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Non-similar solution for unsteady water boundary layer flows over a sphere with non-uniform mass transfer

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Abstract

Purpose – The purpose of this paper is to make an analysis to study the non-similar solution for unsteady water boundary layer flow over sphere with the influence of temperature-dependent viscosity, Prandtl number, non-uniform surface mass transfer and heat transfer.

Design/methodology/approach – The governing quasi-linear partial differential equations have been solved numerically using an implicit finite difference scheme along with a quasi-linearization technique. Non-similar solutions have been obtained from the starting point of the stream-wise coordinate to the point where the skin friction value vanishes.

Findings – It is observed that non-uniform suction causes the point of vanishing skin friction to move downstream. The slot injection causes the vanishing skin friction to move upstream.

Originality/value – The effect of unsteadiness is more significant on the skin friction as compared to the heat transfer.

Keywords Unsteady flow, Sphere, Boundary layer, Variable properties, Non-similar solution, Forced convection flow, Heat and mass transfer, Control surfaces, Flow, Heat transfer

Paper type Research paper

Nomenclature

A	= dimensionless mass transfer parameter	f	= dimensionless stream function
C_f	= skin friction coefficient in the x-direction	F	= dimensionless velocity component in the x-direction
c_p	= specific heat at constant pressure $(\mathrm{kJ\cdot kg^{-1}\cdot K^{-1}})$	G	= dimensionless temperature
Ec	= Eckert number (viscous dissipation parameter)	k	= thermal conductivity $(\mathrm{W\cdot m^{-1}\cdot K^{-1}})$
		L	= characteristic length (m)
		N	= (μ/μ_∞) viscosity ratio

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Nu = Nusselt number
Pr = Prandtl number
$r(x)$ = radius of the section normal to the axis of the sphere (m)
Re_L = Reynolds number
t = dimensional time (s)
t^* = dimensionless time
T = Temperature (K)
u, v = dimensional velocity components in x- and y-directions, respectively, $(m \cdot s^{-1})$
x, y = dimensional meridional and normal distances, respectively, (m)
$\bar{x}$ = dimensionless meridional distance
$\bar{x}_0, \bar{x}_0^*$ = leading and trailing edge of slot, respectively

Greek symbols
α_1 = dimensionless parameters
β = pressure gradient
$\Delta\eta, \Delta\bar{x}, \Delta t^*$ = step sizes in $\bar{x}$- and t^*-directions, respectively
η, ξ = transformed coordinates
ϵ = constant used in the continuous function of time
μ = dynamic viscosity $(kg \cdot m^{-1} \cdot s^{-1})$
ρ = density $(kg \cdot m^{-3})$
$\phi(t^*)$ = continuous function of time
ψ = dimensional stream function $(m^2 s^{-1})$
ω^* = slot length parameter

Subscripts
∞ = conditions in the free stream
e, w = conditions at the edge of the boundary layer and on the surface, respectively
$\bar{x}, \xi, t^*$ = partial derivatives with respect to these variables

1. Introduction

Unsteady as well as non-similar boundary layer flows with temperature-dependent properties have become important in recent years in different categories of fluid mechanics and area of convective heat and mass transfer. In unsteady flow problems, the inclusion of time (independent variable) creates complexity in the solution procedure. Most of the investigators cramped their studies either to steady non-similar flows or to unsteady semi-similar or self-similar flows as a consequence of the mathematical difficulties involved in achieving non-similar solutions. A review of non-similarity solution methods for steady flows along with citations of relevant publications until 1967 is given by Deway and Gross (1967). Since then, several investigators have analyzed about, steady incompressible laminar non-similar flow over two-dimensional and axi-symmetric bodies both approximate and exact methods such as local non-similarity method (Sparrow *et al.*, 1970; Sparrow and Yu, 1971), asymptotic method (Kao and Elrod, 1974, 1976), difference-differential method (Jaffe and Smith, 1971) and the finite difference method (Terrill, 1960; Venkatachala and Nath, 1980). On the other hand, the unsteady incompressible non-similar flows have been discussed by a number of researchers (Watkins, 1976; Cebeci, 1977; Surma Devi and Nath, 1983; Hancock, 1984). Excellent reviews of unsteady flows have been given in Riley (1975), McCorosky (1977) and Telionis (1979, 1981).

