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MELTING EFFECT ON NATURAL CONVECTION ABOUT AXISYMMETRIC STAGNATION POINT ON A SURFACE IN POROUS MEDIA WITH SORET AND DUFOUR EFFECTS AND TEMPERATURE-DEPENDENT VISCOSITY

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ABSTRACT

A three-dimensional boundary layer solution is presented for melting effect on heat and mass transfer by natural convection with temperature-dependent viscosity in the vicinity of an axisymmetric stagnation point on heated vertical surfaces in porous media in the presence of Soret and Dufour effects. The governing equations for the velocity, temperature and concentration fields are solved numerically by the fourth order Runge-Kutta integration scheme. A parametric study illustrating the influence of the Darcy number, melting parameter or Stefan number, viscosity parameter, Dufour number and Soret number on the skin friction coefficient, Nusselt number as well as the Sherwood number are investigated. The results of the parametric study are shown in graphical and tabulated forms. It is found that increasing the Darcy number and the melting parameter within the boundary layer leads to increases in the velocity within the boundary layer and thus, increases the local skin friction coefficient. On the other hand, as the Darcy number and the melting parameter increase, the thermal boundary layer thickness decreases and thus, the rates of heat and mass transfer increase. As the Dufour number increases (or the Soret number decreases), the local skin friction coefficient increases and the local surface concentration decreases and thus increasing the local Sherwood number. Also, increasing the Dufour number tends to increase the local surface temperature and thus, decreasing the local Nusselt number.

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Keywords: Natural convection; melting effect; Dufour effect; Soret effect; porous media; variable viscosity; axisymmetric stagnation flow

NOMENCLATURE

a	constant in equation (7) [1/s]
C	concentration
C_p	specific heat of fluid [J/k]
C_s	concentration susceptibility
DA	Darcy number
D	Molecular diffusivity
D_m	mass diffusivity
D_u	Dufour number
f, F	dimensionless velocity function
g	acceleration due to gravity [m/s^2]
k	thermal conductivity [W/mK]
K	permeability of the porous media [m^2]
K_T	thermal-diffusion ratio.
M	melting parameter
Nu	Nusselt number, $Nu = q_w x / k (T_w - T_\infty)$
m_w	wall mass flux
p	pressure [N/m^2]
Pr_f	Prandtl number, $\Pr_f = C_p \mu_f / k$
q_w	
Sc	wall heat flux [W/ m^2]
Sh	Schmidt number, $Sc = \mu_f / D_m$
S_r	Sherwood number, $Sh = m_w x / D_m (C_w - C_\infty)$
T	
T_m	Soret number
U_∞	temperature [K]
u, v, w	mean fluid temperature [K]
x, y, z	free stream velocity [m/s]
	velocity component in x, y and z-axis [m/s]
	Cartesian coordinates [m]
Greek symbols	
α	thermal diffusivity [m^2/s]
β_T	coefficient of thermal expansion [K^{-1}]
β_C	coefficient of concentration expansion
μ	absolute dynamic viscosity [Ns/m^2]
λ	latent heat of the solid phase
θ	dimensionless temperature
ϕ	dimensionless concentration
γ_f	viscosity parameter [m^2/s]
ρ	density [kg/m^3]

Subscripts	refers to condition at wall
w	refers to condition far from the wall
∞	
Superscript	differentiation with respect to η

1. INTRODUCTION

The subject of convective flow in porous media has attracted considerable attention in the last few decades, due to its numerous applications in a wide variety of industrial processes as well as in many natural circumstances. Examples of such technological applications are geothermal extraction, storage of nuclear waste material, ground water flows, thermal insulation engineering, food processing, fibrous insulation, soil pollution and packed-bed reactors to name just a few. Seddeek [1] studied of the effect of a magnetic field and variable viscosity on steady two Darcy forced convection flow over a flat plate with variable wall temperature in a porous medium in the presence of blowing (suction). Hassanien et al. [2] investigated variable viscosity and thermal conductivity effects on combined heat and mass transfer in mixed convection over a UHF/UMF wedge in porous media in the entire regime. Bagai [3] investigated the effect of temperature dependent viscosity on heat transfer rates in the presence of internal heat generation, a similarity solution is proposed for the analysis of the steady free convection boundary layers over a non-isothermal axisymmetric body embedded in a fluid saturated porous medium. Modather and El-Kabeir [4] analyzed the effect of thermal radiation on free convection flow with variable viscosity and uniform suction velocity along a uniformly heated vertical porous plate embedded in a porous medium in the presence of a uniform transverse magnetic field. Modather [5] studied the effect of thermal radiation on unsteady boundary layer flow with temperature dependent viscosity and thermal conductivity due to a stretching sheet through porous media.

The effects of diffusion-thermo and thermal-diffusion of heat and mass transfer have been examined by Chapman and Cowling [6] and Hirshfelder et al. [7] from the kinetic theory of gases. They explained the phenomena and derived the necessary formulae to calculate the thermal-diffusion coefficient and the thermal-diffusion factor for monatomic gases or polyatomic gas mixtures. The heat and mass transfer simultaneously affecting each other that will cause the cross-diffusion effect. The heat transfer caused by concentration gradient is called the diffusion-thermo or Dufour effect. On the other hand, mass transfer caused by temperature gradients is called Soret or thermal-diffusion effect. Thus, the Soret effect is referred to species differentiation developing in an initial homogeneous mixture submitted to a thermal gradient and the Dufour effect is referred to the heat flux produced by a concentration gradient. The Soret effect, for instance, has been utilized for isotope separation, and in mixture between gases with very light molecular weight (H_2, He) and of medium molecular weight (N_2, air). Kafoussias and Williams [8] considered the boundary layer flows in the presence of Soret and Dufour effects associated with thermal-diffusion and diffusion-thermo for the mixed convection. The similarity equations of the problem considered are obtained by using the usual similarity technique. The Soret and Dufour effects have been found to appreciably influence the flow field in mixed convection boundary layer over a vertical surface embedded in a porous medium [9]. Alam and Rahman [10] investigated the

Dufour and Soret effects on mixed convection flow past a vertical porous flat plate with variable suction. Alam et al. [11] reported the effects of Dufour and Soret on unsteady MHD free Convection and mass transfer flow past a vertical porous plate in a porous medium numerically. Postelnicu [12] examined the heat and mass characteristics of natural convection about a vertical surface embedded in a saturated porous medium subjected to a magnetic field by considering the Dufour and Soret effects. The same author [13] studied influence of chemical reaction on heat and mass transfer by natural convection from vertical surfaces in porous media considering Soret and Dufour effects. Partha et al. [14] examined the Soret and Dufour effects in a non-Darcy porous medium. Mansour et al. [15] studied the multiplicity of solutions induced by thermosolutal convection in a square porous cavity heated from below and submitted to horizontal concentration gradient in the presence of Soret effect. Lakshmi Narayana et al. [16] studied the Soret and Dufour effects in a doubly stratified Darcy porous medium. Lakshmi Narayana and Murthy [17] examined the Soret and Dufour effects on free convection heat and mass transfer from a horizontal flat plate in a Darcy porous medium. Chamkha and Ben-Nakhi [18] considered the mixed convection flow with thermal radiation along a vertical permeable surface immersed in a porous medium in the presence of Soret and Dufour effects. El-Aziz [19] have investigated the combined effects of thermal-diffusion and diffusion-thermo on MHD heat and mass transfer over a permeable stretching surface with thermal radiation. Mahdy [20] studied the problem of MHD non-Darcian free convection from a vertical wavy surface embedded in porous media in the presence of Soret and Dufour effect.

