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UNSTEADY MHD FREE CONVECTION FLOW PAST AN EXPONENTIALLY ACCELERATED VERTICAL PLATE WITH MASS TRANSFER, CHEMICAL REACTION AND THERMAL RADIATION

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ABSTRACT

This paper is concerned with the study of an unsteady, MHD free convective boundary layer flow of a viscous, incompressible and electrically conducting, chemically reacting fluid over an exponentially accelerated infinite vertical plate embedded in a porous medium in the presence of thermal diffusion, radiation and temperature dependent heat source or sink. The fluid considered is a gray, absorbing/emitting radiation but non scattering medium. The dimensionless governing equations for this investigation are solved analytically using the Laplace transform technique. Numerical evaluation of the analytical results is performed and graphical results for the velocity, temperature and concentration profiles within the boundary layer are presented graphically and discussed. Also, the expressions for the skin-friction coefficient, Nusselt number and the Sherwood number have been derived and plotted for different values of the governing parameters.

Keywords: MHD; thermal radiation, chemical reaction, thermal diffusion, heat source or sink, exponentially accelerated plate

NOMENCLATURE

u^*, v^*	Velocity components in x and y directions [ms^{-1}]
u	dimensionless velocity [-]

y^*	Coordinate axis normal to the plate [m]
y	Dimensionless Coordinate axis normal to the plate [-]
C^*	Concentration in the Fluid [kmolm^{-3}]
C_w^*	Concentration at the plate [kmolm-3]
C_∞^*	Concentration far away from the plate [kmolm-3]
C	Dimensionless concentration [-]
C_p	Specific heat at constant pressure [JKg^{-1}K$^{-1]}$
T_w^*	Fluid temperature at the plate [K]
T^*	: Fluid temperature near the plate [K]
T_∞^*	Fluid temperature far away from the plate [K]
K_C	Dimensionless rate of chemical reaction [-]
K_c^*	Rate of chemical reaction [-]
G_m	Modified Grashof number [-]
G_r	Thermal Grashof number [-]
g	Acceleration due to gravity [ms^{-2}]
a^*	Absorption coefficient [-]
a	Accelerated parameter [m]
A	Constant [-]
B_0	External magnetic field [Tesla]
H	Heat absorption Parameter [-]
D	Chemical molecular diffusivity [m^{2}s^{-1}]
k	Dimensionless permeability coefficient of porous medium [-]
k^*	Permeability of porous medium [m^2]
Nu	Nusselt number [-]
P_r	Prandtl number [-]
S_c	Schmidt number [-]
F	Radiation parameter [-]
M	Magnetic parameter [-]
Sh	Sherwood number [-]
q_r	Radiative heat flux in the y direction[-]
t^*	Time [s]
t	Dimensionless time [-]
S_0	Soret number [-]

Greek Symbols

κ	Thermal conductivity of the fluid [Wm^{-1}k^{-1}]
ν	Kinematic viscosity [m^{2}s^{-1}]

σ	Electrical conductivity [sm^{-1}]
μ	Dynamic viscosity [$kgm^{-1}s^{-1}$]
θ	Dimensionless temperature [-]
β_T	Thermal expansion coefficient [K^{-1}]
α	Thermal diffusivity [m^2K^{-1}]
β_c	Concentration expansion coefficient [K^{-1}]
ρ	fluid density [kgm^{-3}]
τ	Skin friction coefficient [-]
erf	Error function [-]
erfc	Complementary error function [-]

1. INTRODUCTION

The study of magneto-hydrodynamics with mass and heat transfer in the presence of radiation and diffusion has attracted the attention of a large number of scholars due to diverse applications in astrophysics and geophysics; it is applied to study the stellar and solar structures, radio-wave propagation through the ionosphere, etc. In engineering, we find its applications like in MHD pumps, MHD bearings, etc. The phenomenon of mass transfer is also very common in the theory of stellar structures and observable effects are detectable on the solar surface. In free convection flow, the study of the effects of magnetic fields plays a major role in liquid metals, electrolytes and ionized gases. In power engineering and thermal physics, hydromagnetic problems with mass transfer have enormous applications. Radiative flows are encountered in many industrial and environmental processes such as heating and cooling chambers, fossil fuel combustion, energy processes, evaporation from large open water reservoirs, astrophysical flows, solar power technology and space vehicle re-entry.

Gupta et al. [1] studied the effects of viscous dissipation on free convection flow past a linearly accelerated vertical plate in the presence of viscous dissipative heat using the perturbation method. Kafousias and Raptis [2] extended this problem to include mass transfer effects subjected to variable suction or injection. Free convection effects on flow past an exponentially accelerated vertical plate was investigated by Singh and Kumar [3]. Soundalgekar et al. [4] considered mass transfer effects on the flow past an impulsively started infinite vertical plate with variable temperature and constant heat flux. The skin friction for accelerated vertical plate has been studied analytically by Hossain and Shayo [5]. Jha et al. [6] analyzed mass transfer effects on exponentially accelerated infinite vertical plate with constant heat flux and uniform mass diffusion.

Muthucumaraswamy et al. [7] studied mass transfer effects on exponentially accelerated isothermal vertical plate. Radiation and mass transfer effects on MHD free convection flow past an exponentially accelerated vertical plate with variable temperature was investigated by Rajesh and Varma [8]. Muthucumaraswamy and Muralidharan [9] studied thermal radiation effects on linearly accelerated vertical plate with variable temperature and uniform mass flux. Again, Rajesh and Varma [10] studied thermal diffusion and radiation effects on MHD flow past an impulsively started infinite vertical plate with variable temperature and mass diffusion. The governing equations are solved by using the Laplace transform technique. Rajput and Sahu [11] investigated the effects of rotation and magnetic field on the flow past an exponentially accelerated vertical plate with constant temperature. Muthucumaraswamy and Visalakshi [12] studied radiative flow past an exponentially accelerated vertical plate with variable temperature and mass diffusion. Recently, Rajput and Kumar [13] investigated radiation effects on MHD flow past an impulsively started vertical plate with variable heat and mass transfer. Chambre and Young [14] reported on the diffusion of chemically reactive species in a laminar flow. Das et al. [15] have studied the effect of homogeneous first-order chemical reaction on the flow past impulsively started vertical plate with uniform heat flux and mass transfer. Again, the mass transfer effects on moving isothermal vertical plate in the presence of chemical reaction was studied by Das et al. [16]. Recently, Kumar and Varma [17] investigated thermal diffusion and radiation effects on unsteady MHD flow past an impulsively started exponentially accelerated vertical plate with variable temperature and variable mass diffusion. Kim [18] investigated unsteady MHD convective heat transfer past a semi-infinite vertical porous moving plate with variable suction. Chamkha and Khaled [19] considered hydromagnetic combined heat and mass transfer by natural convection from a permeable surface embedded in a fluid-saturated porous medium. Chamkha [20] studied the thermal radiation and buoyancy effects on hydromagnetic flow over an accelerating permeable surface with heat source or sink. Kandasamy et al. [21] studied the effects of chemical reaction, heat and mass transfer on MHD flow over a vertical stretching surface with heat source and thermal stratification effects. Srinivas & Muthuraj [22] have investigated MHD flow with slip effects and temperature-dependent heat source in a vertical wavy porous space. Raju et al. [23-27] studied Soret effects due to natural convection between heated inclined plates with magnetic field and the other cases in the presence of radiation and chemical reaction.

