Heat and mass transfer by natural convection flow about a truncated cone in porous media with Soret and Dufour effects

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Abstract

Purpose – The purpose of this paper is to study the effects of chemical reaction, thermal radiation and Soret and Dufour effects of heat and mass transfer by natural convection flow about a truncated cone in porous media.

Design/methodology/approach – The problem is formulated and solved numerically by an accurate implicit finite-difference method.

Findings – It is found that the Soret and Dufour effects as well as the thermal radiation and chemical reaction cause significant effects on the heat and mass transfer charateristics.

Originality/value – The problem is relatively original as it considers Soret and Dufour as well as chemical reaction and porous media effects on this type of problem.

Keywords Porous media, Natural convection, Heat and mass transfer, Soret and Dufour effects, Truncated cone

Paper type Research paper

1. Introduction

Many transport processes occurring both in nature and in industries involve fluid flows with the combined heat and mass transfer. Such flows are driven by the multiple buoyancy effects arising from the density variations caused by the variations in temperature as well as species concentrations. Also, there has been increased interest in studying buoyancy induced flow by simultaneous heat and mass transfer from different geometries embedded in porous media. This interest stems from many engineering and geophysical applications such as geothermal reservoirs, drying of porous solids, thermal insulation, enhanced oil recovery, packed bed catalytic reactors, cooling of nuclear reactors and underground energy transport. Most early studies on porous media have used the Darcy law which is a linear empirical relation between the Darcian velocity and the pressure drop across the porous medium and is limited to slow flows. However, for high velocity flow situations, the Darcy law is inadequate for predicting the proper physical flow behavior since it neglects the porous medium inertia effects which become important. In this situation, the pressure drop across the porous medium is a quadratic function of the flow rate. It has been reported that the high flow situation is established when the Reynolds number based on the pore size is greater than unity. References of comprehensive literature surveys regarding the subject of porous media can be had

in most recent books by Ingham and Pop (1998), Nield and Bejan (2006), Vafai (2005), Pop and Ingham (2001) and Ingham *et al.* (2004) .

Coupled heat and mass transfer problems in presence of chemical reaction are of importance in many processes and have, therefore, received considerable amount of attention in recent times. In processes such as drying, distribution of temperature and moisture over agricultural fields and groves of fruit trees, damage of crops due to freezing, evaporation at the surface of a water body, energy transfer in a wet cooling tower and flow in a desert cooler, heat and mass transfer occur simultaneously. Possible applications of this type of flow can be found in many industries. For instance, in the power industry, among the methods of generating electric power is one in which electrical energy is extracted directly from a moving conducting fluid. Chemical reactions can be modeled as either homogeneous or heterogeneous processes. This depends on whether they occur at an interface or as a single-phase volume reaction. A homogeneous reaction is one that occurs uniformly throughout a given phase. Das *et al.* (1994) considered the effects of a first-order chemical reaction on the flow past an impulsively started infinite vertical plate with constant heat flux and mass transfer. Muthucumarswamy and Ganesan (2001) have studied the first-order chemical reaction on flow past an impulsively started vertical plate with uniform heat and mass flux. Chamkha (2003) presented an analytical solution for heat and mass transfer by laminar flow past uniformly stretched vertical permeable surface embedded in a fluid-saturated porous medium in the presence of a chemical reaction effect. Chamkha *et al.* (2004) have investigated the double-diffusive convective flow of a micropolar fluid over a vertical plate embedded in a porous medium with a chemical reaction. Recently, the heat and mass transfer characteristics of natural convection about a vertical surface embedded in a saturated porous medium subjected to a chemical reaction has been analysed numerically by Postelnicu (2007). Non-Darcian and chemical reaction effects on mixed convective heat and mass transfer past a porous wedge are presented by Kandasamy *et al.* (2008). Rashad and EL-Kabeir (2010) have studied the coupled heat and mass transfer in transient flow by mixed convection past a vertical stretching sheet embedded in a fluid-saturated porous medium in the presence of a chemical reaction effect. The effects of permeability and chemical reaction on heat and mass transfer over an infinite moving permeable plate in a saturated porous medium were reported by Modather *et al.* (2006).

On the other hand, in industrial and chemical engineering processes which involves multi-component fluid, concentrations vary from point to point resulting in mass transfer. Energy flux can be generated not only by temperature gradient but also by concentration gradient as well. The energy flux caused by concentration gradient is called the Dufour effect and the same by temperature gradient is called the Soret effect. These effects are very significant when the temperature and the concentration gradients are high. Li *et al.* (2006) used an implicit finite volume method to investigate thermal-diffusion (Soret) and diffusion-themo (Dufour) effects in a strongly endothermic chemically reacting flow in a porous medium, showing that for low convectional velocity is lower or higher initial temperature of the feeding gas, Soret and Dufour effects have a strong influence on the regime. Partha *et al.* (2006) obtained similarity solutions for double dispersion effects on free convection hydromagnetic heat and mass transfer in a non-Darcy porous medium with Soret and Dufour effects, showing that in both aiding and opposing buoyancies, Dufour and Soret numbers considerably affect the wall mass transfer and heat transfer rates, both of which are also reduced with stronger magnetic field. The thermal-diffusion and diffusion-thermo effects on the

heat and mass transfer characteristics of free convection past a continuously stretching permeable surface in the presence of magnetic field and radiation are studied by Abd El-Aziz (2008) have obtained finite element solutions for Soret and Dufour effects on coupled thermal and species boundary layers in Darcy-Forchheimer porous media. Postelnicu (2004) has discussed the effect of magnetic field, Dufour number and Soret number on combined heat and mass transfer in free convection boundary-layer flow in a Darcian porous medium using a finite-difference procedure.