In the present work, we consider the melting effect on natural convection flow with variable viscosity in the vicinity of an axisymmetric stagnation point on heated vertical surfaces in the presence of Soret and Dufour saturated in porous medium. The effect of Darcy number parameter, melting parameter, viscosity parameter, Dufour number and the Soret number on the velocity, temperature and concentration profiles as well as the skin friction coefficient, Nusselt number and the Sherwood number are investigated.

2. Mathematical Formulation

Consider an axisymmetric stagnation flow normal to a heated surface, as shown in Fig 1. The plate is in the *x*-*y* plane with *z*-axis in the vertical direction. The flow is impinging on the plate at the origin. In the formulation of the problem, the plate is assumed to be at a uniform temperature and concentration are T_w and C_w while the ambient fluid is maintained at the uniform temperature and concentration are T_∞ and C_∞ respectively. The dynamic viscosity μ is taken to be a variable in the equations of motion while the density ρ, the coefficients of thermal and concentration expansion are β_T and β_C respectively, permeability of the porous media is K, the thermal diffusivity, molecular diffusivity and mass diffusivity are α, D and D_m respectively, the specific heat at constant pressure and concentration susceptibility are C_p and C_s respectively, the mean fluid temperature is T_m and K_T is the thermal-diffusion ratio. Under the boundary layer assumptions, the governing equations are given by:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0, \tag{1}$$

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right)=-\frac{\partial p}{\partial x}+\frac{\partial}{\partial x}\left(\mu\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)+\frac{\partial}{\partial z}\left(\mu\frac{\partial u}{\partial z}\right)$$
$$+\rho g\left[\beta_T(T-T_\infty)+\beta_C(C-C_\infty)\right]-\frac{\mu}{K}u, \tag{2}$$

$$\rho\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\right)=-\frac{\partial p}{\partial y}+\frac{\partial}{\partial x}\left(\mu\frac{\partial v}{\partial x}\right)+\frac{\partial}{\partial y}\left(\mu\frac{\partial v}{\partial y}\right)+\frac{\partial}{\partial z}\left(\mu\frac{\partial v}{\partial z}\right)-\frac{\mu}{K}v, \tag{3}$$

$$\rho\left(u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}\right)=-\frac{\partial p}{\partial z}+\frac{\partial}{\partial x}\left(\mu\frac{\partial w}{\partial x}\right)+\frac{\partial}{\partial y}\left(\mu\frac{\partial w}{\partial y}\right)+\frac{\partial}{\partial z}\left(\mu\frac{\partial w}{\partial z}\right)-\frac{\mu}{K}w, \tag{4}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}=\alpha\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right)+\frac{D_m K_T}{C_p C_s}\frac{\partial^2 C}{\partial y^2}, \tag{5}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}+w\frac{\partial C}{\partial z}=D\left(\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial y^2}+\frac{\partial^2 C}{\partial z^2}\right)+\frac{D_m K_T}{T_m}\frac{\partial^2 T}{\partial y^2}. \tag{6}$$

The boundary conditions are given by:

$$u=v=0,\quad \mathrm{k}\frac{\partial \mathrm{T}}{\partial \mathrm{z}}=\rho\left[\lambda+C_p(T_w-T_\infty)\right]w,\quad T=T_w,\ \mathrm{C}=C_w.\ \text{on } z=0$$
$$u=ax, v=0, w=-az,\ T=T_\infty,\ \mathrm{C}=C_\infty,$$
$$p=p_0-\frac{1}{2}\rho a^2(x^2+y^2).\quad \text{at } z=\infty \tag{7}$$

Here u, v and w are the velocity components along x, y and z-axes; p is the pressure in the flow field; p_0 is the stagnation pressure; g is the acceleration due to gravity, λ is latent heat of the solid phase and the constant a is directly proportional to the free stream velocity (U_∞) far from the plate and inversely proportional to a characteristic length (L) of the plate.

Following Carey and Mollendorf [26], we assume that the absolute viscosity can be expressed as

$$\mu=\mu_f\left[1+\frac{1}{\mu_f}\left(\frac{d\mu}{dT}\right)_f(T-T_f)\right] \tag{8}$$

where μ_f is the value of μ at the film temperature of the flow. The following transformations would reduce the governing equations (1-6) to ordinary differential equations:

$$u = \frac{g\beta(T_\infty - T_w)}{a} F(\eta) + axf'(\eta);\ \ v = ayf'(\eta) \tag{9}$$

$$w = -2\sqrt{\frac{a\mu_f}{\rho}} f(\eta),\ \ T - T_\infty = (T_w - T_\infty)\theta(\eta),\ C - C_\infty = (C_w - C_\infty)\phi(\eta) \tag{10}$$

$$p = p_0 - \frac{\rho}{2}\left[a^2(x^2 + y^2) + w^2 - 2\frac{\mu_f}{\rho}\frac{dw}{dz}\right] \tag{11}$$

and

$$\eta = \sqrt{\frac{a\rho}{\mu_f}} z \tag{12}$$

where F, f, θ and ϕ are related to the non-dimensional velocity, temperature and concentration functions.

Using (10), relation (8) can be written as

$$\mu = \mu_f[1 + \gamma_f(\theta - 0.5)] \tag{13}$$

where $\gamma_f = \left(\frac{1}{\mu}\frac{d\mu}{dT}\right)_f (T_w - T_\infty)$ (14)

It can be verified that with transformations (9) to (12) the continuity equation (1) is automatically satisfied and the momentum, angular momentum and energy equations (2) to (6) reduce to:

$$[1 + \gamma_f(\theta - 0.5)]f''' + (2f + \gamma_f\theta')f'' + 1 - f'^2 - \frac{[1 + \gamma_f(\theta - 0.5)]}{DA} f' = 0 \tag{15}$$

$$[1 + \gamma_f(\theta - 0.5)]F'' + (2f + \gamma_f\theta')F' + f'F - N\phi - \theta - \frac{[1 + \gamma_f(\theta - 0.5)]}{DA} F = 0 \tag{16}$$

$$\frac{1}{\Pr_f}\theta'' + 2f\theta' + D_u\phi'' = 0 \tag{17}$$

$$\frac{1}{Sc}\phi'' + 2f\phi' + S_r\theta'' = 0 \tag{18}$$

where a prime denotes differentiation with respect to η only and $DA = \dfrac{K\rho a}{\mu_f}$ is Darcy number, $N = \dfrac{\beta_C (C_w - C_\infty)}{\beta_T (T_w - T_\infty)}$ is concentration to thermal buoyancy parameter, $\Pr_f = \dfrac{C_p \mu_f}{k}$ is the film Prandtl number, $Sc = \dfrac{\mu_f}{D_m}$ is Schmidt number, $D_u = \dfrac{D_m K_T (C_w - C_\infty)}{C_p C_s \mu_f (T_w - T_\infty)}$ is the Dufour number and $S_r = \dfrac{D_m K_T (T_w - T_\infty)}{\mu_f T_m (C_w - C_\infty)}$ is the Soret number.

The transformed boundary conditions may be written as:

$$\begin{aligned} &2\Pr_f\, f(0) + M\theta'(0) = 0, \quad f'(0) = F(0) = 0, \;\; \theta(0) = 1, \;\; \phi(0) = 1, \\ &f'(\infty) = 1, \;\; F(\infty) = \theta(\infty) = \phi(\infty) = 0. \end{aligned} \tag{19}$$

where M is the Stefan number or the melting parameter.