The thermo physical properties of fluid often vary significantly with respect to the temperature in the circumstances where large or moderate temperature gradients exists across the fluid medium. If the temperature of the laminar flow of a liquid increases there will be shear stresses due to molecular interchange and also there are substantial attractive and cohesive forces between the molecules of liquid. Both molecular interchange and cohesion contribute to viscous shear stress in liquids, causing reduction in the viscosity across the momentum boundary-layer leading to a local increase in the transport phenomena. Also, the process has impact on the thermal conductivity across the thermal boundary-layer which, in turn, affects the heat transfer rate at the wall. The effect of variable viscosity and Prandtl number for steady non-similar axi-symmetric water boundary layer flows over rotating sphere have been discussed in Saikrishnan and Roy (2002) and for unsteady non-similar axi-symmetric water boundary layer flows have been done in Eswara and Nath (1994). Recently different studies have been reported with variable viscosity and Prandtl number (Adekojo Waheed, 2006; Chamkha *et al.*, 2011).

Mass transfer from a wall slot into the boundary layer is of interest for various prospective applications together with thermal protections, energizing the inner portion of boundary layer in adverse pressure gradient and skin friction reduction on control surfaces. The effect of slot injection (suction) into laminar compressible boundary layer over a flat plate by taking the interaction between the boundary layer and oncoming stream have been studied in Smith and Stewartson (1973), Napolitano and Messick (1980) and Riley (1981). Despite of uniform mass transfer, finite discontinuities arise at the leading and trailing edges of the slot and those can be evaded by choosing a non-uniform mass transfer distribution along a stream-wise slot and it has been discussed in Minkowycz *et al.* (1988). More recently, different studies have reported the influence of non-uniform mass transfer on steady boundary layer flows over a sphere and a cylinder (Saikrishnan and Roy, 2003).

The extensive literature on non-similar solution and mass transfer technique shows that the studies confined only for steady water boundary layer flows with non-uniform mass transfer. The outcome of the literature leads to study the simultaneous effects of non-uniform slot suction (injection) and temperature dependent fluid properties on unsteady water boundary layer flow over a sphere. The non-similar solutions have been obtained, starting from the origin of the streamwise co-ordinate to the point where the skin friction value vanishes, to discuss the simultaneous effects on skin friction and heat transfer parameters. These studies are useful for an engineer or a designer in determining surface heat requirements for stabilizing the laminar water boundary layer flow over a sphere. The problem has plentiful applications in engineering, astrophysics and meteorology.

2. Analysis

Consider unsteady, incompressible laminar non-similar water boundary-layer flow with temperature-dependent viscosity and Prandtl number over a sphere (Figure 1).

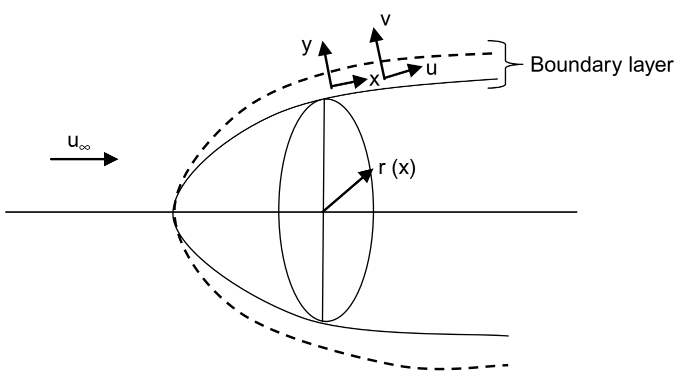


Figure 1.
Flow model and co-ordinate system

Let the stream velocity and mass transfer (injection/suction) vary with the axial distance (x) along the surface and with the time (t). The following assumptions are made in present analysis:

- The temperature difference between the wall and the free stream is small ($0°C < T < 40°C$).
- In spite of the variation of the both density (ρ) and the specific heat (C_p) with temperature by less than 1 per cent in the above mentioned temperature range, they are taken as constants (Saikrishnan and Roy, 2003).
- It is considered that the injected fluid posses the same physical properties as the boundary-layer fluid.
- The blowing rate of the fluid is assumed to be small and it does not affect the inviscid flow at the edge of the boundary layer.
- The viscosity and the Prandtl number are assumed to vary as an inverse function of temperature and is given below (Yao, 1978; Ling and Dybbs, 1987; Pop *et al.*, 1992):

$$\mu = \frac{1}{(b_1 + b_2 T)} \quad \text{and} \quad Pr = \frac{1}{c_1 + c_2 T} \tag{1}$$

where:

$$b_1 = 53.41, \quad b_2 = 2.43, \quad c_1 = 0.068 \ \text{and} \ c_2 = 0.004 \tag{2}$$

The numerical data, utilized for these correlations, are taken from Lide (1990) (Table I).