Finally, the effect of radiation on flow and heat transfer in porous media has become more important in certain application, including waste heat storage in aquifers and gasification of oil shale which is of interest on combined convection since fluid is pumped into porous region. In addition, in the case of gasification, large temperature gradients exist in the neighborhood of the combustion, hence radiation effects may become important. Radiation heat transfer in porous media has also been studied by many researchers. Whitaker (1980) discussed radiative heat transfer in porous media. Chamkha (1997), Chamka and Khanafer (1999) studied solar radiation assisted free convection in the boundary layer adjacent to a vertical flat plate in a uniform porous medium considering amore general Darcy-Forchheimer-Brinkman flow model. Chamkha *et al.* (2001) investigated the effects of thermal radiation on the combined convection flow along an vertical flat plate embedded in a porous medium of variable porosity using Forchheimer-Darcy flow model. EL-Hakim and Rashad (2007) used Rosseland diffusion approximation in studying the effect of radiation on free convection from a vertical cylinder embedded in a fluid-saturated porous medium. Rashad (2008) studied the problem of thermal radiation effect on heat and mass transfer by free convection over a vertical flat plate embedded in a fluid-saturated porous media. The combined heat and mass transfer by non-Darcy natural convection about an impermeable horizontal cylinder in a power law fluid embedded in porous medium under coupled thermal and mass diffusion and thermal radiation effects was studied. EL-Kabeir *et al.* (2008), Chamkha and Ben-Nakhi (2008) considered the simultaneous heat and mass transfer by mixed convection from a permeable vertical plate embedded in a fluid-saturated non-Darcian porous medium in the presence of thermal radiation, and Dufour and Soret effects. Bakier *et al.* (2009) investigated the effect of the thermal radiation on mixed convection along vertical plate in a liquid saturated porous medium in the presence of melting effect. The objective of this paper is to generalize the works of Yih (1999) and Chamkha (2001) and consider simultaneous heat and mass transfer by natural convection over a permeable isothermal truncated cone embedded in a fluid-saturated porous medium in the presence of thermal radiation, chemical reaction, Soret and Dufour effects and using Brinkman-Forchheimer extended Darcy model.

2. Governing equations
Consider steady, laminar, heat and mass transfer by natural convection, boundary-layer flow of an optically dense fluid about a truncated permeable cone with a half angle g embedded in a non-Daracian porous medium in the presence of thermal radiation, chemical reaction, thermal-diffusion and the diffusion-thermo effects as shown in Figure 1. The origin of the coordinate system is placed at the vertex of the full cone where x represents the distance along the cone and y represents the distance normal to the surface of the cone. The cone surface is maintained at a constant temperature T_w and a constant concentration C_w, and the ambient temperature and concentration far away from the surface of the cone T_∞ and C_∞ are assumed to be uniform. For $T_w \succ T_\infty$ and $C_w \succ C_\infty$ an
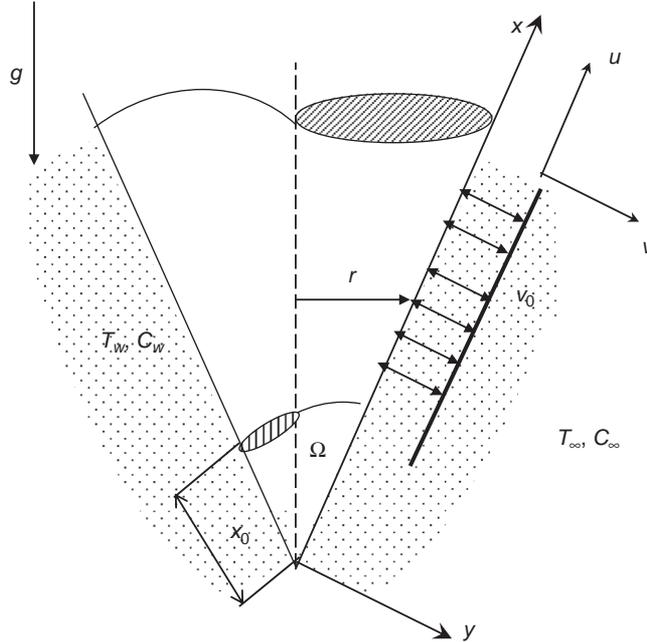
Figure 1.
Flow model and physical
coordinate system

upward flow is induced as a result of the thermal and concentration buoyancy effects. Fluid suction or injection is imposed at the surface. For the flow in the porous medium we adopt the Brinkman-Forchheimer extended Darcy model proposed by Vafai and Tien (1981), including the non-Darcian boundary and inertia effects. All physical properties are assumed constant except the density in the buoyancy force term a first-order homogeneous chemical reaction is assumed to take place in the flow. By invoking all of the boundary layer, Boussineq and Rosseland diffusion approximations the governing equations for this investigation can be written as (see Yih, 1999; Chamkha, 2001):

$$\frac{\partial(ru)}{\partial x} + \frac{\partial(rv)}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = v\frac{\partial^2 u}{\partial y^2} + g\cos\Omega(\beta(T - T_\infty) + \beta^*(C - C_\infty)) - \frac{v}{K}u - K^*u^2 \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{16}{3(a_r + \sigma_s)\rho C_p}\frac{\partial}{\partial y}\left(T^3\frac{\partial T}{\partial y}\right) + \frac{Dk_T}{C_s C_p}\frac{\partial^2 C}{\partial y^2} \tag{3}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} + \frac{Dk_T}{T_m}\frac{\partial^2 T}{\partial y^2} - K_1(C - C_\infty) \tag{4}$$

The boundary conditions for this problem are defined as follows:

$$y = 0 : u = 0, v = -v_0, \qquad T = T_w, \, C = C_w$$
$$y \rightarrow \infty : u \rightarrow 0, T \rightarrow T_\infty, \, C \rightarrow C_\infty \tag{5}$$

where r is the radius of the truncated cone. u, v, T, and C are the x-component of velocity, y-component of velocity, temperature, and concentration, respectively. K and K^* are the permeability of porous medium and inertia coefficient (reflect the Darcian and Forchheimer flows, respectively). ρ, v, C_p, α and D are the fluid density, kinematic viscosity, specific heat at constant pressure, thermal diffusivity, and mass diffusivity, respectively. K_1, Ω, β_T and β_C are the dimensional of chemical reaction, half angle of the truncated cone, thermal expansion coefficient, and concentration expansion coefficient, respectively. g, σ_s, and a_r are the acceleration due to gravity, scattering coefficient, and the Rosseland mean extinction coefficient, respectively. v_0 (>0) is the wall suction velocity. The coefficient of the last term of Equation (3) is sometimes called radiative conductivity.