The present investigation is aimed at analyzing the effects of thermal diffusion and radiation on unsteady MHD flow past an impulsively started infinite accelerated vertical plate with variable temperature and mass diffusion in the presence of homogenous chemical reaction under the influence of transverse applied magnetic field. The governing equations are solved by using the Laplace transform technique and the solutions are expressed in terms of exponential and complementary error functions.

2. Problem Formulation

We have considered the unsteady flow of an incompressible and electrically-conducting viscous fluid past an infinite vertical plate with variable temperature embedded in a porous medium in the presence of chemical reaction and heat absorption. A magnetic field of uniform strength B_0 is applied transversely to the plate. The induced magnetic field is neglected as the magnetic Reynolds number of the flow is taken to be very small. The viscous dissipation is assumed to be negligible. The flow is assumed to be in the x^*-direction which is taken along the vertical plate in the upward direction. The y^* -axis is taken to be normal to the plate. Initially, the plate and the fluid are at the same temperature T_∞^* in the stationary condition with concentration level C_∞^* at all points. At time $t^* > 0$, the plate is exponentially accelerated with a velocity $u = u_0 e^{a^* t^*}$ in its own plane and the plate temperature is raised linearly with time t and the level of concentration near the plate is raised to C_w^*. The fluid considered here is a gray, absorbing/emitting radiation but a non-scattering medium. It is assumed that there is no applied voltage which implies the absence of an electrical field. The transversely applied magnetic field and magnetic Reynolds number are assumed to be very small so that the induced magnetic field and the Hall Effect are negligible. The viscous dissipation and Joule heating terms are neglected. As the plate is infinite in extent, the physical variables are functions of y′ and t′ only.

Using the above assumptions and the usual Bossinesq's approximation, the unsteady flow is governed by the following equations:

Momentum equation:

$$\frac{\partial u^*}{\partial t^*} = \nu \frac{\partial^2 u^*}{\partial y^{*2}} + g\beta_T (T^* - T^*_\infty) + g\beta_C (C^* - C^*_\infty) - \frac{\sigma B_0^2}{\rho} u^* - \frac{\vartheta}{k^*} u^* \tag{1}$$

Energy equation:

$$\frac{\partial T^*}{\partial t^*} = \frac{\kappa}{\rho C_P} \frac{\partial^2 T^*}{\partial y^{*2}} - \frac{1}{\rho C_P} \frac{\partial q_r}{\partial y^*} - \frac{Q}{\rho C_P}\left(T^* - T^*_\infty\right) \tag{2}$$

Concentration equation**:**

$$\frac{\partial C^*}{\partial t^*} = D \frac{\partial^2 C^*}{\partial y^{*2}} + D_1 \left(\frac{\partial^2 T^*}{\partial y^{*2}} \right) - K_c^* (C^* - C^*_\infty) \tag{3}$$

The initial and boundary conditions are

$$\begin{aligned} & u^* = 0, T^* = T^*_\infty, C^* = C^*_\infty \qquad for\ \ all\ \ y^*, t^* \le 0 \\ & u^* = u_0\, \mathrm{e}^{a^* t^*}, T^* = T^*_\infty + (T^*_w - T^*_\infty) A t^*, C^* = C^*_w \quad at\ \ y^* = 0, t^* > 0 \\ & u^* = 0, T^* = T^*_\infty, C^* = C^*_\infty \quad as\ \ y \to \infty, t^* > 0 \end{aligned} \tag{4}$$

To reduce the above equations into non-dimensional form, we introduce the following dimensionaless variables and parameters:

$$y = \frac{U_0 y^*}{\nu},\ u = \frac{u^*}{U_0},\ \theta = \frac{T^* - T^*_\infty}{T_W^* - T^*_\infty},\ t = \frac{t^* U_0^{\ 2}}{\nu}, k = \frac{U_0^2 k^*}{\nu^2},$$

$$F = \frac{16\sigma a^* \nu^2 T_\infty^{*3}}{\kappa U_0^2},\ \ C = \frac{C^* - C^*_\infty}{C_W^* - C^*_\infty},\ \ \mathrm{P_r} = \frac{\mu c_p}{\kappa}, M = \frac{\sigma B_0^2 \nu}{\rho U_0^2},$$

$$G_r = \frac{vg\beta_T(T_W^* - T_\infty^*)}{U_0^3},\ G_m = \frac{vg\beta_c(C_W^* - C_0^*)}{U_0^3},\ K_C = \frac{vK_C^{\ *}}{U_0^2},$$

$$S_c = \frac{v}{D},\ \ s_0 = \frac{D_1(T_W^{\ *} - T_\infty^*)}{v(C_W^{\ *} - C_\infty^*)},\ \ H = \frac{Qv^2}{\kappa U_0^2}, a = \frac{a^* v}{U_0^2} \tag{5}$$

where $A = \dfrac{u_0^2}{v}$, . The local radiant for the case of an optically thin gray gas is expressed by

$$\frac{\partial q_r}{\partial y^*} = -4a^*\sigma(T_\infty^{*4} - T^{*4}) \tag{6}$$