The unsteady boundary-layer equations in non-dimensional form become (Eswara and Nath, 1994):

$$(NF')' + \phi[fF' + \beta(1 - F^2)] - P[F_{t^*} - \phi^{-1}\phi_{t^*}(1 - F)] = 2\xi\phi(FF_\xi - f_\xi F') \tag{3}$$

$$(N\,Pr^{-1}G')' + \phi fG' + NEc(u_e/u_\infty)^2(F')^2 - PG_{t^*} = 2\xi\phi(FG_\xi - f_\xi G') \tag{4}$$

Temperature (T) (°C)	Density (ρ) (g/cm^3)	Specific heat (c_p) (J·10^7/kg K)	Thermal conductivity (k) (erg·10^5/cm s K)	Viscosity (μ) (g·10^{-2}/cm s)	Prandtl number (Pr)
0	1.00228	4.2176	0.5610	1.7930	13.48
10	0.99970	4.1921	0.5800	1.3070	9.45
20	0.99821	4.1818	0.5984	1.0060	7.03
30	0.99565	4.1784	0.6154	0.7977	5.12
40	0.99222	4.1785	0.6305	0.6532	4.32
50	0.98803	4.1806	0.6435	0.5470	3.55

Source: Lide (1990)

Table I. Values of thermo-physical properties of water at different temperature with boundary conditions:

$$\left.\begin{aligned} F(\bar{x},0,t^*)=0 \quad G(\bar{x},0,t^*)=1 \\ F(\bar{x},\infty,t^*)=1 \quad G(\bar{x},\infty,t^*)=0 \end{aligned}\right\} \tag{5}$$

where:

$$N=\frac{\mu}{\mu_\infty}=\frac{(b_1+b_2T_\infty)}{(b_1+b_2T)}=\frac{1}{(1+a_1G)}$$

$$Pr=\frac{1}{(c_1+c_2T)}=\frac{1}{(a_2+a_3G)}$$

$$a_1=[b_2/(b_1+b_2T_\infty)]\Delta T_w, \quad a_2=c_1+c_2T_\infty$$

$$a_3=c_2\Delta T_w, \quad \Delta T_w=T_w-T_\infty$$

$$\xi=\int_0^x (U/u_\infty)(r/L)^2 d(x/L), \quad t^*=(3/2)Re_L(\mu_e/\rho L^2)t,$$

$$\eta=(U/u_\infty)(Re_L/2\xi)^{1/2}(r/L)(y/L), \quad Re_L=u_\infty\frac{L}{\nu'}$$

$$\psi(x,y,t)=u_\infty L(2\xi/Re_L)^{1/2}\phi(t^*)f(\xi,\eta,t^*)$$

where:

$$\phi(t^*)=1+\epsilon t^{*2}, \quad \epsilon=0.25$$

$$u=(L/r)\psi_y, \quad v=-(L/r)\psi_x,$$

$$G=(T-T_\infty)/(T_w-T_\infty)$$

$$\frac{u}{u_e}=f'=F, \quad u_e=U\phi(t^*)$$

$$v=-(r/L)(2\xi Re_L)^{-1/2}U\phi[f+2\xi f_\xi+(\beta+\alpha_1-1)\eta F]$$

$$\beta(\xi)=(2\xi/U)(dU/d\xi)$$

$$P=3\xi(L/r)^2(u_\infty/U)^2$$

$$\alpha_1=(2\xi/r)(dr/d\xi)$$

$$Ec=u_\infty^2/[c_p(T_w-T_\infty)], \quad f=\int_0^\eta F d\eta+f_w$$

$$f_w=-(\xi)^{-1/2}(Re_L/2)^{1/2}\phi^{-1}\int_0^x (v_w/u_\infty)(r/L)d(x/L) \tag{6}$$

Here f_w is the surface mass transfer. For a sphere, the unsteadiness as well as non-similarity are both due to the external velocity at the edge of the boundary layer, $u_e(\bar{x},t^*)$ (where $\bar{x}$ is the dimensionless distance along the surface) and the normal component of the velocity at the surface, $f_w(\bar{x},t)$. The free-stream velocity distribution for the case of axi-symmetric flow over a sphere and the distance from the axis of the body are given by:

$$u_e/u_\infty = (3/2)\sin\bar{x}\phi(t^*), \quad U/u_\infty = (3/2)\sin\bar{x}$$
$$\bar{x} = x/L, \quad r/L = \sin\bar{x} \tag{7}$$

The expressions for $\xi, \beta, f_w, P, \alpha_1, C_f$ and Nu can be written, respectively, as:

$$\xi = [(1-\cos\bar{x})^2(2+\cos\bar{x})]/2,$$
$$\beta = (2/3)[\cos\bar{x}(2+\cos\bar{x})/(1+\cos\bar{x})^2],$$
$$P = (2/3)(2+\cos\bar{x})(1+\cos\bar{x})^{-2}, \quad \alpha_1 = \beta$$