It is convenient to transform Equations (1)-(4) by using the following non-similarity transformations reported earlier by Yih (1999) and Chamkha (2001):

$$\xi = \frac{x^*}{x_0} = \frac{x - x_0}{x_0}, \, \eta = \frac{y}{x^*}(Gr_{x^*})^{1/4}, \, Gr_{x^*} = g\beta_T(T_w - T_\infty)x^{*3}/v^2$$
$$\psi = rv(Gr_{x^*})^{1/4}f(\xi, \eta), \theta(\xi, \eta) = (T - T_\infty)/(T_w - T_\infty), \, \phi(\xi, \eta)$$
$$= (C - C_\infty)/(C_w - C_\infty) \tag{6}$$

where Gr_x^* is the Grashof number and ψ is the stream function defined as $ru = \partial\psi/\partial y$ and $rv = -\partial\psi/\partial y$, therefore the continuity equation is identically satisfied. In addition the velocities components are:

$$u = \frac{v(Gr_{x^*})^{1/2}}{x^*}f', \, v = -\frac{v(Gr_{x^*})^{1/4}}{x^*}\left[\left(\frac{\xi}{1 + \xi} + \frac{3}{4}\right)f + \xi\frac{\partial f}{\partial\xi} - \frac{1}{4}\eta f'\right] \tag{7}$$

Substituting Equations (6) into Equations (1)-(5) yields the following non-similar equations and boundary conditions:

$$f''' + \left(\frac{\xi}{1 + \xi} + \frac{3}{4}\right)ff'' - \left(\frac{1}{2} + \xi\Gamma\right)f'^{2} - \frac{\xi^{1/2}}{Da}f' + \theta + N\phi$$
$$= \xi\left(f'\frac{\partial f'}{\partial\xi} - f''\frac{\partial f}{\partial\xi}\right) \tag{8}$$

$$\frac{1}{\Pr}\theta'' + \left(\frac{\xi}{1 + \xi} + \frac{3}{4}\right)f\theta' + \frac{4R_d}{3\Pr}\left\{\theta'[(rt - 1)\theta + 1]^3\right\}' + D_f\phi''$$
$$= \xi\left(f'\frac{\partial\theta}{\partial\xi} - \theta'\frac{\partial f}{\partial\xi}\right) \tag{9}$$

$$\frac{1}{Sc}\phi'' + \left(\frac{\xi}{1+\xi} + \frac{3}{4}\right)f\phi' + S_r\theta'' - \gamma\xi^{1/2}\phi = \xi\left(f'\frac{\partial\phi}{\partial\xi} - \phi'\frac{\partial f}{\partial\xi}\right) \qquad (10)$$

The dimensionless boundary conditions become:

$$\eta = 0: \quad f' = 0, \; f_w\xi^{1/4} = \left(\frac{\xi}{1+\xi} + \frac{3}{4}\right)f + \frac{\partial f}{\partial\xi} \; \; \theta = 1 \,, \; \phi = 1$$
$$\eta \to \infty: \quad \eta = 0, \; \theta \to 0, \; \; \phi \to 0 \qquad (11)$$

where a prime denotes partial differentiation with respect to η and:

$$Da = K(Gr_{x_0})^{1/2}/x_0^2, \; \Gamma = K^*x_0, \; N = \beta^*(C_w - C_\infty)/\beta(T_w - T_\infty)$$
$$f_w = v_0 x_0/v(Gr_{x_0})^{1/4}, R_d = \frac{4\sigma T_\infty^3}{[k(a_r + \sigma_s)]}, \;\; rt = T_w/T_\infty, \;\; \gamma = K_1 x_0^2/v(Gr_{x_0})^{1/2}$$
$$\Pr = v/\alpha, \; Sc = v/D, \; Gr_{x_0} = g\beta_T(T_w - T_\infty)x_0^3/v^2$$
$$D_f = \frac{k_T(C_w - C_\infty)D}{C_s C_p(T_w - T_\infty)v}, \;\; S_r = \frac{k_T(T_w - T_\infty)D}{T_m(C_w - C_\infty)v} \qquad (12)$$

are the Darcy number, Forchheimer number, ratio of the buoyancy force due to mass diffusion to the buoyancy force due to the thermal diffusion, mass transfer coefficient, radiation-conduction parameter, surface temperature parameter, dimensionless of chemical reaction parameter, Prandtl number, Schmidt number, Grashof number based on x_0, Dufour number, Soret number, and k is the thermal conductivity.