The shear stress, heat and mass flux at the plate are important physical characteristics for this type of heat and mass transfer problem. They are given by:

$$\begin{aligned} \tau_{wx} &= \left(\mu \frac{\partial u}{\partial z}\right)_{z=0} \\ &= \mu_f \left(1 + \frac{1}{2}\gamma_f\right)\sqrt{\frac{a\rho}{\mu_f}}\left[\frac{g\beta(T_\infty - T_w)}{a} F'(0) + axf''(0)\right] \end{aligned} \tag{20}$$

$$\begin{aligned} \tau_{wy} &= \left(\mu \frac{\partial v}{\partial z}\right)_{z=0} \\ &= \mu_f \left(1 + \frac{1}{2}\gamma_f\right)\sqrt{\frac{a\rho}{\mu_f}}\, ayf''(0) \end{aligned} \tag{21}$$

$$q_w = -k\left(\frac{\partial T}{\partial z}\right)_{z=0} = -k\sqrt{\frac{a\rho}{\mu_f}}(T_w - T_\infty)\theta'(0). \tag{22}$$

$$m_w = -D_m\left(\frac{\partial C}{\partial z}\right)_{z=0} = -D_m\sqrt{\frac{a\rho}{\mu_f}}(C_w - C_\infty)\phi'(0).$$

One can write Nusselt number and Sherwood number as:

$$Nu = -\sqrt{\frac{a\rho}{\mu_f}}\theta'(0) \tag{23}$$

$$Sh = -\sqrt{\frac{a\rho}{\mu_f}}\phi'(0) \tag{24}$$

where

$$Nu = \frac{q_w}{T_w - T_\infty}\frac{x}{k} \text{ and } Sh = \frac{m_w}{C_w - C_\infty}\frac{x}{D_m}. \tag{25}$$

where q_w and m_w are wall heat and mass flux, respectively.

3. Numerical Method

The set of Equations (15) to (18) under the boundary conditions (19) has been solved numerically using the Runge–Kutta integration scheme with the shooting method. We let

$$f = x_1,\ f' = x_2,\ f'' = x_3,\ F = x_4,\ F' = x_5,\ \theta = x_6,\ \theta' = x_7,\ \phi = x_8,\ \phi' = x_9 \tag{26}$$

Equations (15) to (18) are transformed into systems of first-order differential equations as follows:

$$\begin{aligned}
x_1' &= x_2\\
x_2' &= x_3\\
x_3' &= -\frac{1}{\left(1+\gamma_f\left(x_6-0.5\right)\right)}\left((2x_1+\gamma_f x_7)x_3 + 1 - x_2^2 - \frac{[1+\gamma_f(x_6-0.5)]}{DA}x_2\right)\\
x_4' &= x_5\\
x_5' &= -\frac{1}{\left(1+\gamma_f\left(x_6-0.5\right)\right)}\left((2x_1+\gamma_f x_7)x_5 + x_2x_4 - Nx_8 - x_6 - \frac{[1+\gamma_f(x_6-0.5)]}{DA}x_4\right)\\
x_6' &= x_7\\
x_7' &= -\mathrm{Pr}_f\left(2x_1x_7 + D_u x_9'\right)\ x_8' = x_9\\
x_9' &= -Sc\left(2x_1x_9 + S_r x_7'\right)
\end{aligned} \tag{27}$$

subject to the following initial conditions:

$$2\,\mathrm{Pr}_f\ x_1(0) + Mx_7(0) = 0,\quad x_2(0) = x_4(0) = 0,\ x_6(0) = 1,\ \ x_8(0) = 1,$$

$$x_2(\infty)=1,\ x_4(\infty)=x_6(\infty)=x_8(\infty)=0. \tag{28}$$

Equation (27) is then integrated numerically as an initial-value problem to a given terminal point. The accuracy of the assumed missing initial condition is then checked by comparing the calculated value of the dependent variable at the terminal point with its given value. If a difference exists, improved values of the missing initial conditions must be obtained and the process is repeated. A step size of $\eta = 0.001$ was selected to be satisfactory for a convergence criterion of 10^{-6} in nearly all cases. The maximum value of η_∞ to each group of parameters DA, Pr_f, Sc, D_u , S_r, M, N and γ_f are determined when the values of unknown boundary conditions at $\eta = 0$ do not change to successful loop with error less than 10^{-6}. From the process of numerical computation, the local skin friction coefficient, the local Nusselt number and the local Sherwood number, which are respectively proportional to $f''(0)$, $-F'(0)$, $-\theta'(0)$ and $-\phi'(0)$ are worked out and their numerical values presented in tabular forms.

4. Results and Discussion

Based on the fourth-order Runge-Kutta integration scheme described above, calculations were carried out for the values of Prandtl number Pr=0.733, Schmidt number Sc=0.66, Darcy number DA ranging from 1.0 to ∞, variable viscosity parameter γ_f ranging from -0.8 to 0.8, Soret number S_r ranging from 0.06 to 0.8, Dufour number D_u ranging from 0.075 to 1.0 (the product of S_r and D_u must stay constant i.e. 0.06) and melting parameter M ranging from 0.0 to 1.5 with N=0.5.

It should be noted here that in the case of γ_f=0, M=0, Sc=0, DA=∞ and D_u=S_r=0, Wang [27] has derived the solution of this problem. On the other hand, for the variable viscosity case (M=0, Sc=0, DA=∞ and D_u=S_r=0) solution of the above equations (15-18) has been obtained by Takhar [28] for Pr_f=0.7,1,10 and 100 and viscosity variation parameter ($-1.6 \le \gamma_f \le 1.6$). The present results were compared with the results reported by Takhar [28] and they were found to be in good agreement.

Tables 1-3 display the results for the flow from heated vertical surface which show the surface values of velocity, temperature and concentration gradient components. These are proportional to the friction factor, Nusselt number and Sherwood number respectively. Table 1 shows the effect of the melting parameter M and the Darcy number DA on the local skin friction coefficient, Nusselt number and the Sherwood number.

The results indicate that as the melting parameter M increases, there is an increase in the local skin friction coefficient while the local Nusselt and Sherwood numbers have the opposite behavior. The Darcy number parameter also has a noticeable effect on the local skin friction coefficient. Increasing the Darcy number within the boundary layer leads to an increase in the velocity within the boundary layer and thus, increases the local skin friction coefficient. On the other hand, as the Darcy number increases, the thermal boundary layer thickness decreases and thus, the rates of heat and mass transfer increase.

Table 1. Values of $f''(0), F'(0), \theta'(0)$ and $\phi'(0)$ with *Pr* =0.733, *Sc*=0.66 and *N*=0.5.

DA	M	$f''(0)$	$-F'(0)$	$-\theta'(0)$	$-\phi'(0)$	$-F'(0)/f''(0)$
1.0	0.0	0.72729	0.78105	0.51764	0.51065	1.07391
	0.4	0.77812	0.77188	0.58483	0.57518	0.99199
	0.8	0.85050	0.75704	0.67785	0.66441	0.89011
	1.0	0.90000	0.74621	0.73975	0.72374	0.82912
	1.5	1.10432	0.70007	0.98286	0.95659	0.63394
2.0	0.0	0.85556	0.80577	0.55306	0.54549	0.94180
	0.4	0.91999	0.79256	0.62485	0.61444	0.86149
	0.8	1.01098	0.77261	0.72405	0.70959	0.76422
	1.0	1.07269	0.75870	0.78995	0.77275	0.70729
	1.5	1.32327	0.70299	1.04780	1.01971	0.53125
5.0	0.0	0.94762	0.81794	0.57558	0.56764	0.86315
	0.4	1.02127	0.80222	0.65014	0.63925	0.78551
	0.8	1.12468	0.77919	0.75304	0.73795	0.69281
	1.0	1.19443	0.76350	0.82132	0.80338	0.63922
	1.5	1.47481	0.70253	1.08785	1.05863	0.47636
20.0	0.0	0.99816	0.82320	0.58711	0.57899	0.82471
	0.4	1.07670	0.80622	0.66306	0.65192	0.74879
	0.8	1.18663	0.78169	0.76781	0.75239	0.65875
	1.0	1.26057	0.76515	0.83727	0.81895	0.60699
	1.5	1.55633	0.70174	1.10808	1.07828	0.45090
50.0	0.0	1.00864	0.82418	0.58944	0.58128	0.81712
	0.4	1.08817	0.80695	0.66566	0.65448	0.74157
	0.8	1.19943	0.78213	0.77078	0.75529	0.65209
	1.0	1.27423	0.76543	0.84048	0.82208	0.60070
	1.5	1.57309	0.70154	1.11213	1.08223	0.44596
100.0	0.0	1.01216	0.82450	0.59022	0.58204	0.81460
	0.4	1.09202	0.80719	0.66653	0.65533	0.73917
	0.8	1.20373	0.78227	0.77178	0.75626	0.64988
	1.0	1.27881	0.76551	0.84155	0.82312	0.59861
	1.5	1.57871	0.70147	1.11349	1.08354	0.44433
∞	0.0	1.01569	0.82482	0.59099	0.58281	0.81208
	0.4	1.09589	0.80743	0.66740	0.65618	0.73678
	0.8	1.20804	0.78241	0.77277	0.75723	0.64767
	1.0	1.28341	0.76560	0.84262	0.82417	0.59653
	1.5	1.58435	0.70140	1.11484	1.08486	0.44270

