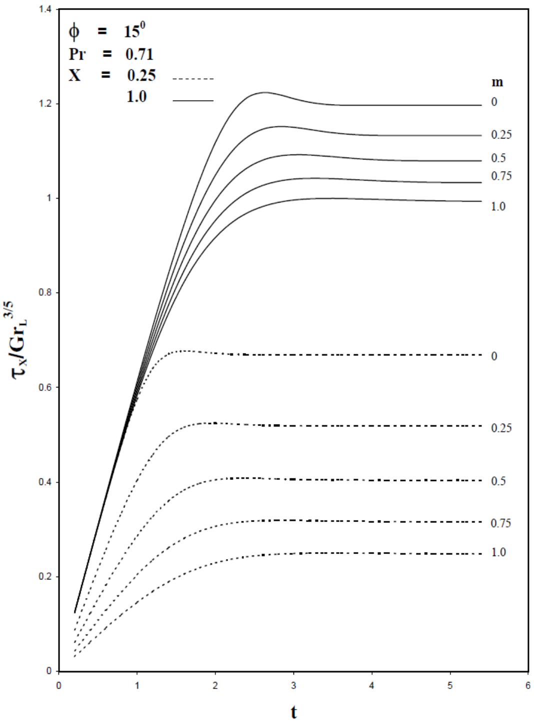

Fig. 13. Local skin fraction at X=0.25 and 1.0 for different values of m in transient state

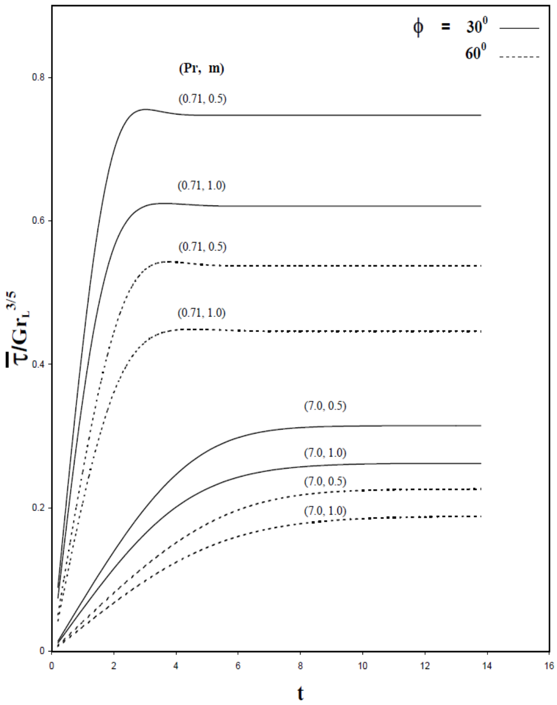

Fig. 15. Average skin friction for different values of m, ϕ and Pr in transient state

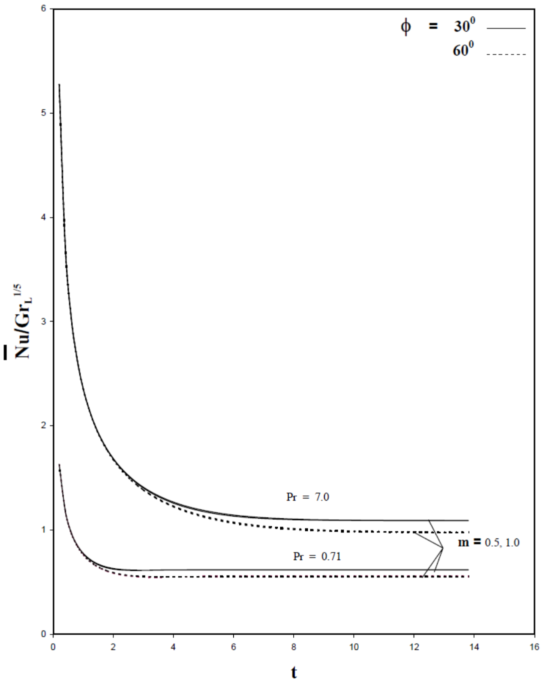

Fig. 16. Average Nusselt number for different values of m, ϕ and Pr in transient state

5. CONCLUSION

A numerical study has been carried out for the unsteady laminar free convection from a vertical cone with non-uniform surface heat flux. The dimensionless governing boundary layer equations are solved by an implicit finite-difference method of Crank-Nicolson type. Present results are compared with available results in literature and are found to be in good agreement. The following conclusions are made:

1. The time taken to reach steady state increases with increasing Pr, ϕ or m.
2. The velocity reduces when the parameters ϕ, Pr or m are increased.
3. Temperature increases with increasing ϕ and decreasing Pr, m values.
4. Momentum boundary layers become thick when ϕ is increased.
5. Thermal boundary layer becomes thin when ϕ is reduced or Pr is increased.
6. The difference between temporal maximum values and steady state values (for both velocity and temperature) become less when Pr, ϕ or m increases.
7. The influence of ϕ over the local skin friction τ_X and local Nusselt number Nu_X are less near the vertex of the cone and then increases slowly with increasing distance from the vertex.
8. Local and average skin-frictions increase when the value of ϕ, Pr or m is reduced.
9. Local and average Nusselt numbers reduce with increasing ϕ or decreasing Pr.
10. In transient period, the local Nusselt number reduces with increasing m near the apex but that trend is changed and reversed as the distance increases from it.
11. The effect of m on average Nusselt number $\overline{Nu}$ is almost negligible.

Table 1 Comparison of steady state local skin-friction and temperature values at X=1.0 with those of Lin [16].

Pr	Temperature			Local skin friction		
	Lin(1976) results		Present results	Lin (1976) results		Present results
	$-\theta(0)$	$-\left(\frac{20}{9}\right)^{1/5}\theta(0)$	T	$f''(0)$	$\left(\frac{20}{9}\right)^{2/5} f''(0)$	τ_X
0.72	1.52278 1.52278 [a]	1.7864	1.7796	0.88930 0.88930 [a]	1.224	1.2154
1	1.39174	1.6327 1.6329 [aa]	1.6263	0.78446	1.0797	1.0721
2	1.16209	1.3633	1.3578	0.60252	0.8293	0.8235
4	0.98095	1.1508	1.1463	0.46307	0.6373	0.6328
6	0.89195	1.0464	1.0421	0.39688	0.5462	0.5423
8	0.83497	0.9796	0.9754	0.35563	0.4895	0..4859
10	0.79388	0.9314	0.9272	0.32655	0.4494	0.4460
100	0.48372	0.5675	0.5604	0.13371	0.184	0.1813

[a] Values taken from Pop and Watanabe (1992) when suction/injection is zero.

[aa] Values taken from Na and Chiou (1979a) when solutions for flow over a full cone.

Table 2 Comparison of steady state local skin-friction and local Nusselt number values at X=1.0 with those of Hossain and Paul (2001) for different values of Pr when $m=0.5$ and suction is zero.

Pr	Local skin-friction		Local Nusselt number	
	Hossain(2001) results $F_0^{''}(0)$	Present results $\tau_X / \left(Gr_L^*\right)^{3/5}$	Hossain (2001) results $1/\Phi_0(0)$	Present results $Nu_X / \left(Gr_L^*\right)^{1/5}$
0.01	5.13457	5.1155	0.14633	0.1458
0.05	2.93993	2.9297	0.26212	0.2630
0.1	2.29051	2.2838	0.33174	0.3324

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