The local skin-friction coefficient C_f, local Nusselt number Nu_x^* and the local Sherwood number Sh_x^* are important physical properties. These can be defined in dimensionless form below as:

$$C_f = -2(Gr_{x^*})^{-1/4}f''(\xi,0), \; Nu_{x^*} = -\left(1 + \frac{4R_d rt^3}{3}\right)(Gr_{x^*})^{1/4}\theta'(\xi,0)$$
$$Sh_{x^*} = -(Gr_{x^*})^{1/4}\phi'(\xi,0) \qquad (13)$$

3. Numerical method

The problem represented by Equations (8)-(10) is non-linear and has no closed-form solution. Therefore, it must be solved numerically. The implicit, tri-diagonal, finite-difference method discussed by Blottner (1970) has proven to be adequate for the solution of boundary-layer equations accurately. For this reason, it is adopted in this work. All first-order derivatives with respect to ξ are replaced by two-point backward difference quotients while the derivatives with respect to η are discretized using three-point central-difference quotients and, as a consequence, a set of algebraic equations results at each line of constant ξ. These algebraic equations are then solved by the well-known Thomas algorithm Blottner, 1970 with iteration to deal with the non-linearities of the problem. When the solution at a specific line of constant ξ is

obtained, the same solution procedure is used for the next line of constant ξ. This marching process continues until the desired value of ξ is reached. At each line of constant ξ, when f′ is known, the trapezoidal rule is used to solve for f. The convergence criterion employed was based on the relative difference between the current and the previous iterations. When this difference reached 10^{-5} the solution was assumed converged and the iteration procedure was terminated. Constant step sizes in the ξ direction were used whereas variable step sizes in the η direction were utilized in order to accommodate the sharp changes in the dependent variables especially in the immediate vicinity of the truncated cone surface. The (ξ,η) computational domain consisted of 101 and 196 points, respectively. The initial step sizes in $\Delta\xi_1$ and $\Delta\eta_1$ were taken to be equal to 10^{-2} and 10^{-3}, respectively, and the growth factor for the η direction was taken to be 1.0375. This gave $\xi_\infty = 1$ and $\eta_\infty = 35$. These values were found to give accurate grid-independent results as verified by the comparisons mentioned below.

In order to access the accuracy of the numerical results, various comparisons with previously published work for the cases of a vertical plate $(\xi = 0)$ and a full cone $(\xi = \infty)$ were performed. These comparisons are presented in Tables I and II . It is obvious from these tables that excellent agreement between the results exist. These favourable comparisons lend confidence in the graphical results to be reported in Section 4.

4. Results and discussions
In this section, a representative set of numerical results for the velocity, temperature and concentration profiles as well as the local skin-friction coefficient, local Nusselt

Pr	f″(0,0)			−θ′(0,0)		
	Yih (1999)	Chamkha (2001)	Present results	Yih (1999)	Chamkha (2001)	Present results
0.0001	1.4998	1.4997	1.4997	0.0060	0.0059	0.0059
0.001	1.4728	1.4727	1.4727	0.0189	0.0188	0.0188
0.01	1.3968	1.3965	1.3965	0.0570	0.0574	0.0574
0.1	1.2144	1.2151	1.2151	0.1629	0.1630	0.1630
1	0.9084	0.9081	0.9081	0.4012	0.4015	0.4015
10	0.5927	0.5927	0.5927	0.8266	0.8274	0.8274
100	0.3559	0.3558	0.3558	1.5493	1.5503	1.5503
1000	0.2049	0.2049	0.2049	2.8035	2.8044	2.8044
10,000	0.1161	0.1161	0.1161	5.0127	5.0131	5.0131

Table I.
Comparison of values of f″(0,0) and −θ′(0,0) for various values of Pr with Da = Γ = 0, f_w = 0, N = 0 and R_d = 0

Pr	f″(∞,0)			−θ′(∞,0)		
	Yih (1999)	Chamkha (2001)	Present results	Yih (1999)	Chamkha (2001)	Present results
0.0001	1.6006	1.6005	1.6005	0.0079	0.0078	0.0078
0.001	1.5135	1.5133	1.5133	0.0246	0.0245	0.0245
0.01	1.3551	1.3549	1.3549	0.0749	0.0751	0.0751
0.1	1.0960	1.0962	1.0962	0.2116	0.2116	0.2116
1	0.7699	0.7697	0.7697	0.5109	0.5111	0.5111
10	0.4877	0.4877	0.4877	1.0339	1.0342	1.0342
100	0.2896	0.2895	0.2895	1.9226	1.9230	1.9230
1000	0.1661	0.1661	0.1661	3.4696	3.4700	3.4700
10,000	0.0940	0.0940	0.0940	6.1984	6.1988	6.1988

Table II.
Comparison of values of f″(∞,0) and -θ′(∞,0) for various values of Pr Da = Γ = 0, f_w = 0, N = 0 and R_d = 0