Here, α_1 and P are the dimensionless parameters. The function $\phi(t^*)$ is an arbitrary function of time with a continuous first derivative for $t^* \geq 0$, representing the unsteadiness in the free stream:

$$f_w = \begin{cases} 0, & \bar{x} \leq \bar{x}_0 \\ A\phi^{-1}Q_1^{-1}Q_3^{-1/2}C(\bar{x}, \bar{x}_0), & \bar{x}_0 \leq \bar{x} \leq \bar{x}_0^*, \\ A\phi^{-1}Q_1^{-1}Q_3^{-1/2}C(\bar{x}_0^*, \bar{x}_0), & \bar{x} \geq \bar{x}_0^*, \end{cases} \tag{8}$$

where the function:

$$C(\bar{x}, \bar{x}_0) = \frac{\sin\{(\omega^*-1)\bar{x} - \omega^*\bar{x}_0\} + \sin\bar{x}_0}{(\omega^*-1)} - \frac{\sin\{(\omega^*+1)\bar{x} - \omega^*\bar{x}_0\} - \sin\bar{x}_0}{(\omega^*+1)}$$

and:

$$Q_1 = 1-\cos\bar{x}, \quad Q_2 = 1+\cos\bar{x}, \quad Q_3 = 2+\cos\bar{x} \tag{9}$$

Here v_w is taken as:

$$v_w = \begin{cases} -u_\infty\left(\frac{Re_L}{2}\right)^{-1/2} 2^{1/2} A\sin\{\omega^*(\bar{x}-\bar{x}_0)\}, & \bar{x}_0 \leq \bar{x} \leq \bar{x}_0^* \\ 0, & \bar{x} \leq \bar{x}_0 \ \textit{and} \ \bar{x} \geq \bar{x}_0^* \end{cases}$$

where ω^* and $\bar{x}_0$ are the free parameters which determine the slot length and slot locations, respectively. The function $v_w(\bar{x})$ is a continuous function for all values of $\bar{x}$ representing velocity at wall in η-direction and it has non-zero values only in the interval $\left[\bar{x}_0, \bar{x}_0^*\right]$ and zero value at all points outside interval $\left[\bar{x}_0, \bar{x}_0^*\right]$ (i.e. only the part of surface of sphere $\left[\bar{x}_0, \bar{x}_0^*\right]$ used to transfer mass and all other parts of surface are to be treated as solid wall). The reason for taking such a function is that it allows the mass transfer to change slowly in the neighbourhood of the leading and the trailing edge of the slot. The parameter $A > 0$ or $A < 0$ according to whether there is a suction or an injection. It is convenient to express equations (3) and (4) in terms of $\bar{x}$ instead of ξ. Equation (7) gives the relation between ξ and $\bar{x}$ as:

$$\xi \frac{\partial}{\partial \xi} = B(\bar{x}) \frac{\partial}{\partial \bar{x}} \quad (10)$$

where:

$$B(\bar{x}) = 3^{-1} \tan\left(\frac{\bar{x}}{2}\right) Q_3 Q_2^{-1}.$$

Substituting equation (10) into equations (3) and (4), we obtain:

$$(NF')' + \phi[fF' + b(\bar{x})(1 - F^2)] - P[F_{t^*} - \phi^{-1}\phi_{t^*}(1 - F)] = 2\phi B(\bar{x})(FF_{\bar{x}} - f_{\bar{x}}F') \quad (11)$$

$$N(Pr^{-1}G')' + \phi f G' + N\,Ec(u_e/u_\infty)^2(F')^2 - PG_{t^*} = 2\phi B(\bar{x})(FG_{\bar{x}} - f_{\bar{x}}G') \quad (12)$$

The boundary conditions become:

$$F(\bar{x}, 0, t^*) = 0 \quad G(\bar{x}, 0, t^*) = 1 \quad (13)$$
$$F(\bar{x}, \infty, t^*) = 1 \quad G(\bar{x}, \infty, t^*) = 0$$

The skin-friction coefficient at the wall can be expressed in the form:

$$C_f(Re_L)^{1/2} = (9/2)\sin\bar{x}(1 + \cos\bar{x})(2 + \cos\bar{x})^{-1/2}\phi(t^*)N_w F'_w \quad (14)$$

where:

$$C_f = \frac{2[\mu(\partial u/\partial y)]_w}{\rho u_\infty^2} \text{ and } N_w = \frac{1}{1 + \alpha_1 G_w} = \frac{1}{1 + \alpha_1} = \text{constant}.$$

Similarly, the heat transfer coefficient in terms of Nusselt number can be written as:

$$Nu(Re_L)^{-1/2} = (3/2)(1 + \cos\bar{x})(2 + \cos\bar{x})^{-1/2}G'_w \quad (15)$$

where:

$$Nu = \frac{L(\partial T/\partial y)_w}{(T_\infty - T_w)}.$$

From equations (14) and (15), it is clear that $(F')_w$ and $(G')_w$ are the critical parameters which characterize the skin friction and the heat transfer of the fluid flow.