Table 2. Values of $f''(0), F'(0), \theta'(0)$ and $\phi'(0)$ with *Pr* =0.733, *Sc*=0.66 and *N*=0.5

D_u	S_r	M	$f''(0)$	$-F'(0)$	$-\theta'(0)$	$-\phi'(0)$	$-F'(0)/f''(0)$
0.075	0.80	0.0	1.05053	0.90275	0.64464	0.42769	0.85932
		0.4	1.14758	0.88995	0.75422	0.44354	0.77550
		0.8	1.29893	0.86655	0.92379	0.46212	0.66713
		1.0	1.41419	0.84727	1.05150	0.47300	0.59912
		1.5	2.07204	0.73621	1.75681	0.51365	0.35531
0.15	0.40	0.0	1.05245	0.87204	0.62712	0.53183	0.82858
		0.4	1.14598	0.85481	0.72501	0.59028	0.74592
		0.8	1.28636	0.82727	0.87057	0.67473	0.64311
		1.0	1.38847	0.80685	0.97525	0.73422	0.58111
		1.5	1.88960	0.71170	1.47490	1.01152	0.37664
0.30	0.20	0.0	1.05627	0.86620	0.59208	0.58382	0.82006
		0.4	1.14298	0.84965	0.66872	0.65742	0.74337
		0.8	1.26419	0.82527	0.77440	0.75877	0.65280
		1.0	1.34560	0.80861	0.84445	0.82589	0.60093
		1.5	1.67026	0.74380	1.11735	1.08723	0.44532
0.60	0.10	0.0	1.06384	0.88249	0.52192	0.60963	0.82953
		0.4	1.13772	0.87103	0.56383	0.67962	0.76559
		0.8	1.22892	0.85562	0.61402	0.76558	0.69624
		1.0	1.28297	0.84607	0.64308	0.81628	0.65946
		1.5	1.45244	0.81512	0.73163	0.97386	0.56120
1.0	0.06	0.0	1.07381	0.91325	0.42827	0.61972	0.85047
		0.4	1.13199	0.90787	0.43772	0.67620	0.80201
		0.8	1.19452	0.90106	0.44650	0.73689	0.75433
		1.0	1.22736	0.89713	0.45062	0.76874	0.73094
		1.5	1.31390	0.88585	0.46017	0.85254	0.67422

Table 3. Values of $f''(0), F'(0), \theta'(0)$ and $\phi'(0)$ with *Pr* =0.733, *Sc*=0.66 and *N*=0.5

γ_f	DA	$f''(0)$	$-F'(0)$	$-\theta'(0)$	$-\phi'(0)$	$-F'(0)/f''(0)$
-0.80	1.00	1.51537	1.52585	0.55310	0.69844	1.00692
	2.0	1.84679	1.52912	0.60507	0.76323	0.82799
	5.0	2.10996	1.51824	0.64044	0.80730	0.71956
	20.0	2.26229	1.50930	0.65912	0.83059	0.66715
	50.0	2.29450	1.50724	0.66292	0.83533	0.65689
	100.0	2.30537	1.50654	0.66419	0.83692	0.65349
	∞	2.31631	1.50583	0.66547	0.83851	0.65010
-0.60	1.0	1.35138	1.35169	0.54899	0.69153	1.00023
	2.0	1.64403	1.35863	0.59916	0.75410	0.82640
	5.0	1.87312	1.35214	0.63296	0.79624	0.72187
	20.0	2.00471	1.34597	0.65072	0.81839	0.67140
	50.0	2.03245	1.34451	0.65433	0.82289	0.66152
	100.0	2.04181	1.34401	0.65554	0.82440	0.65825

Table 3. (Continued)

γ_f	DA	$f''(0)$	$-F'(0)$	$-\theta'(0)$	$-\phi'(0)$	$-F'(0)/f''(0)$
	∞	2.05122	1.34350	0.65675	0.82591	0.65498
0.0	1.0	1.04181	1.01970	0.54076	0.67690	0.97877
	2.0	1.25907	1.03380	0.58562	0.73302	0.82108
	5.0	1.42232	1.03596	0.61494	0.76968	0.72836
	20.0	1.51404	1.03527	0.63014	0.78869	0.68378
	50.0	1.53321	1.03499	0.63322	0.79254	0.67505
	100.0	1.53967	1.03489	0.63425	0.79383	0.67215
	∞	1.54615	1.03478	0.63528	0.79511	0.66926
0.60	1.0	0.86564	0.82678	0.53687	0.66853	0.95511
	2.0	1.03683	0.84538	0.57620	0.71802	0.81535
	5.0	1.16070	0.85291	0.60121	0.74948	0.73482
	20.0	1.22892	0.85562	0.61402	0.76558	0.69624
	50.0	1.24307	0.85608	0.61660	0.76883	0.68869
	100.0	1.24783	0.85623	0.61746	0.76992	0.68618
	∞	1.25260	0.85638	0.61832	0.77100	0.68368
0.80	1.0	0.82261	0.77877	0.53631	0.66677	0.94670
	2.0	0.98179	0.79858	0.57371	0.71395	0.81340
	5.0	1.09563	0.80752	0.59729	0.74369	0.73704
	20.0	1.15794	0.81113	0.60932	0.75887	0.70049
	50.0	1.17084	0.81178	0.61174	0.76192	0.69333
	100.0	1.17518	0.81200	0.61255	0.76294	0.69096
	∞	1.17952	0.81221	0.61335	0.76396	0.68859

The effects of the Soret S_r and Dufour D_u numbers with the melting parameter on the local skin friction coefficient, Nusselt number and Sherwood number are shown in Table 2. The Soret number S_r represents the effect of temperature gradients on mass (species) diffusion while the Dufour number D_u simulates the effect of concentration gradients on thermal energy flux in the flow domain. Table 2 indicates that as the Dufour number increases (and the Soret number decreases), significant changes in the local heat and mass transfer coefficients and a slight change in the value of local skin friction coefficient are caused. As the Dufour number increases (and the Soret number decreases), the local skin friction coefficient increases and the local surface concentration decreases and thus increasing the local Sherwood number. Increasing the Dufour number tends to increase the local surface temperature and thus, decreasing the local Nusselt number.