number and the local Sherwood number is presented graphically in Figures 2-19. These results illustrate the effects of the Darcy number Da, Forchheimer number Γ, radiation-conduction parameter R_d, dimensionless of chemical reaction parameter γ, Dufour number D_f, Soret number S_r and the suction or injection parameter f_w on the solutions. Throughout the calculations, These conditions are intended for water ($Pr = 7.0$) polluted by the species Benzene ($Sc = 1.60$) that represents a diffusion chemical species of most common interest in water. The values of the corresponding buoyancy force parameter (ratio of the buoyancy force due to mass diffusion to the buoyancy force due to the thermal diffusion) N takes the value 1.0 for low concentration and the values of Dufour number and Soret number are chosen in such a way that their product is constant provided that the mean temperature T_m is kept constant as well.
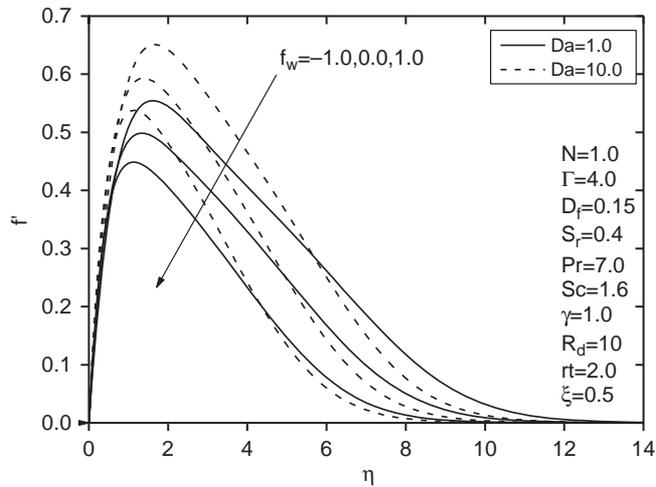
Figure 2.
Effects of Darcy number Da and the suction/injection parameter f_w on the velocity profiles
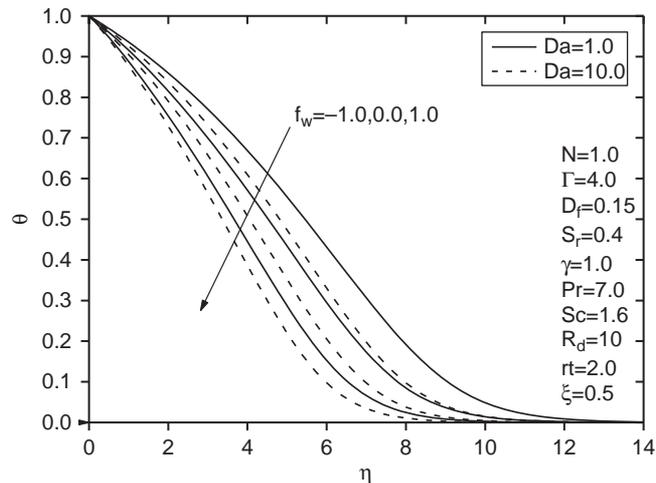
Figure 3.
Effects of Darcy number Da and the suction/injection parameter f_w on the temperature profiles
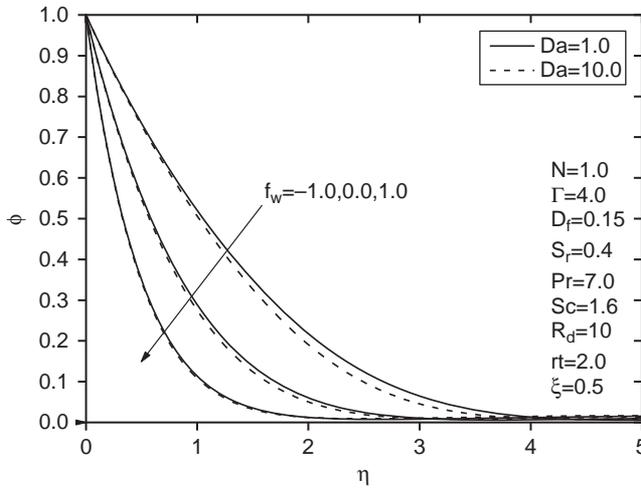
Figure 4.
Effects of Darcy number
Da and the suction/
injection parameter f_w on
the concentration profiles
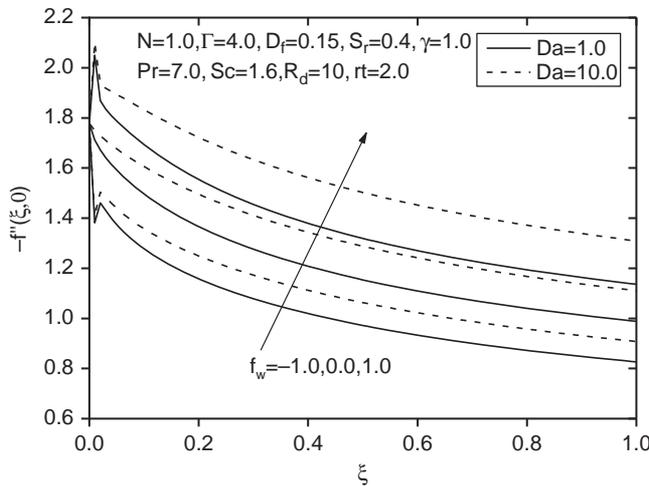
Figure 5.
Effects of Darcy number
Da and the suction/
injection parameter f_w on
the local skin-friction
coefficient

Figures 2-4 present typical profiles for the velocity along the cone f′, temperature θ and concentration φ for various values of the suction/injection parameter f_w and two values of the Darcy number (permeability of porous medium) Da, respectively. It is noted that the presence of a porous medium in the flow presents resistance to flow. Thus, the resulting resistive force tends to slow down the motion of the fluid along the cone surface and causes increases in its temperature and concentration. This is depicted by the increases in the values of f′ and decreases in the values of both θ and φ as the Darcy number *Da* increases shown in Figures 2-4. These behaviours in f′, θ and φ are accompanied by decreases in all of the flow, thermal and concentration boundary layers as Darcy number increases. Further, imposition of fluid suction ($f_w > 0$) at the wall has a tendency to reduce all of the flow, thermal and concentration boundary layers. This causes all of the velocity, temperature and concentration to decrease at every point far from the surface. On the other hand, injection of fluid ($f_w < 0$) through
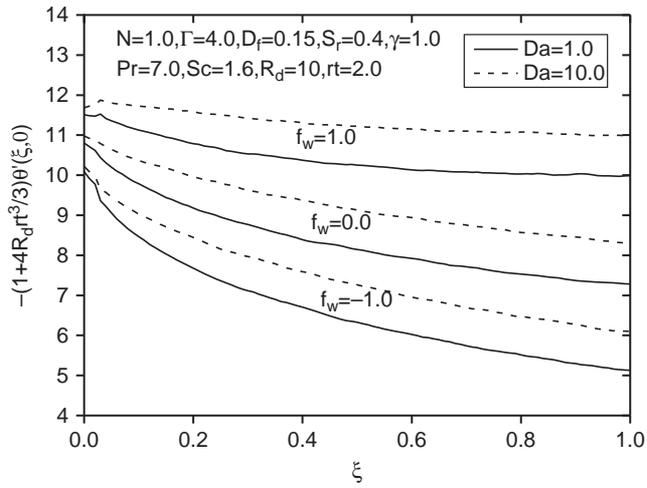
Figure 6.
Effects of Darcy number
Da and the suction/
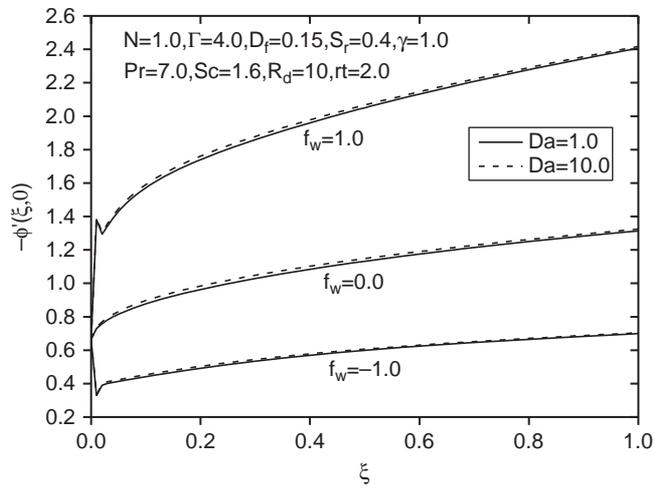injection parameter f_w on
the local Nusselt number