3. Results and discussion

The partial differential equations represented by equations (11) and (12) with the boundary conditions (13) have been solved using the implicit finite-difference method along with the quasi-linearization technique. Since the above mentioned technique is given ornately in Inouye and Tate (1974), the description is omitted here for the sake of brevity. The grid sizes $\Delta\eta, \Delta\bar{x}, \Delta t^*$ have been optimized and taken as $\Delta\eta = 0.01$, $\Delta t^* = 0.01$ throughout the computation and $\Delta\bar{x} = 0.01$ for $\bar{x} \leq 1.5$ and $\Delta\bar{x} = 0.0001$, for $\bar{x} > 1.5$, in order to ensure the convergence of the numerical solution to the exact solution. The grid sizes have been optimized because the convergence becomes slower when the point of vanishing skin friction in the chord-wise direction is approached.

Further reduction of grid sizes will not affect the result up to the fourth decimal places. The value of η_∞ (i.e. the edge of the boundary layer) has been taken as 6.0. The convergence criteria based on the relative difference between the current and previous iteration values of the velocity and temperature gradients at wall are used. When the difference reaches less than 10^{-4} the solution is assumed to have converged and the iterative process is terminated.

With the purpose of getting the validity of the present method, in steady and unsteady cases ($t^* = 0$ and $t^* > 0$) the values of the skin-friction coefficient $C_f(Re_L)^{1/2}$ have been calculated for constant and variable viscosities without mass transfer. In Figure 2, the effects of variable fluid properties on the skin-friction coefficient have been compared with constant fluid property, for different times $t^* = 0$ and $t^* = 1$. The present results obtained for the particular case ($t^* = 0$) is compared with those of Eswara and Nath (1994) and Saikrishnan and Roy (2003) and it shows very good agreement in numerical results.

Also, the skin friction and the heat transfer parameters (F'_w, G'_w) at different times ($t^* = 0$, $t^* = 2$), for constant and variable properties, have been calculated and those results are compared with those of Surma Devi and Nath (1983) and Eswara and Nath (1994). The present results are matching well with previous results available in the literature and comparisons are shown in Figure 3.

It is appropriate to mention here that the point of zero skin friction in unsteady axi-symmetric flow on a fixed wall does not imply separation in contrast to the steady flow. Therefore, it is necessary to distinguish between points of separation and the points of zero skin friction in unsteady flows. When the imposed pressure gradient is adverse, the thickness of the boundary layer increases as the momentum is consumed by both wall shear and pressure gradient, and at some point, the viscous layer breaks away from the surface. Downstream of this point of breakaway, the original boundary layer fluid passes over a region of re-circulating flow. The point at which the boundary layer breaks away from the surface and which divides the region of downstream

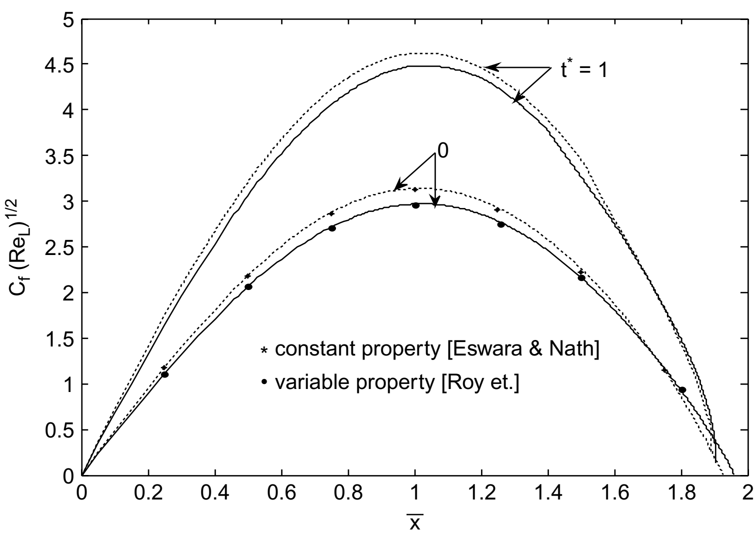


Figure 2. Comparison of skin friction coefficient on time (t^*) for variable fluid properties (_____) and constant fluid properties (……..) at time $t^* = 0$ and 1, with $\varphi(t^*) = 1 + \varepsilon(t^*)^2$, $\varepsilon = 0.25$, $Ec = 0$, $Pr = 0.7$