The effects of the temperature-dependent viscosity with Darcy number on the local skin friction coefficient, Nusselt number and Sherwood number are shown in Table 3. It can be seen from this table that the viscosity variation parameter has a significant effect on the local Nusselt number. As the viscosity variation parameter decreases, the thermal boundary layer thickness decreases and, thus, the rate of heat transfer increases. The viscosity variation parameter also has a noticeable effect on the local skin friction coefficient and a slight effect on the local mass transfer coefficient. It is observed that decreasing the viscosity within the boundary layer leads to an increase in the velocity within the layer and, thus, increases the local skin friction coefficient while the local mass transfer coefficient has the opposite behavior.

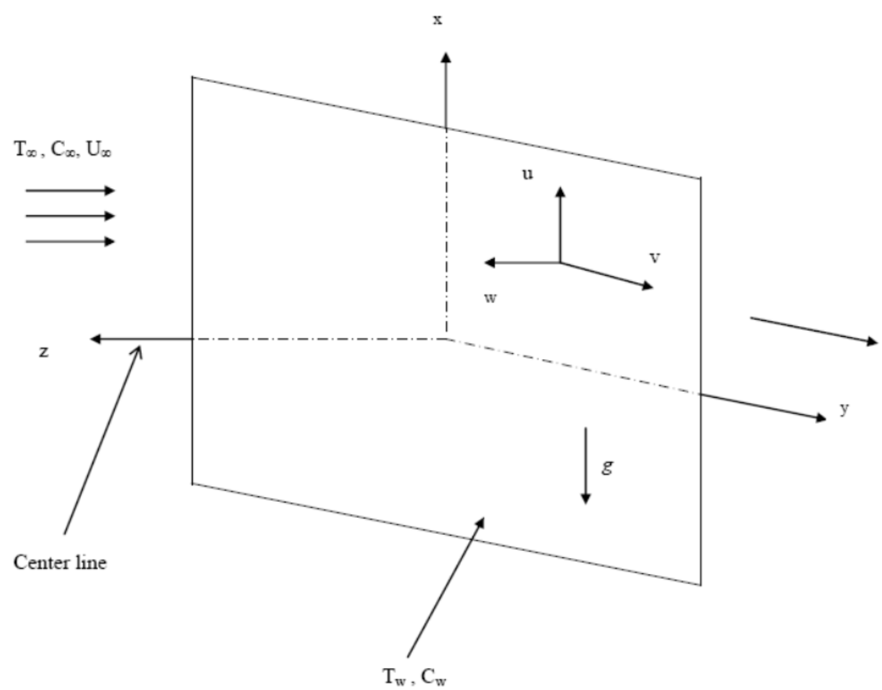

Figure 1. Physical model and coordinate system.

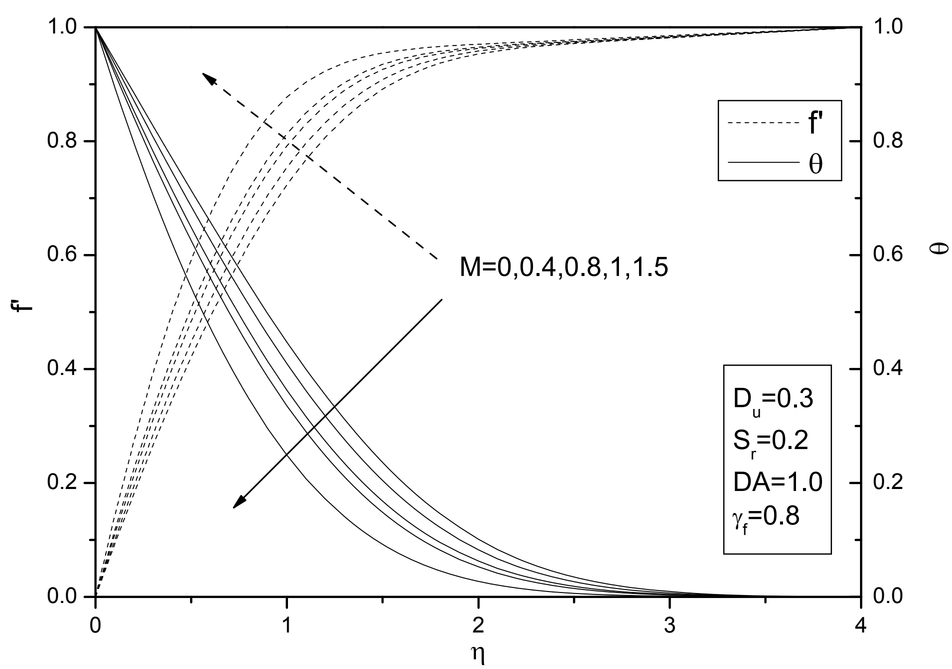

Figure 2. Effect of melting patameter M on velocity (f') and temperature (θ) profiles.

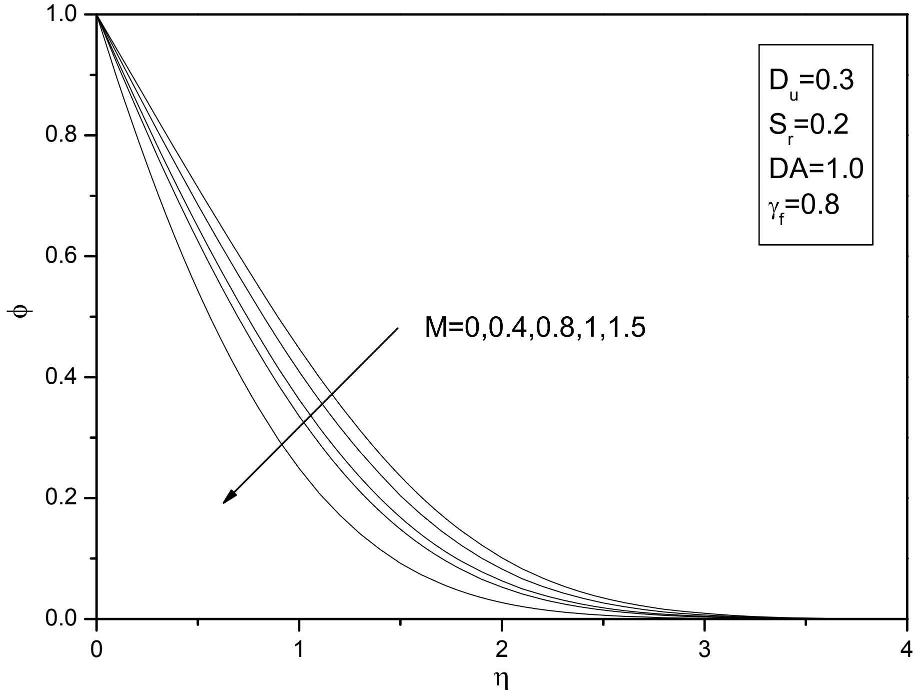

Figure 3. Effect of melting patameter M on concentration profile (ϕ).