Figure 7.
Effects of Darcy number
Da and the suction/
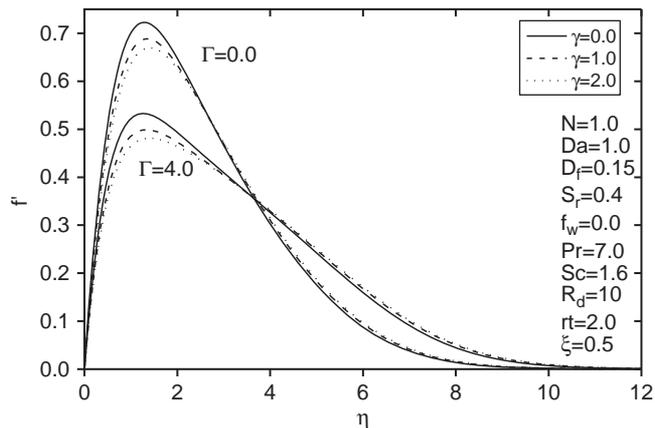injection parameter f_w
on the local Sherwood
number

Figure 8.
Effects of Forchheimer
number Γ and the
dimensionless of chemical
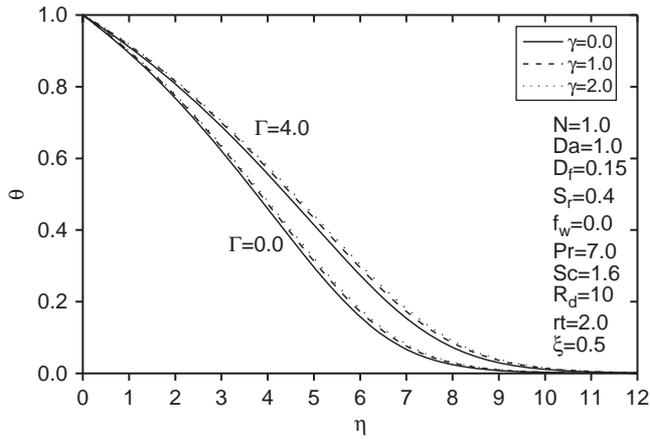reaction parameter γ on
the velocity profiles

Figure 9.
Effects of Forchheimer
number Γ and the
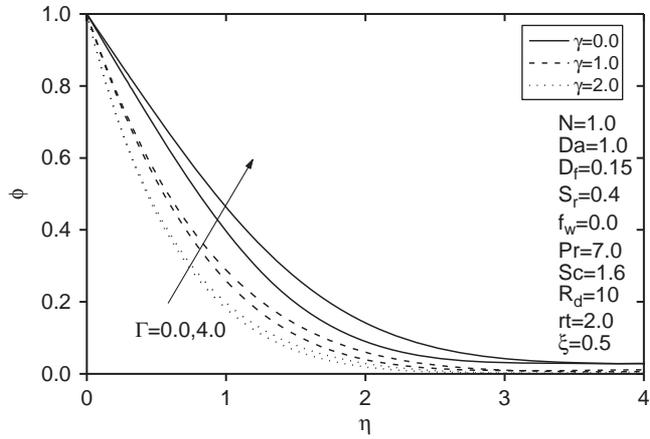dimensionless of chemical
reaction parameter γ on
the temperature profiles

Figure 10.
Effects of Forchheimer
number Γ and the
dimensionless of chemical
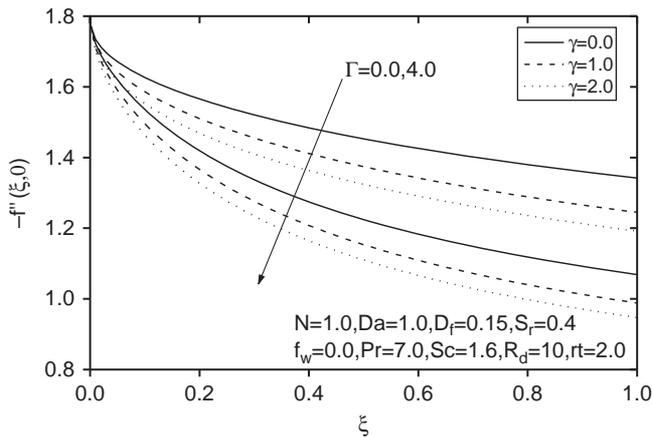reaction parameter γ on
the concentration profiles