directed flow, in which the viscous effects are quite limited in extent, from the region of re-circulating flow is known as the separation point (Schlichting, 1979). The physical symptom of separation in unsteady two-dimensional flow over a fixed wall is the simultaneous vanishing of velocity and shear stress (skin friction) away from the wall in a coordinate system moving with separation. Detailed discussions of the phenomenon of separation for unsteady flows have been presented in Williams (1980), Ingham (1984), Cebeci (1984) and Sarpkaya (1992).

At different stream-wise locations, the variation of the skin friction and the heat transfer coefficients $[C_f(Re_L)^{1/2}, Nu(Re_L)^{-1/2}]$ over time in the presence of variable viscosity and Prandtl number have been showed in Figure 4. It is observed that as time increases, both $C_f(Re_L)^{1/2}$ and $Nu(Re_L)^{-1/2}$ are increased. The response is the same at all stream-wise locations. In fact, the percentage of increase in $C_f(Re_L)^{1/2}$ at $\bar{x} = 1$, for an increase of t^* from 0 to 2 is 60 per cent whereas in the case of $Nu(Re_L)^{-1/2}$ at $\bar{x} = 1$, it is about 4 per cent.

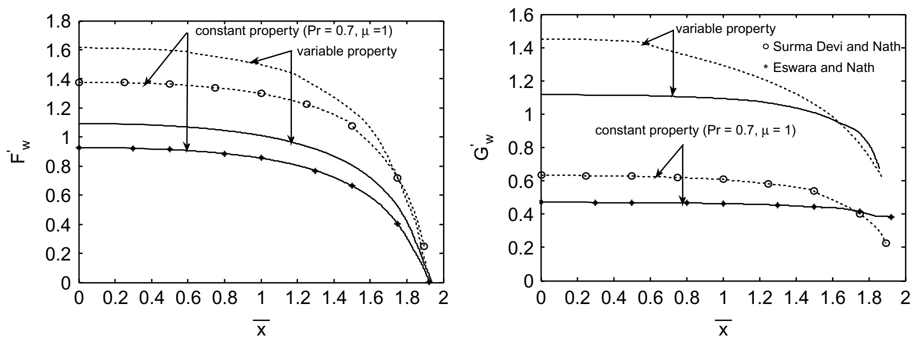


Figure 3. Effect of constant and variable fluid properties (viscosity and Prandtl number) on (a) skin friction parameter and (b) heat transfer parameters at time $t^* = 0$ (______) and $t^* = 2$ (……..), with $\varphi(t^*) = 1 + \varepsilon(t^*)^2$, $\varepsilon = 0.25$, $Ec = 0$

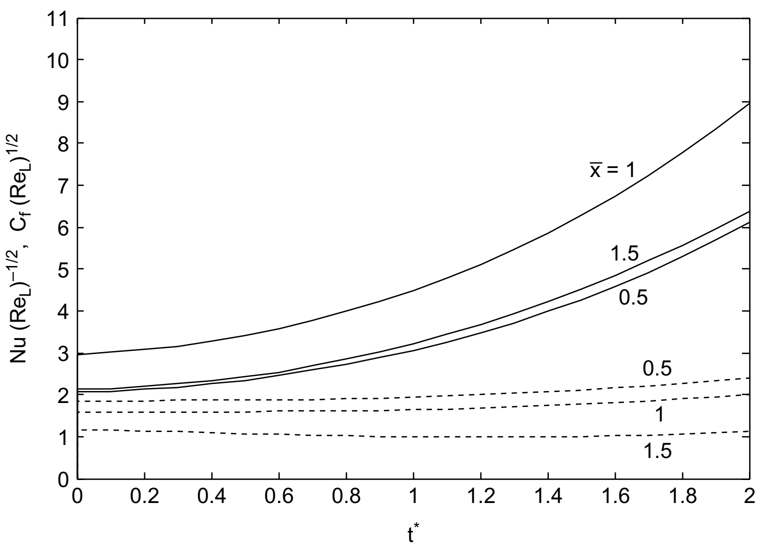
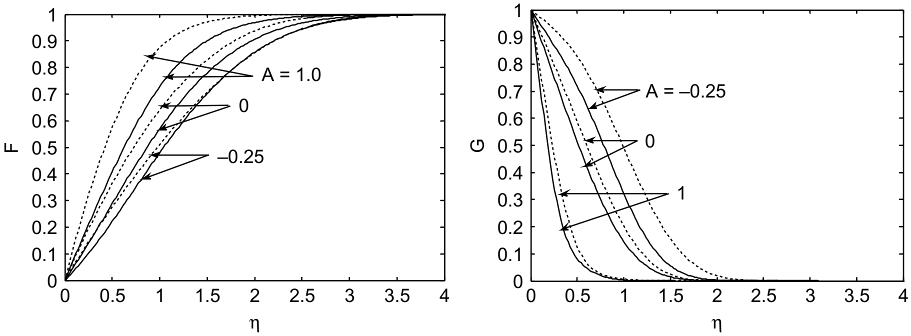