It is assumed that the temperature differences with in the flow are sufficiently small that T^{*4} may be expressed as a linear function of the temeperature. This is accomplished by expanding T^{*4} in a Taylor series about T^*_∞ and neglecting the higher-order terms, thus

$$T^{*4} = 4T_\infty^{*3} T^* - 3T_\infty^{*4} \tag{7}$$

By using equation (6) and (7), equation (2) reduces to

$$\rho C_P \frac{\partial T^*}{\partial t^*} = K\frac{\partial^2 T^*}{\partial y^{*2}} + 16a^*\sigma T_\infty^{*3}(T_\infty^* - T^*) - Q\left(T^* - T_\infty^*\right) \tag{8}$$

With the help of equation (8), the governing equations (1) to (3) are reduced to

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} - G_r\theta - G_m C - M_1 u \tag{9}$$

where $M_1 = M + \dfrac{1}{K}$

$$\mathrm{P_r}\frac{\partial \theta}{\partial t} = \frac{\partial^2\theta}{\partial y^2} - F_1\theta \tag{10}$$

where $F_1 = F + H$

$$S_C \frac{\partial C}{\partial t} = \frac{\partial^2 C}{\partial y^2} + S_0 S_c \frac{\partial^2 \theta}{\partial y^2} - K_c S_C C \tag{11}$$

The corresponding initial and boundary conditions in non-dimensional form are

$$u = 0, \theta = 0, C = 0 \text{ for all } y^* \geq 0, t^* \leq 0$$

$$u = e^{at}, \theta = 1, C = 1 \text{ at } y = 0, t^* > 0$$

$$u \to 0, \theta \to 0, C \to 0 \quad \text{as } y \to \infty, t^* > 0 \tag{12}$$

All the physical parameters are defined in the Nomenclature section.

3. Problem Solution

We solve the governing equations in exact form by the Laplace transform technique. The Laplace transforms of equations (9) to (11) and the boundary conditions (12) are given by

$$\frac{d^2\overline{\theta}}{dy^2} - (sP_r + F_1)\overline{\theta} = 0 \tag{13}$$

$$\frac{d^2\overline{C}}{dy^2} - (sS_c + K_c S_c)\overline{C} + S_0 S_c (sP_r + F_1)\overline{\theta} = 0 \tag{14}$$

$$\frac{d^2\overline{u}}{dy^2} - (M_1 + s)\overline{u} = -G_r \overline{\theta} - G_m \overline{C} \tag{15}$$

The corresponding boundary conditions are

$$\overline{u} = \frac{1}{s-a}, \ \overline{\theta} = \frac{1}{s^2}, \ \overline{C} = \frac{1}{s} \qquad at \ y = 0, \ t > 0$$

$$\overline{u} = 0, \ \overline{\theta} = 0, \ \overline{C} = 0 \qquad at \ y \to \infty, t > 0 \tag{16}$$

Solving the equations (13)-(15) with the help of equation (16), we get

$$\overline{\theta}(y,s) = \left(\frac{1}{s^2}\right) e^{-y\sqrt{s\,\mathrm{P_r} + F_1}} \tag{17}$$

$$\overline{C}(s,y) = \left[\begin{array}{l} \left[\frac{1}{s} - \frac{(b_2+b_4)}{s} - \frac{(b_2+b_3)}{s+a_2} - \frac{b_5}{s^2}\right] e^{-y\sqrt{sS_c + sK_c}} + \\ \left[\frac{b_2+b_4}{s} + \frac{b_2+b_3}{s+a_2}\right] e^{-y\sqrt{sP_r+F_1}} \end{array}\right] \tag{18}$$

$$\begin{aligned} \overline{u}(s,y) = & \left[\frac{1}{s-a} - \frac{z_{29}}{s} - \frac{z_{30}}{s^2} - \frac{z_{31}}{s+a_2} - \frac{z_{11}}{s+z_2} - \frac{z_{28}}{s+z_{19}}\right] e^{-y\sqrt{s+M_1}} \\ & + \left[\frac{z_{10}}{s} + \frac{z_8}{s^2} + \frac{z_{11}}{s+z_2} + \frac{z_4}{s+a_2} - \frac{z_{11}}{s+z_2} - \frac{z_{28}}{s+z_{19}}\right] e^{-y\sqrt{sS_c+S_cK_c}} \\ & + \left[\frac{z_{26}}{s} - \frac{z_{27}}{s^2} - \frac{z_{28}}{s+z_{19}} + \frac{z_{18}}{s+z_2}\right] e^{-y\sqrt{sP_r+F_1}} \end{aligned} \tag{19}$$

where ‘s’ is the Laplace transformation parameter. On taking the inverse Laplace transform of equations (17), (18) and (19), we get the general solution of the present problem for the temperature $\theta(y,t)$, the velocity $u(y,t)$ and the concentration $C(y,t)$ for $t > 0$ which are given by

$$\begin{aligned} u(y,t) = & \frac{e^{at}}{2}\left[e^{-y\sqrt{a+M_1}} erfc\left(\eta - \sqrt{(M_1+a)t}\right) + e^{y\sqrt{a+M_1}} erfc\left(\eta + \sqrt{(M_1+a)t}\right)\right] - \\ & \frac{z_{29}}{2}\left[e^{-y\sqrt{M_1}} erfc\left(\eta - \sqrt{(M_1)t}\right) + e^{y\sqrt{M_1}} erfc\left(\eta + \sqrt{(M_1)t}\right)\right] - \\ & z_{30}\left[\begin{array}{l} \left(\frac{t}{2} - \frac{y}{4\sqrt{M_1}}\right)\left(e^{-y\sqrt{M_1}} erfc\left(\eta - \sqrt{(M_1)t}\right)\right) + \\ \left(\frac{t}{2} + \frac{y}{4\sqrt{M_1}}\right)\left(e^{y\sqrt{M_1}} erfc\left(\eta + \sqrt{(M_1)t}\right)\right) \end{array}\right] - \\ & \frac{z_{31}e^{-a_2t}}{2}\left[\begin{array}{l} e^{-y\sqrt{M_1-a_2}} erfc\left(\eta - \sqrt{(M_1-a_2)t}\right) + \\ e^{y\sqrt{M_1-a_2}} erfc\left(\eta + \sqrt{(M_1-a_2)t}\right) \end{array}\right] - \frac{z_{11}e^{-z_2t}}{2}\left[\begin{array}{l} e^{-y\sqrt{M_1-z_2}} erfc\left(\eta - \sqrt{(M_1-z_2)t}\right) + \\ e^{y\sqrt{M_1-z_2}} erfc\left(\eta + \sqrt{(M_1-z_2)t}\right) \end{array}\right] - \\ & \frac{z_{28}e^{-z_{13}t}}{2}\left[\begin{array}{l} e^{-y\sqrt{M_1-z_{13}}} erfc\left(\eta - \sqrt{(M_1-z_{13})t}\right) + \\ e^{y\sqrt{M_1-z_{13}}} erfc\left(\eta + \sqrt{(M_1-z_{13})t}\right) \end{array}\right] + \frac{z_{10}}{2}\left[\begin{array}{l} e^{-y\sqrt{S_cK_c}} erfc\left(\eta\sqrt{S_c} - \sqrt{K_ct}\right) + \\ e^{y\sqrt{S_cK_c}} erfc\left(\eta\sqrt{S_c} + \sqrt{K_ct}\right) \end{array}\right] + \end{aligned}$$