Figure 11.
Effects of Forchheimer
number Γ and the
dimensionless of chemical
reaction parameter γ on
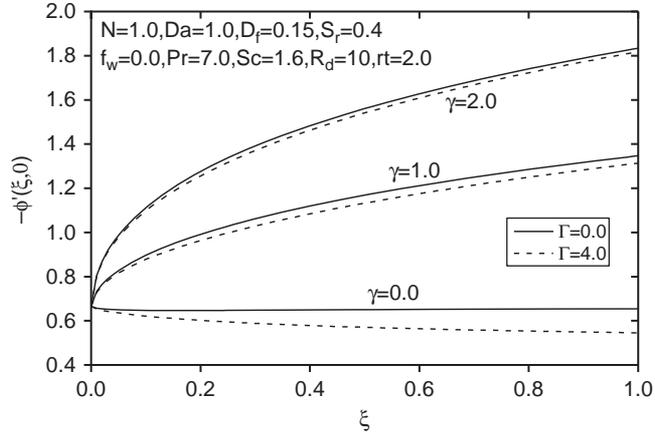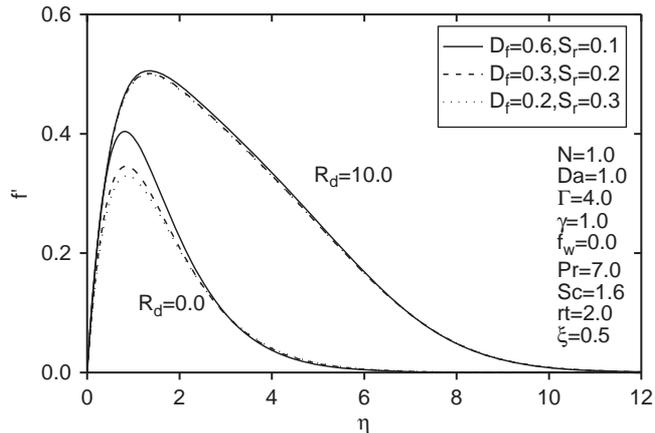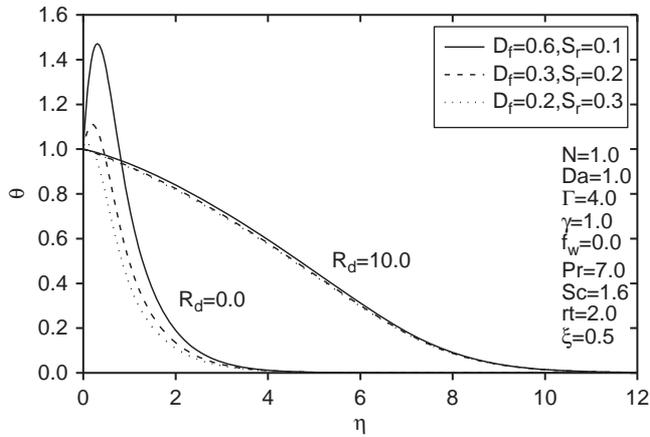the local skin-friction
coefficient

Figure 15.
Effects of the radiation
parameter R_d, Soret
number S_r and Dufour
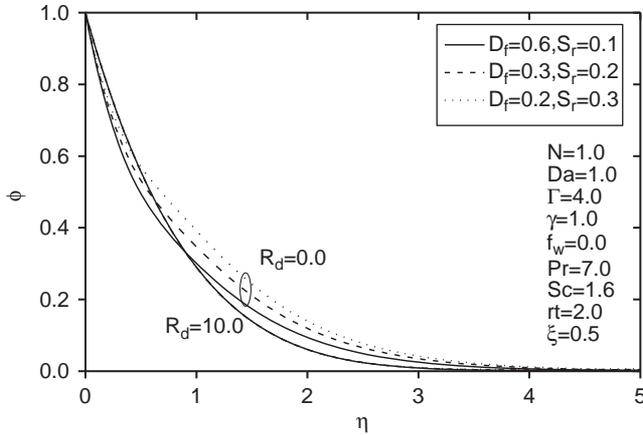number D_f on the
temperature profiles

Figure 16.
Effects of the radiation
parameter R_d, Soret
number S_r and Dufour
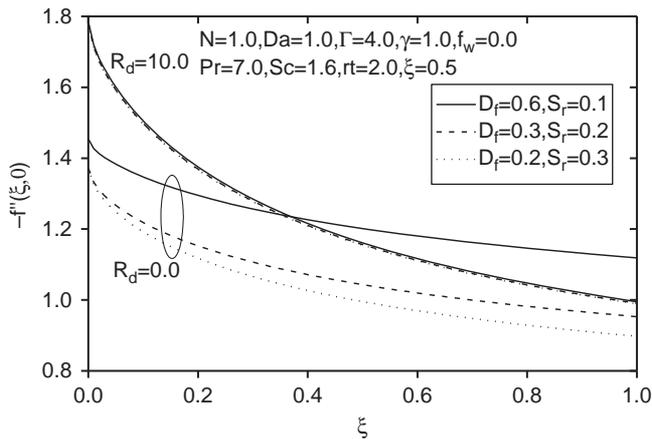number D_f on the
concentration profiles

Figure 17.
Effects of the radiation
parameter R_d, Soret
number S_r and Dufour
number D_f on the local
skin-friction coefficient

the surface produces the opposite effect, namely increases in all of the velocity, temperature and concentration. These behaviours are clearly shown in Figures 2-4.