Figure 4. Effect of time on skin friction (______) and heat transfer (−−−−−) coefficients for $A = Ec = 0$, $\bar{x} = 0.5, 1.0, 1.5$, $T_\infty = 18.7$, $\Delta T_w = 10$

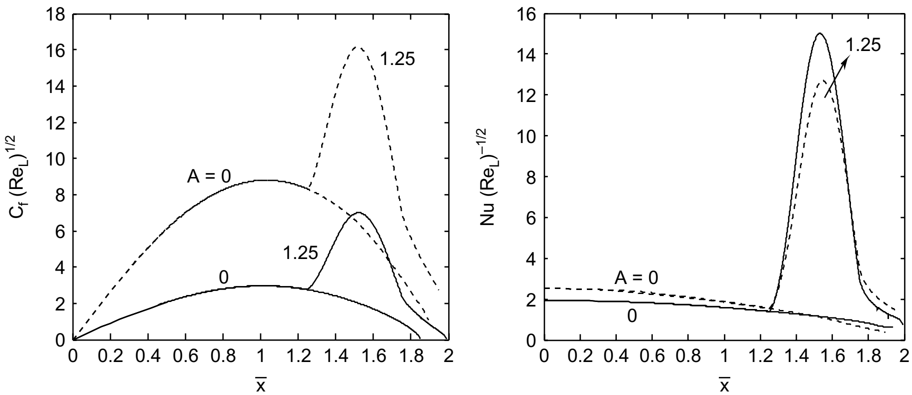
In Figure 5, the velocity and temperature profiles have been shown and it has been found that for $A \geq 0$, as time increases, the thickness of both momentum and thermal boundary layer decrease, whereas wall injection ($A < 0$) does the opposite. Thus, it is found that, unsteadiness decreases the thickness of both momentum and thermal boundary layer when the free stream is accelerating.

The effects of suction (or injection) parameter ($A > 0$ or $A < 0$) on $C_f(Re_L)^{1/2}$ and $Nu(Re_L)^{-1/2}$ for different times $t^* = 0$ and $t^* = 2$ have been shown in Figures 6 and 7, respectively. In the case of slot suction, it is found that as time increases, whatever may be the value of A, both the skin friction ($C_f(Re_L)^{1/2}$) and the heat transfer ($Nu(Re_L)^{-1/2}$) increase. When $t^* \geq 0$, in the case of non-uniform slot suction, the skin friction and the heat transfer coefficients increase as the slot starts and attain their maximum value before the trailing edge of the slot. Finally, the values of $C_f(Re_L)^{1/2}$ and $Nu(Re_L)^{-1/2}$ decrease from their maximum value and remain finite for the unsteady case whereas, for the steady case, $C_f(Re_L)^{1/2}$ reaches zero but $Nu(Re_L)^{-1/2}$ remains finite. Suction causes the location of zero skin friction to move downstream, because of the low energy fluid particles in the boundary layer is removed and behind the slot a new boundary



Note: Slot position (1.2-1.7)

Figure 5. Effect of mass transfer parameter (A) on (a) velocity profile and (b) temperature profile for $t^* = 0$ (______) and $t^* = 2$ (………) with $\varphi(t^*) = 1 + \varepsilon(t^*)^2$, $\varepsilon = 0.25$, $Ec = 0$, $\bar{x} = 1.75$, $T_\infty = 18.7$°C, $\Delta T_w = 10.0$°C, $w^* = 2\pi$, variable viscosity and Prandtl number



Note: Slot position (1.25-1.75)