$$\frac{z_{11}e^{-z_2 t}}{2}\left[\begin{array}{l} e^{-y\sqrt{S_c(K_c-z_2)}}erfc\left(\eta\sqrt{S_c}-\sqrt{(K_c-z_2)t}\right)+ \\ e^{y\sqrt{S_c(K_c-z_2)}}erfc\left(\eta\sqrt{S_c}+\sqrt{(K_c-z_2)t}\right) \end{array}\right]+$$

$$\frac{z_4 e^{-a_2 t}}{2}\left[\begin{array}{l} e^{-y\sqrt{S_c(K_c-a_2)}}erfc\left(\eta\sqrt{S_c}-\sqrt{(K_c-a_2)t}\right)+ \\ e^{y\sqrt{S_c(K_c-a_2)}}erfc\left(\eta\sqrt{S_c}+\sqrt{(K_c-a_2)t}\right) \end{array}\right]+$$

$$z_8\left[\begin{array}{l} \left(\frac{t}{2}-\frac{y\sqrt{S_c}}{4\sqrt{K_c}}\right)\left(e^{-y\sqrt{S_cK_c}}erfc\left(\eta\sqrt{S_c}-\sqrt{K_c t}\right)\right)+ \\ \left(\frac{t}{2}+\frac{y\sqrt{S_c}}{4\sqrt{K_c}}\right)\left(e^{y\sqrt{S_cK_c}}erfc\left(\eta\sqrt{S_c}+\sqrt{K_c t}\right)\right) \end{array}\right]+$$

$$\frac{z_{26}}{2}\left[\begin{array}{l} e^{-y\sqrt{F_1}}erfc\left(\eta\sqrt{\mathrm{P_r}}-\sqrt{\frac{F_1}{\mathrm{P_r}}t}\right)+ \\ e^{y\sqrt{F_1}}erfc\left(\eta\sqrt{\mathrm{P_r}}+\sqrt{\frac{F_1}{\mathrm{P_r}}t}\right) \end{array}\right]+$$

$$z_{27}\left[\begin{array}{l} \left(\frac{t}{2}-\frac{y\,\mathrm{P_r}}{4\sqrt{F_1}}\right)\left(e^{-y\sqrt{F_1}}erfc\left(\eta\sqrt{\mathrm{P_r}}-\sqrt{\frac{F_1}{\mathrm{Pr}}t}\right)\right)+ \\ \left(\frac{t}{2}+\frac{y\,\mathrm{Pr}}{4\sqrt{F_1}}\right)\left(e^{y\sqrt{F_1}}erfc\left(\eta\sqrt{\mathrm{P_r}}+\sqrt{\frac{F_1}{\mathrm{P_r}}t}\right)\right) \end{array}\right]+$$

$$\frac{z_{28}e^{-z_{13}t}}{2}\left[\begin{array}{l} \left(e^{-y\sqrt{F_1-z_{13}P_r}}erfc\left(\eta\sqrt{\mathrm{P_r}}-\sqrt{\left(\frac{F_1}{\mathrm{P_r}}-z_{13}\right)t}\right)\right)+ \\ \left(e^{y\sqrt{F_1-z_{13}Pr}}erfc\left(\eta\sqrt{\mathrm{P_r}}+\sqrt{\left(\frac{F_1}{\mathrm{P_r}}-z_{13}\right)t}\right)\right) \end{array}\right]+$$

$$\frac{z_{18}e^{-a_2 t}}{2}\left[\begin{array}{l} \left(e^{-y\sqrt{F_1-a_2P_r}}erfc\left(\eta\sqrt{\mathrm{P_r}}-\sqrt{\left(\frac{F_1}{\mathrm{P_r}}-a_2\right)t}\right)\right)+ \\ \left(e^{y\sqrt{F_1-a_2P_r}}erfc\left(\eta\sqrt{\mathrm{P_r}}+\sqrt{\left(\frac{F_1}{\mathrm{P_r}}-a_2\right)t}\right)\right) \end{array}\right] \qquad (20)$$

$$\theta(y,t) = \begin{bmatrix} \left(\frac{t}{2} - \frac{y\,\mathrm{P_r}}{4\sqrt{F_1}}\right)\left(e^{-y\sqrt{F_1}} erfc\left(\eta\sqrt{\mathrm{P_r}} - \sqrt{\frac{F_1}{\mathrm{P_r}}t}\right)\right) + \\ \left(\frac{t}{2} + \frac{y\,\mathrm{P_r}}{4\sqrt{F_1}}\right)\left(e^{y\sqrt{F_1}} erfc\left(\eta\sqrt{\mathrm{P_r}} + \sqrt{\frac{F_1}{\mathrm{P_r}}t}\right)\right) \end{bmatrix} \tag{21}$$

where $F_1 = F + H$

$$C(y,\mathrm{t}) = \frac{(1-(b_2+\mathrm{b}_4))}{2}\begin{bmatrix} e^{-y\sqrt{S_cK_c}} erfc(\eta\sqrt{S_c} - \sqrt{K_ct}) + \\ e^{y\sqrt{S_cK_c}} erfc(\eta\sqrt{S_c} + \sqrt{K_ct}) \end{bmatrix} -$$