The effects of both the suction/injection parameter f_w and two values of the Darcy number Da on the development of the local skin-friction coefficient (or $f''(\xi,0)$), the local Nusselt number (or $-\theta'(\xi,0)$) and the local Sherwood number (or $-\phi'(\xi,0)$) are displayed in Figures 5-7, respectively. It was seen from Figures 2-4 that the wall slope of the velocity profile increases while the slopes of both the temperature and concentration profiles decrease as Da increases. This produces enhancing in all of C_f, $Nu_x \cdot$ and $Sh_x \cdot$ as Da increases as depicted in Figures 5-7. On other hand, an increase in the suction/injection parameter f_w produces increases in the local skin-friction coefficient, the local Nusselt and Sherwood numbers. This is consistent with the results reported earlier by Chamkha (2001).

Figures 8-10 display the effects of the chemical reaction parameter γ and two values of the Forchheimer number Γ, on the velocity, temperature and concentration profiles, respectively. It is obvious that, the velocity decreases as the Forchheimer number (inertial parameter) increases. The reason for this behaviour is that the inertia of the

porous medium provides an additional resistance to the fluid flow mechanism, which causes the fluid to move at a retarded rate with reduced temperature. Thus, it is observed that the temperature and concentration of the fluid are almost affected a decrease with increase of the Forchheimer number. Also, It is seen that the concentration of the fluid decrease with increase of chemical reaction parameter, while the velocity and temperature profiles increase with increase of chemical reaction parameter. It is evident to note that the increase of chemical reaction significantly alter the concentration boundary layer thickness but not momentum and thermal boundary layers.

Figures 11-13 depict the variations in the values of C_f, Nu_x and Sh_x as a result of changing the values of Forchheimer number Γ and chemical reaction parameter γ, respectively. It is found that the values of the local skin-friction coefficient, local Nusselt and Sherwood numbers decrease due to increase in the value of Γ, i.e. the increase in Γ implies that the medium is offering more resistance to the fluid motion. Hence, the fluid motion is decelerated which results hence the inertia effect tends to in thicker momentum, thermal and concentration boundary layers. Further, it can be seen that as γ increases, the local Sherwood number increases, while the opposite effect is found for both of the local skin-friction coefficient and the local Nusselt number. This is because as γ increases, the concentration difference between the cone surface and the fluid decreases and so the rate of mass transfer at the cone surface must increase, while both of the skin-friction coefficient and the rate of heat transfer decrease as a result of the decrease in the flow velocity and fluid temperature, respectively.

Figures 14-16 present representative velocity, temperature and concentration profiles for various values of Dufour number D_f, Soret number S_r and two values of the the radiation parameter R_d, respectively. As shown, the concentration of the fluid are increasing with increasing Sr (or decreasing D_f), but the velocity and temperature decrease as Sr increases (or D_f decreases). This behaviour is a direct consequence of the Soret effect, which produces a mass flux from lower to higher solute concentration driven by the temperature gradient. Also, when D_f is high enough, the thermal and the solutal buoyancy forces combine their actions to enhance convection velocity, which leads to increase the velocity of the fluid. Consistent with the behaviour reported by Yih (1999) and Chamkha (2001), increasing the value of R_d results in increases in both the velocity, and temperature distribution and the maximum velocity tends to move away from the surface. However, the concentration distribution tends to decrease as a result of increasing the radiation effect as observed from Figure 16.

The effects of Dufour number D_f, Soret number S_r and R_d on the development of the local skin-friction coefficient (or C_f), the local Nusselt number (or Nu_x) and the local Sherwood number (or Sh_x) are displayed in Figures 17-19, respectively. We can observe that as the Dufour number D_f increases (Sr decreases), both of the skin-friction coefficient and the Sherwood number enhance, while the Nusselt number reduces. This is because either increase in temperature difference or decrease in concentration difference leads to an increase in the value of D_f resulting trends similar to the above observation. Similarly, either an increase in concentration difference or a decrease in temperature difference leads to an increase in the value of the Soret number Sr. Therefore, increasing the parameter Sr causes decreases in the skin-friction coefficient and the local Sherwood number while it produces increases in the local Nusselt number. It is noted that the changes in the local skin-friction coefficient and Sherwood number as a result of varying D_f for ($R_d = 0$) in the absence of thermal radiation are much more significant than those obtained for (($R_d = 10$) in the presence of the thermal radiation. On other hand, it should be mentioned that, the skin-friction coefficient and

the local Nusselt number increases with increasing in the value of R_d whereas the opposite effect with the local Sherwood number. The reason for this trend can be explained as follows, higher values of R_d implies higher values of wall temperature, so as the presence of thermal radiation works as a heat source and so the quantity of heat added to the fluid increases, and accelerated the motion of the fluid along the cone surface. Consequently, both of the velocity, temperature and concentration gradients and hence all of these physical parameters increases.

5. Conclusions
In this study, the effects of thermal radiation and chemical reaction on heat and mass transfer by non-Darcy natural convection boundary-layer flow around a permeable truncated cone embedded in porous media using Brinkman-Forchheimer extended Darcy model in the presence of Dufour and Soret effects were considered. A set of non-similar governing differential equations was obtained and solved numerically by an implicit finite-difference methodology. Comparisons with previously published work on various special cases of the general problem were performed and the results were found to be in excellent agreement. A representative set of numerical results for the velocity, temperature and concentration profiles as well as the local skin-friction coefficient, local Nusselt number and the local Sherwood number was presented graphically and discussed. It was found that, in general, all of the local skin-friction coefficient, local Nusselt number and the local Sherwood number enhanced as the Darcy number thermal radiation parameter were increased. Also, while all of these physical parameters decreased with the distance along the cone surface in the presence of the inertial parameter (Forchheimer number), they increased with it in the absence of the inertial parameter. It was also found that owing the presence of chemical reaction effects, both of the local skin-friction coefficient and local Nusselt number decreased wile the local Sherwood number increased. Also, all of these physical parameters were found to increase as the suction/injection parameter was increased. However, both of the local skin-friction coefficient and the local Sherwood number were enhanced, whereas local Nusselt number reduced with an increase of the Dufour number and the decrease in the Soret number.

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