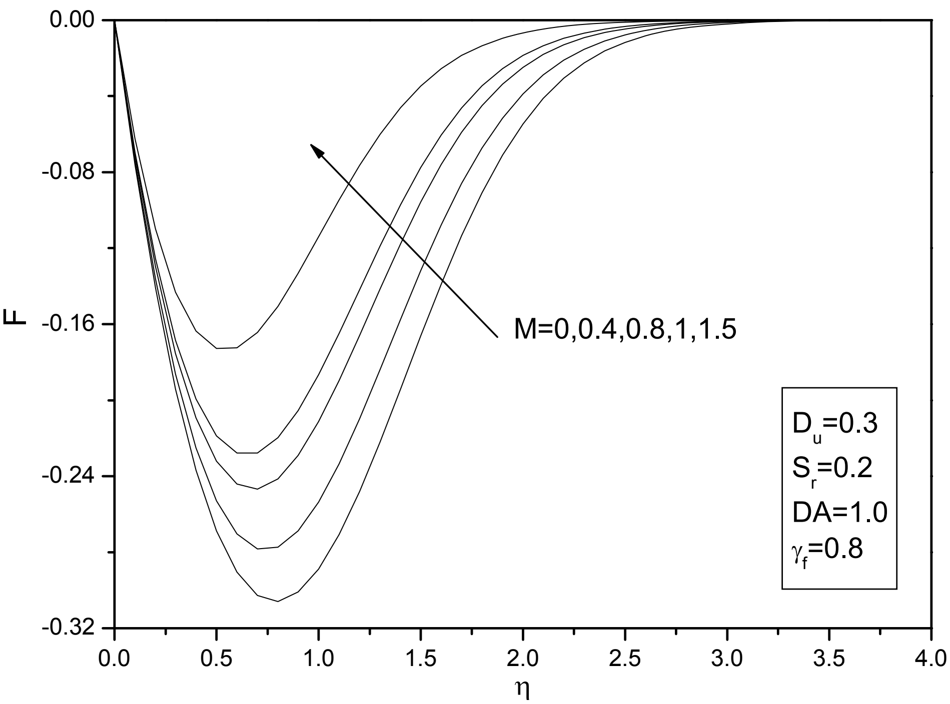

Figure 4. Effect of melting patameter M on velocity profile (F).

Figures 2-4 display typical dimensionless velocity, temperature and concentration profiles for various values of the melting parameter M, respectively. It can be seen that the velocity of the fluid increases as the melting parameter M increases while temperature and concentration decrease.

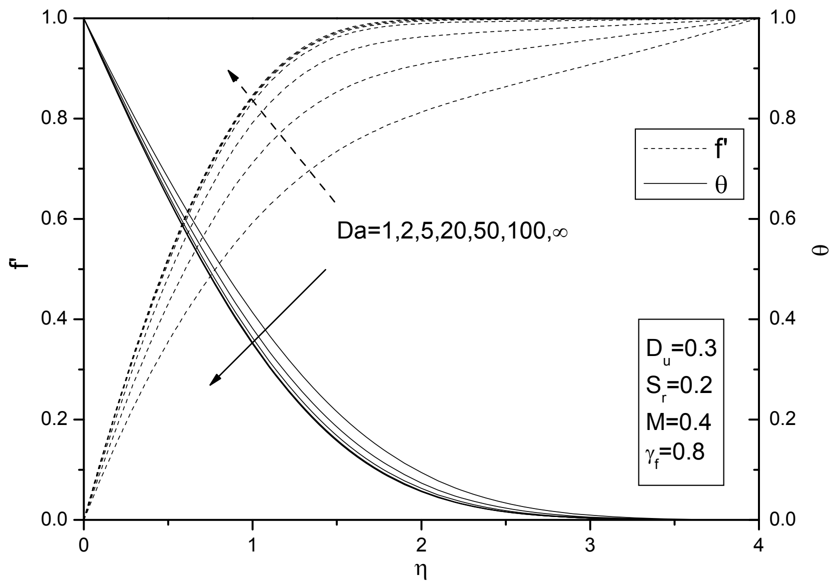

Figure 5. Effect of Darcy number Da on Velocity (f') and temperature (θ) profiles.

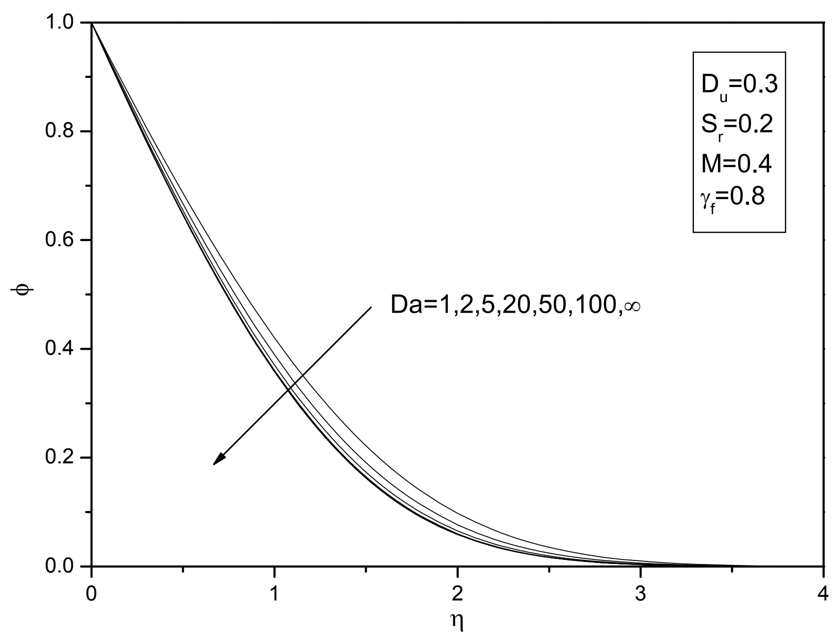

Figure 6. Effect of Darcy number Da on concentration profile (ϕ).

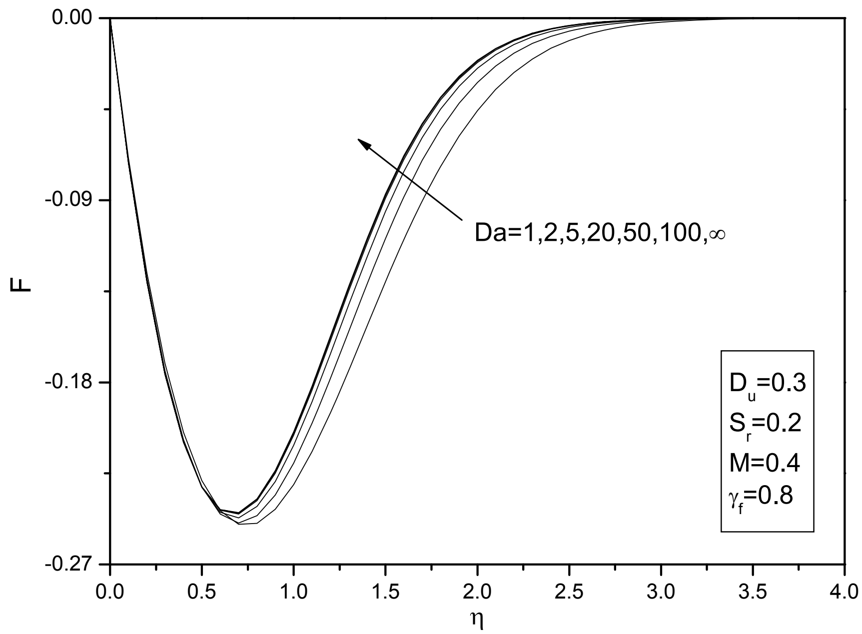

Figure 7. Effect of Darcy number Da on Velocity profile (F).

Figures 5-7 describe the behavior of the velocity, temperature and concentration fields for various values of the Darcy number parameter DA, respectively. It can be seen that the mass concentration and the fluid temperature fields decrease as the Darcy number DA increases, while the velocity increases as the Darcy number DA increases.

Figures 8-10 display the dimensionless velocity, temperature and concentration profiles for various values of variable viscosity γ_f, respectively. It is observed that the temperature and concentration across the boundary layer increase while the fluid velocity decreases with increasing values of the viscosity variation parameter γ_f.