$$\frac{(b_1+b_3)e^{-a_2t}}{2}\begin{bmatrix} e^{-y\sqrt{S_c(K_c-a_2)}} erfc\left(\eta\sqrt{S_c} - \sqrt{(K_c-a_2)t}\right) + \\ e^{y\sqrt{S_c(K_c-a_2)}} erfc\left(\eta\sqrt{S_c} + \sqrt{(K_c-a_2)t}\right) \end{bmatrix} -$$

$$b_5\begin{bmatrix} \left(\frac{t}{2} - \frac{y\sqrt{S_c}}{4\sqrt{K_c}}\right)\left(e^{-y\sqrt{S_cK_c}} erfc\left(\eta\sqrt{S_c} - \sqrt{K_ct}\right)\right) + \\ \left(\frac{t}{2} + \frac{y\sqrt{S_c}}{4\sqrt{K_c}}\right)\left(e^{y\sqrt{S_cK_c}} erfc\left(\eta\sqrt{S_c} + \sqrt{K_ct}\right)\right) \end{bmatrix} + \frac{(b_2+b_4)}{2}\begin{bmatrix} e^{-y\sqrt{F_1}} erfc\left(\eta\sqrt{\mathrm{P_r}} - \sqrt{\frac{F_1}{\mathrm{P_r}}t}\right) + \\ e^{y\sqrt{F_1}} erfc\left(\eta\sqrt{\mathrm{P_r}} + \sqrt{\frac{F_1}{\mathrm{P_r}}t}\right) \end{bmatrix} +$$

$$\frac{(b_1+b_3)e^{-a_2t}}{2}\begin{bmatrix} \left(e^{-y\sqrt{F_1-a_2P_r}} erfc\left(\eta\sqrt{\mathrm{P_r}} - \sqrt{\left(\frac{F_1}{\mathrm{P_r}} - a_2\right)t}\right)\right) + \\ \left(e^{y\sqrt{F_1-a_2Pr}} erfc\left(\eta\sqrt{\mathrm{P_r}} + \sqrt{\left(\frac{F_1}{\mathrm{P_r}} - a_2\right)t}\right)\right) \end{bmatrix} + \tag{22}$$

$$b_5\begin{bmatrix} \left(\frac{t}{2} - \frac{y\,\mathrm{P_r}}{4\sqrt{F_1}}\right)\left(e^{-y\sqrt{F_1}} erfc\left(\eta\sqrt{\mathrm{P_r}} - \sqrt{\frac{F_1}{\mathrm{P_r}}t}\right)\right) + \\ \left(\frac{t}{2} + \frac{y\,\mathrm{P_r}}{4\sqrt{F_1}}\right)\left(e^{y\sqrt{F_1}} erfc\left(\eta\sqrt{\mathrm{P_r}} + \sqrt{\frac{F_1}{\mathrm{P_r}}t}\right)\right) \end{bmatrix}$$

4. Skin Friction Coefficient

Knowing the velocity field, we now study the effect of t, Pr, F, M, etc. on the skin-friction coefficient. In the dimensionless form, it is given by

$$\tau = -\left(\frac{\partial u}{\partial y}\right)_{y=0} = e^{at}\left[\frac{1}{\sqrt{\pi t}}e^{-(a+M_1)t} + \sqrt{a+M_1}\,erf\left(\sqrt{(M_1+a)t}\right)\right] + z_{29}\left[\frac{1}{\sqrt{\pi t}}e^{-M_1t} + \sqrt{M_1}\,erf\left(\sqrt{(M_1)t}\right)\right] +$$

$$z_{30}\left[\left(\frac{2tM_1+1}{2\sqrt{M_1}}\right)erf\left(\sqrt{(M_1)t}\right)+\sqrt{\frac{t}{\pi}}e^{-M_1t}\right]+z_{31}e^{-a_2t}\left[\sqrt{(M_1-a_2)}\,erf\left(\sqrt{(M_1-a_2)t}\right)+\frac{1}{\sqrt{\pi t}}e^{-(M_1-a_2)t}\right]+$$
$$z_{10}\left[\sqrt{\frac{S_c}{\pi t}}e^{-K_ct}+\sqrt{S_cK_c}\,erf\left(\sqrt{K_ct}\right)\right]-z_{11}e^{-z_2t}\left[\sqrt{\frac{S_c}{\pi t}}e^{-(K_c-z_2)t}+\sqrt{S_c(K_c-z_2)}\,erf\left(\sqrt{(K_c-z_2)t}\right)\right]-$$
$$z_4e^{-a_2t}\left[\sqrt{\frac{S_c}{\pi t}}e^{-(K_c-a_2)t}+\sqrt{S_c(K_c-a_2)}\,erf\left(\sqrt{(K_c-a_2)t}\right)\right]-$$
$$z_8\left[\left(\frac{2tK_c+\sqrt{S_c}}{2\sqrt{K_c}}\right)erf\left(\sqrt{(K_c)t}\right)+\sqrt{\frac{tS_c}{\pi}}e^{-\sqrt{K_ct}}\right]-z_{26}\left[\sqrt{\frac{\mathrm{P_r}}{\pi t}}e^{-\frac{F_1}{\mathrm{P_r}}t}+\sqrt{F_1}\,erf\left(\sqrt{\frac{F_1}{\mathrm{P_r}}t}\right)\right]-$$
$$z_{27}\left[\left(\frac{2tF_1+P_r}{2\sqrt{F_1}}\right)erf\left(\sqrt{(\frac{F_1}{P_r})t}\right)+\sqrt{\frac{tP_r}{\pi}}e^{\frac{F_1}{P_r}t}\right]-z_{28}\left[\sqrt{\frac{tP_r}{\pi}}e^{-\left(\frac{F_1}{P_r}-z_{12}\right)t}+\sqrt{F_1-z_{13}P_r}\,erf\sqrt{\left(\frac{F_1}{P_r}-z_{13}\right)t}\right]$$

(23)

5. NUSSELT NUMBER

An important phenomenon in this study is to understand the effect of t, F and H on the Nusselt number.