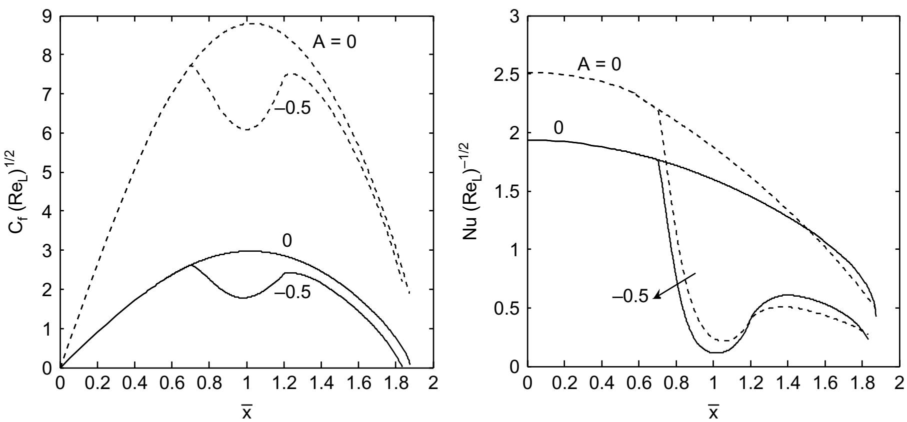
Figure 6. Effect of mass transfer parameter ($A > 0$) on (a) skin friction coefficient and (b) heat transfer coefficient at times $t^* = 0$ (_____) and $t^* = 2$ (– – – – –) with $\varphi(t^*) = 1 + \varepsilon(t^*)^2$, $\varepsilon = 0.25$, $Ec = 0$, $T_\infty = 18.7$°C, $\Delta T_w = 10.0$°C, $w^* = 2\pi$, variable viscosity and Prandtl number

layer forms which can overcome a certain pressure increase and hence the reverse flow region moves downstream as suction increases. The effect of injection is just the opposite of the effect of suction.

In Figure 8, the effect of slot movement on the skin friction and the heat transfer is shown for different times. It has been observed that if we move the location of the slot downstream, the adverse pressure gradient region also moves in the downstream direction. Thus, the reverse flow region can be moved downstream by imposing non-uniform suction and also by moving the slot downstream.

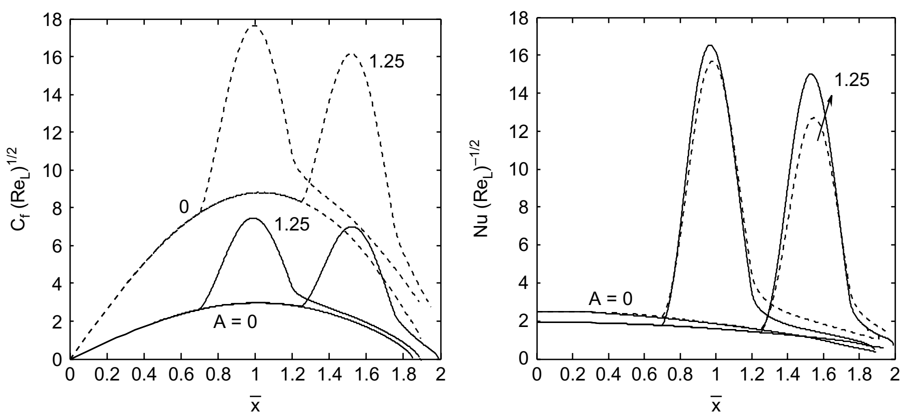
4. Conclusion

- Non-similar solutions of an unsteady water boundary layer flow over sphere with non-uniform slot suction (injection) have been obtained.



Note: Slot position (0.7-1.2)

Figure 7.
Effect of mass transfer parameter ($A < 0$) on (a) skin friction coefficient and (b) heat transfer coefficient at times $t^* = 0$ (_____) and $t^* = 2$ (-----) with $\varphi(t^*) = 1 + \varepsilon(t^*)^2$, $\varepsilon = 0.25$, $Ec = 0$, $T_\infty = 18.7°C$, $\Delta T_w = 10.0°C$, $w^* = 2\pi$, variable viscosity and Prandtl number



Note: Slot position (0.7-1.2) and (1.25-1.75)

Figure 8.
Effect of slot locations on (a) skin friction coefficient, (b) heat transfer coefficient when $t^* = 0$ (_____) and $t^* = 2$ (-----) with $\varphi(t^*) = 1 + \varepsilon(t^*)^2$, $\varepsilon = 0.25$, $Ec = 0$, $T_\infty = 18.7°C$, $\Delta T_w = 10.0°C$, $w^* = 2\pi$, variable viscosity and Prandtl number

- It has been observed that the vanishing skin friction can be delayed by non-uniform slot suction and also by moving the slot downstream.
- The effect of injection is just the opposite of imposing suction.
- As time increases, there is a significant increase in the skin friction and heat transfer parameters.
- The effect of unsteadiness is more significant on the skin friction as compared to the heat transfer.
- It is found that the unsteadiness decreases the thickness of both momentum and thermal boundary layer as time increases.

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