Figures 11-13 show the velocity, temperature and concentration distributions with collective variation in the Soret number S_r, and the Dufour number D_u, respectively. We observe from these figures that a decrease in the value of D_u from 1.0 to 0.075 strongly increases the fluid velocity and decreases the fluid temperature values in the regime. Decreasing the value of D_u clearly reduces the influence of species gradients on the temperature field, so that the temperature function values are clearly lowered and the boundary layer regime is cooled. The S_r values rise from 0.06 to 0.8 over this range (the product of S_r and D_u must stay constant i.e. 0.06). On the other hand, the concentration function in the boundary layer regime is increased as D_u is decreased from 1.0 to 0.075 (and S_r simultaneously increased from 0.06 to 0.8). The mass diffusion is evidently enhanced in the domain as a result of the contribution of the temperature gradients.

From equation (20) it follows that the stagnation point is at:

$$x_s = \frac{g\beta(T_w - T_\infty)}{a^2}\frac{F'(0)}{f''(0)} \quad \text{at y=z=0} \tag{29}$$

Since $F'(0)$ is negative, the stagnation point is below the centre line of the flow when $T_W > T_\infty$ (heated wall) and above the centre line of the flow for $T_W < T_\infty$ (cooled wall).

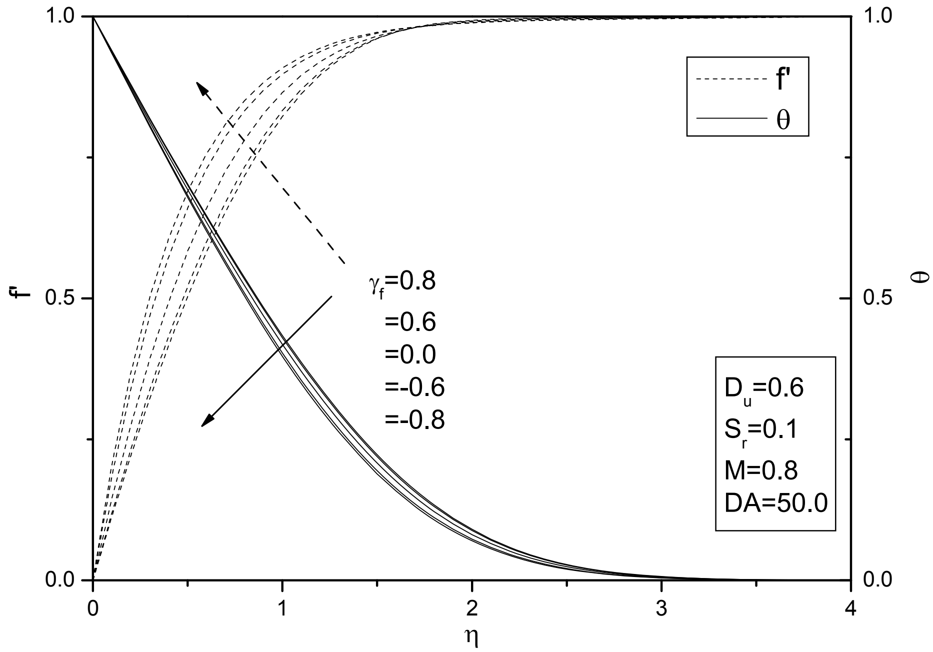

Figure 8. Effect of variable viscosity Y on velocity (f) and temperature (θ) profiles.

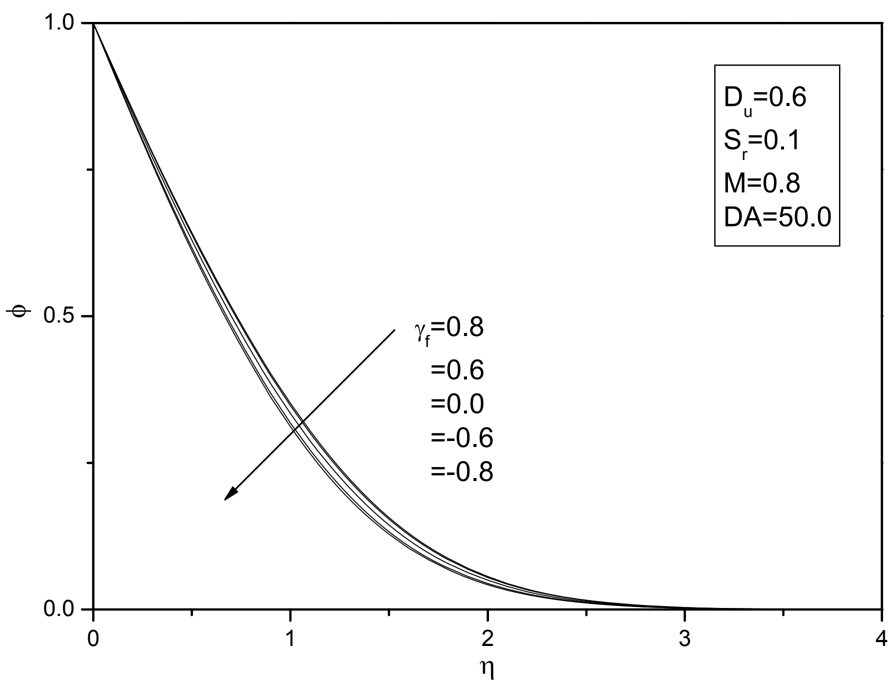

Figure 9. Effect of variable viscosity y_f on concentration profile (ϕ).

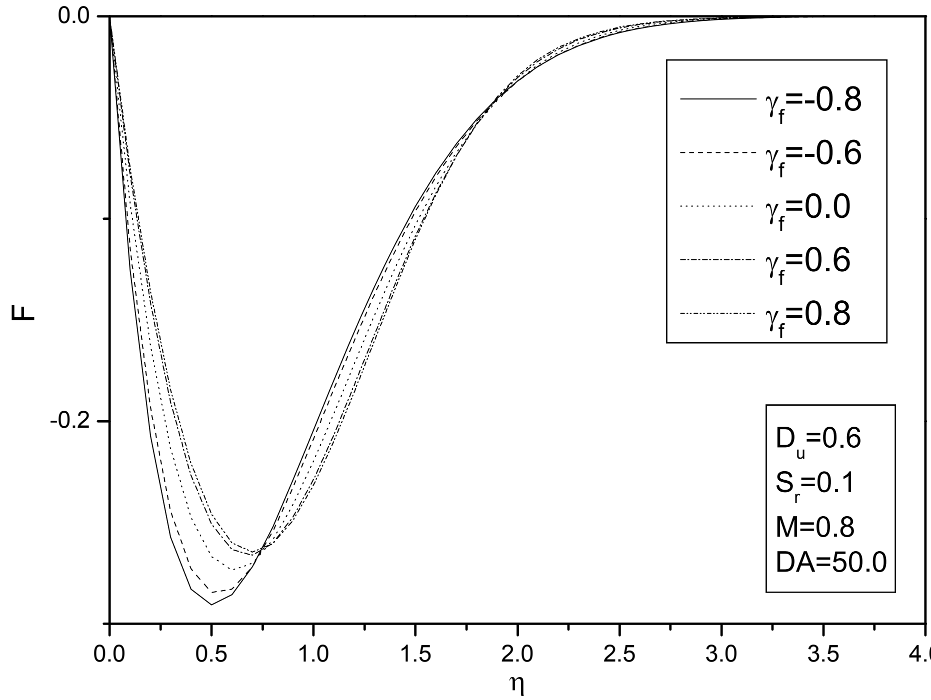

Figure 10. Effect of variable viscosity γ_f on Velocity profile (F).