In non-dimensional form, the rate of heat transfer is given

$$Nu=-\left(\frac{\partial\theta}{\partial y}\right)_{y=0}$$
$$=\left(\frac{2tF_1+\mathrm{P_r}}{2\sqrt{F_1}}\right)erf\left(\sqrt{\left(\frac{F_1}{\mathrm{P_r}}\right)t}\right)+\sqrt{\frac{t\,\mathrm{P_r}}{\pi}}e^{-\frac{F_1}{\mathrm{P_r}}t} \quad (24)$$

6. SHERWOOD NUMBER

Another important phenomenon in this study is to understand the effect of t, S_c *and* K_c on the Sherwood Number. In dimensionless form, the rate of mass transfer is given by

$$Sh=-\left(\frac{\partial C}{\partial y}\right)_{y=0}=(1-(b_2+b_4))\left[\sqrt{\frac{S_c}{\pi t}}e^{-K_ct}+\sqrt{S_cK_c}\,erf\left(\sqrt{K_ct}\right)\right]-$$

$$(b_2+b_4)\left[\sqrt{\frac{P_r}{\pi t}}e^{-\frac{F_1}{P_r}t}+\sqrt{F_1}erf\left(\sqrt{\frac{F_1}{P_r}t}\right)\right]+$$
$$(b_1+b_3)e^{-a_2t}\left[\sqrt{\frac{P_r}{\pi t}}e^{-\left(\frac{F_1}{P_r}-a_2\right)t}+\sqrt{F_1-a_2\,P_r}\;erf\left(\sqrt{\left(\frac{F_1}{P_r}-a_2\right)t}\right)\right]+$$
$$b_5\left[\left(\frac{2tF_1+P_r}{2\sqrt{F_1}}\right)erf\left(\sqrt{\left(\frac{F_1}{P_r}\right)t}\right)+\sqrt{\frac{t\,P_r}{\pi}}e^{-\frac{F_1}{P_r}t}\right]$$
$$(b_1+b_3)\,e^{-a_2t}\left[\sqrt{\frac{S_c}{\pi t}}e^{-(K_c-a_2)t}+\sqrt{S_c(K_c-a_2)}erf\left(\sqrt{(K_c-a_2)t}\right)\right]-$$
$$b_5\left[\left(\frac{2tK_c+\sqrt{S_c}}{2\sqrt{K_c}}\right)erf\left(\sqrt{(K_c)t}\right)+\sqrt{\frac{tS_c}{\pi}}e^{-\sqrt{K_c t}}\right]+ \tag{25}$$

7. Results and Discussion

Numerical evaluation of the analytical results reported in the previous section was performed and a selective representative set of results is reported graphically in Figures 1-11. These results are obtained to illustrate the influence of the radiation parameter F, heat absorption parameter H, Soret number S_o and the chemical reaction parameter K_c. The values of the Prandtl number are chosen such that they represent water (P_r = 7.0) and air (P_r = 0.71).

The values of the Schmidt number are chosen to represent the presence of species by Hydrogen (0.22), water vapor (0.60), ammonia (0.78), Ethyl benzene (2.01) and carbon dioxide (0.96), [see Ref. 18].

In the present study, the boundary condition for $y \to \infty$ is replaced by y_{max} which is a sufficiently large value of y where the velocity profile approaches the relevant free stream velocity. A span-wise step distance Δy of 0.01 is used with y_{max} = 2. In order to assess the accuracy of our method, we have compared our results with accepted data sets for the velocity and the skin friction profiles for an exponentially accelerated vertical plate corresponding to the case computed by Rajesh and Varma [16]. The results of this comparison are found to be in a very good agreement.

The influences of the heat absorption parameter H, chemical reaction parameter K_c and the Soret number S_0 are displayed through the velocity profiles in Figures 1-3, respectively. The velocity distribution attains a distinctive maximum value in the vicinity of the plate surface and then decreases properly to approach the free stream value of zero.

From these figures, it is seen that an increase in either of the heat absorption parameter or the chemical reaction parameter leads to a decay in the velocity field while it enhances with an increase in the value of the Soret number.

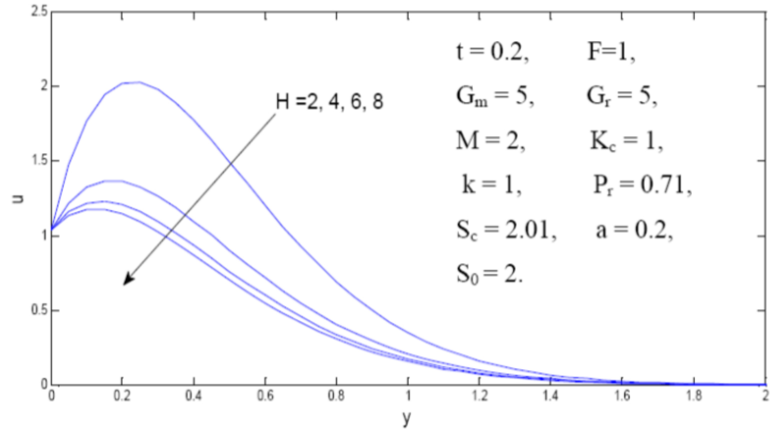

Figure 1. Effects of H on velocity profiles.

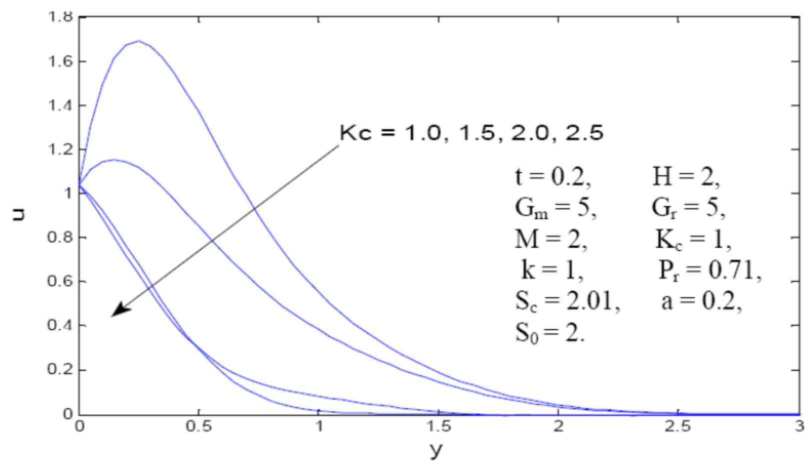

Figure 2. Effects of K_c on velocity profiles.

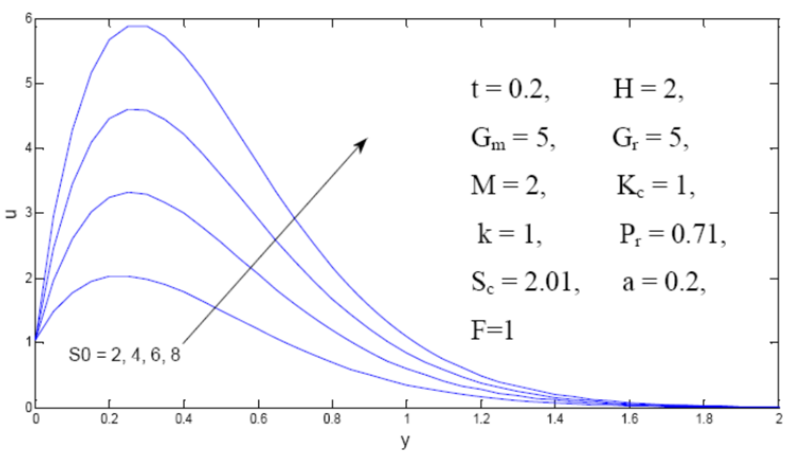

Figure 3. Effects of S_o on velocity profiles.

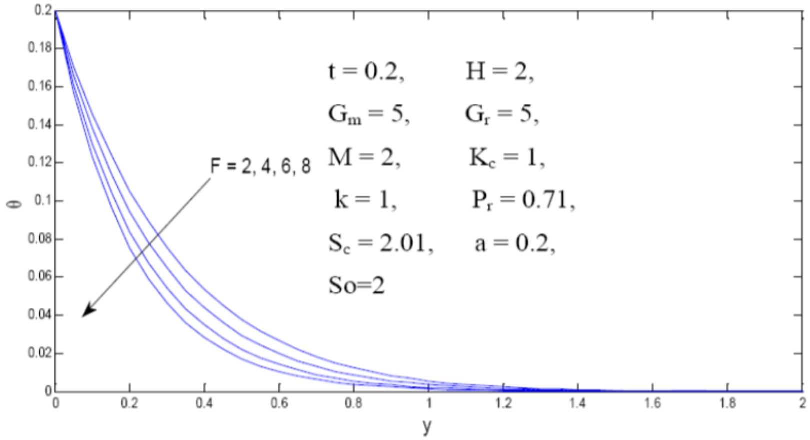

Figure 4. Effects of F on temperature profiles.

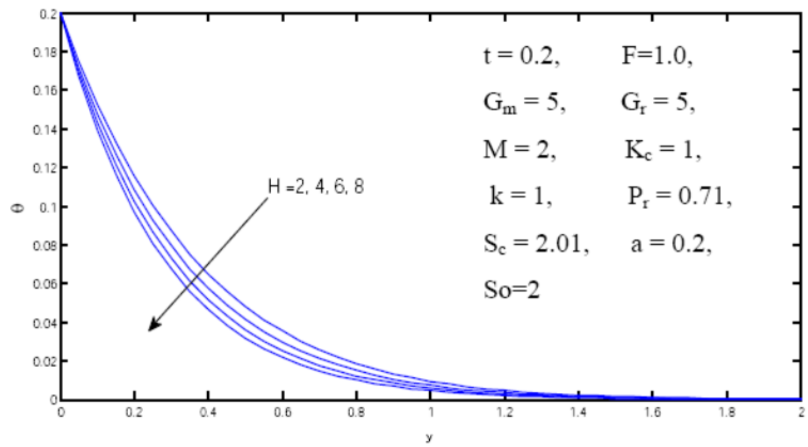

Figure 5. Effects of H on temperature profiles.

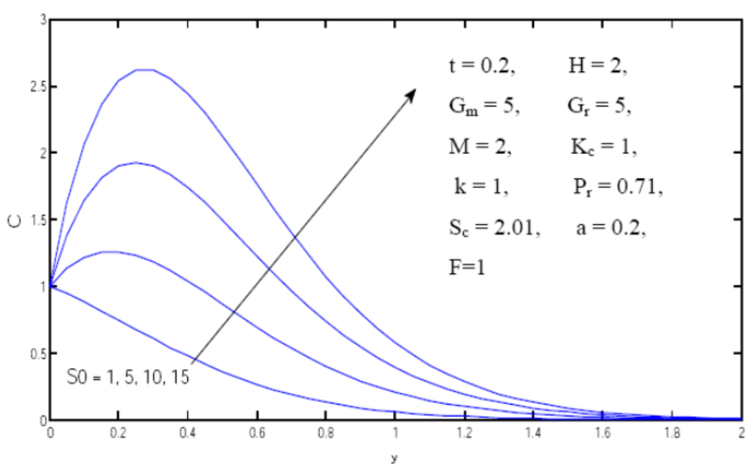

Figure 6. Effects of S_o on concentration profiles.

The temperature profiles for different values of the radiation parameter F, and the heat absorption parameter H are exhibited through Figures 4 and 5, respectively. It is observed that there is decay in the temperature field with increasing values of either F or H.

The numerical results show that the effect of increasing values of F results in decreasing the thermal boundary layer thickness and more uniform temperature distribution across the boundary layer and that the heat absorption effects has the tendency to decrease the fluid temperature.

The concentration field is studied for different values of parameters in Figures 6 and 7. From these figures, it is seen that the concentration increases with increasing values of either the Soret number S_0 or the chemical reaction parameter K_c. The skin-friction coefficient is presented for different values of flow parameters against time t in Figures 8 and 9. From these figures, it is observed that the skin-friction coefficient decreases with an increase in the chemical reaction parameter K_c. It is interesting to note that the skin-friction coefficient decreases up to a certain value of t (approximately 0.275) and increases later moving away from the plate with the increasing values of Soret number S_0. Finally, The Sherwood number is presented in Figures 10 and 11 against time t. From these figures, it is noticed that the Sherwood number increases with an increase in the chemical reaction parameter K_c. Also, it is observed that with an increase in the Soret number S_0, the Sherwood number increases up to certain value of t (0.275) and decreases later moving away from the plate.