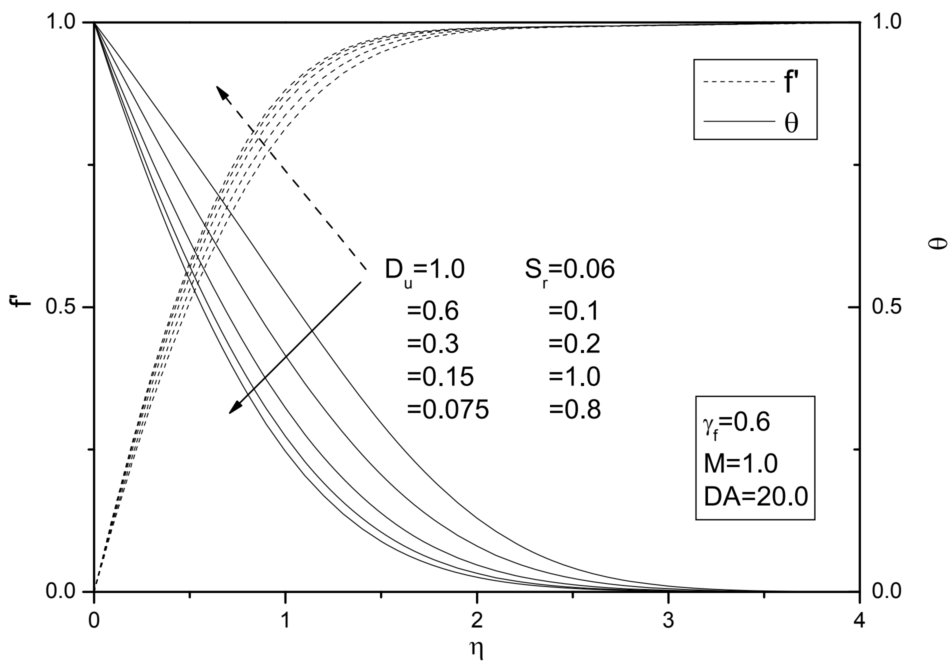

Figure 11. Effect of Dufour D_u and Soret S_r on velocity (f') and temperature (θ) profiles.

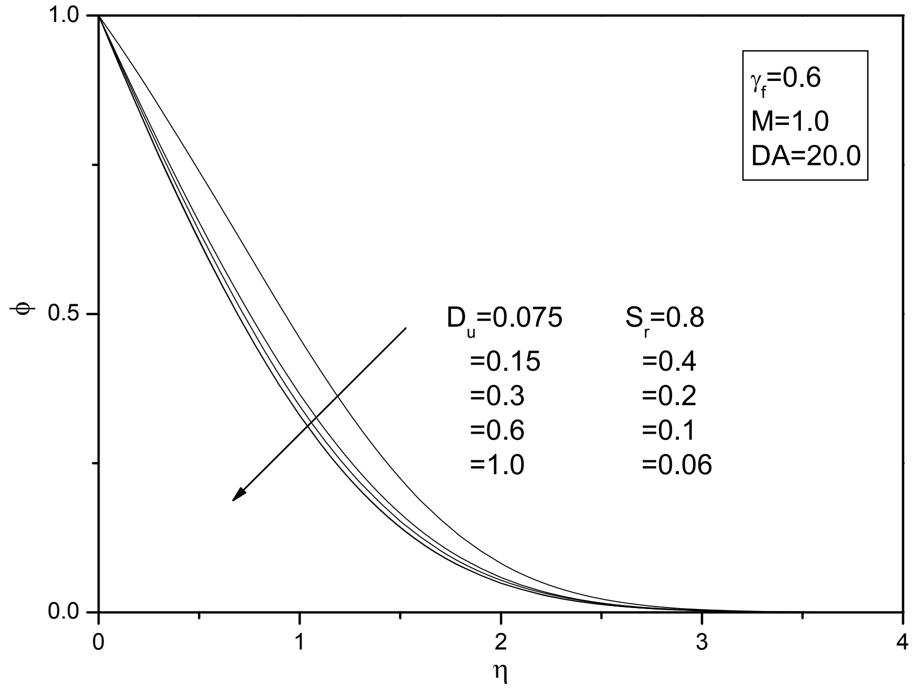

Figure 12. Effect of Dufour D_u and Soret S_r on concentration profile (ϕ).

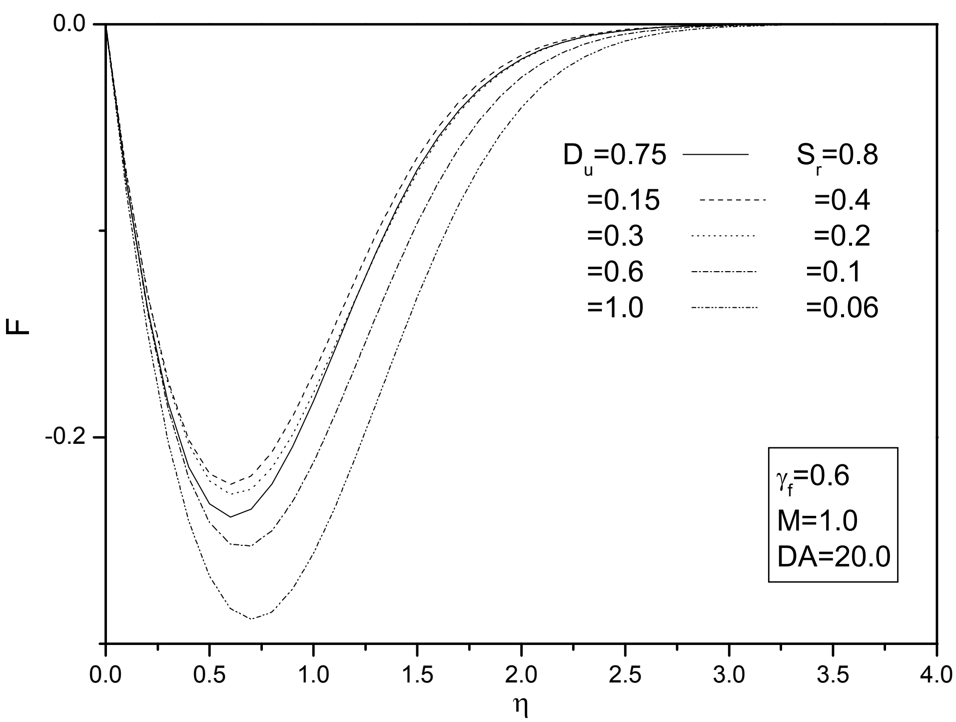

Figure 13. Effect of Dufour D_u and Soret S_r on Velocity profile (F).

CONCLUSION

In the present work, the melting effect on natural convection flow with variable viscosity in the vicinity of an axisymmetric stagnation point on heated vertical surfaces in saturated porous media in the presence of Soret and Dufour effects was studied. The viscosity of the fluid was taken as a function of temperature. The governing equations were developed and transformed into a set of similarity equations. These equations were solved by the fourth order Runge-Kutta numerical method. Favorable comparisons with previously published work were performed and the results were found to be in good agreement. The effects of Darcy number parameter, melting parameter, viscosity parameter, Dufour number and the Soret number on the velocity, temperature, and concentration profiles as well as the skin friction coefficient, Nusselt number and the Sherwood number were investigated. It was concluded that as the melting parameter increased, the local skin friction coefficient was increased while the local Nusselt number and Sherwood number had the opposite behavior. The Darcy number had a noticeable effect on the local skin friction coefficient and the local heat and mass transfer coefficients. Increasing the Dufour number (with the Soret number decreasing) led to significant changes in the local heat and mass transfer coefficients and a slight change in the value of local skin friction coefficient. As the Dufour number increased (and the Soret number decreased), the local skin friction coefficient increased and the local surface concentration decreased and thus, increased the local Sherwood number. Increasing the Dufour number increased the local surface temperature and thus, decreased the local Nusselt number.

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