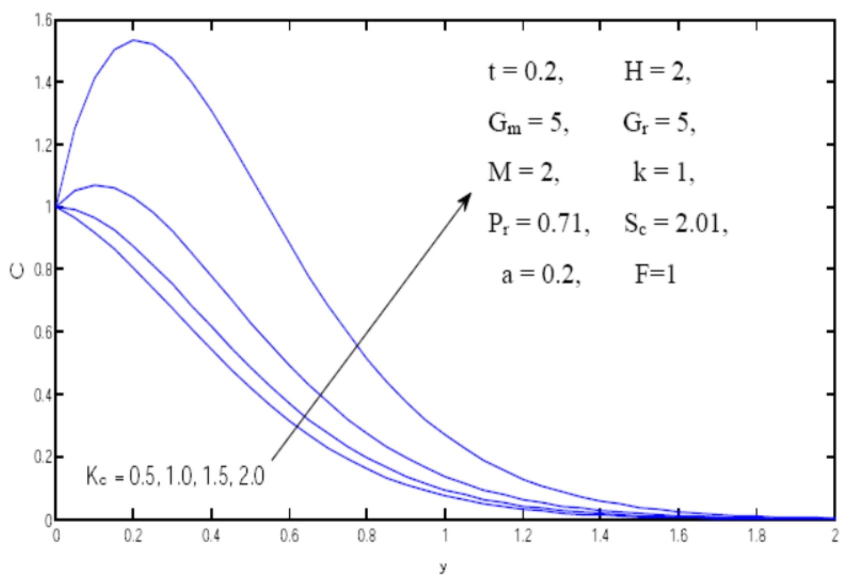

Figure 7. Effects of K_c on concentration profiles.

The influences of the heat absorption parameter H, chemical reaction parameter K_c and the Soret number S_0 are displayed through the velocity profiles in Figures 1-3, respectively. The velocity distribution attains a distinctive maximum value in the vicinity of the plate surface and then decreases properly to approach the free stream value of zero. From these figures, it is seen that an increase in either of the heat absorption parameter or the chemical reaction parameter leads to a decay in the velocity field while it enhances with an increase in the value of the Soret number.

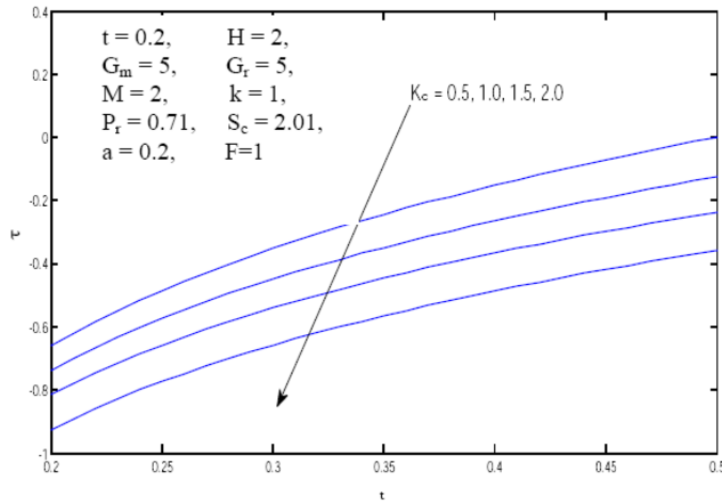

Figure 8. Effects of K_c on skin-friction coefficient profiles.

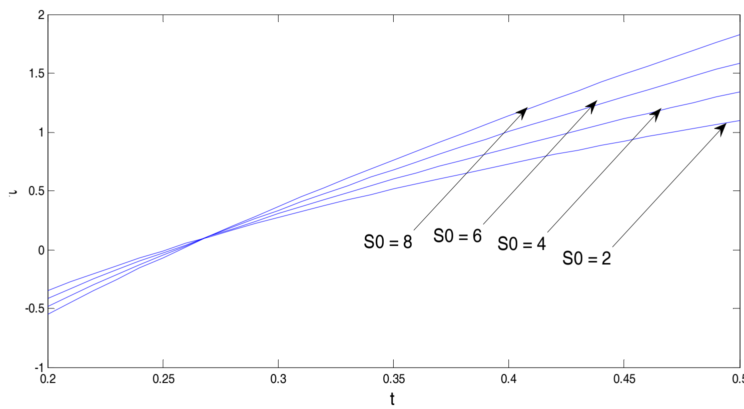

Figure 9. Effects of S_o on skin-friction coefficient profiles.

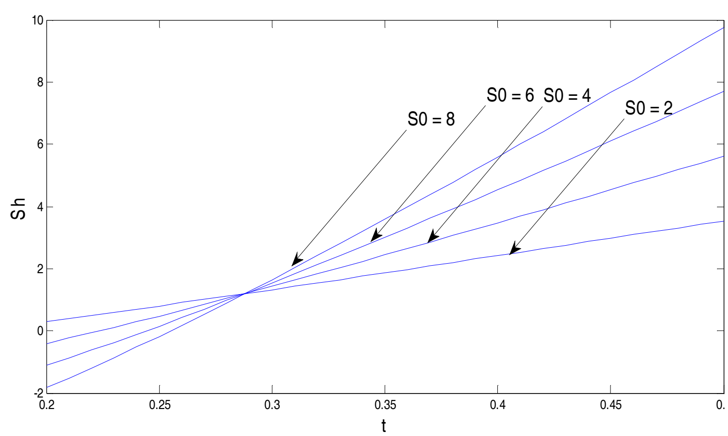

Figure 10. Effects of Sherwood number profiles.

The temperature profiles for different values of the radiation parameter F, and the heat absorption parameter H are exhibited through Figures 4 and 5, respectively. It is observed that there is decay in the temperature field with increasing values of either F or H. The numerical results show that the effect of increasing values of F results in decreasing the thermal boundary layer thickness and more uniform temperature distribution across the boundary layer and that the heat absorption effects has the tendency to decrease the fluid temperature.

The concentration field is studied for different values of parameters in Figures 6 and 7. From these figures, it is seen that the concentration increases with increasing values of either the Soret number S_0 or the chemical reaction parameter K_c. The skin-friction coefficient is presented for different values of flow parameters against time t in Figures 8 and 9. From these figures, it is observed that the skin-friction coefficient decreases with an increase in the chemical reaction parameter K_c. It is interesting to note that the skin-friction coefficient decreases up to a certain value of t (approximately 0.275) and increases later moving away from the plate with the increasing values of Soret number S_0. Finally, The Sherwood number is presented in Figures 10 and 11 against time t. From these figures, it is noticed that the Sherwood number increases with an increase in the chemical reaction parameter K_c. Also, it is observed that with an increase in the Soret number S_0, the Sherwood number increases up to certain value of t (0.275) and decreases later moving away from the plate.

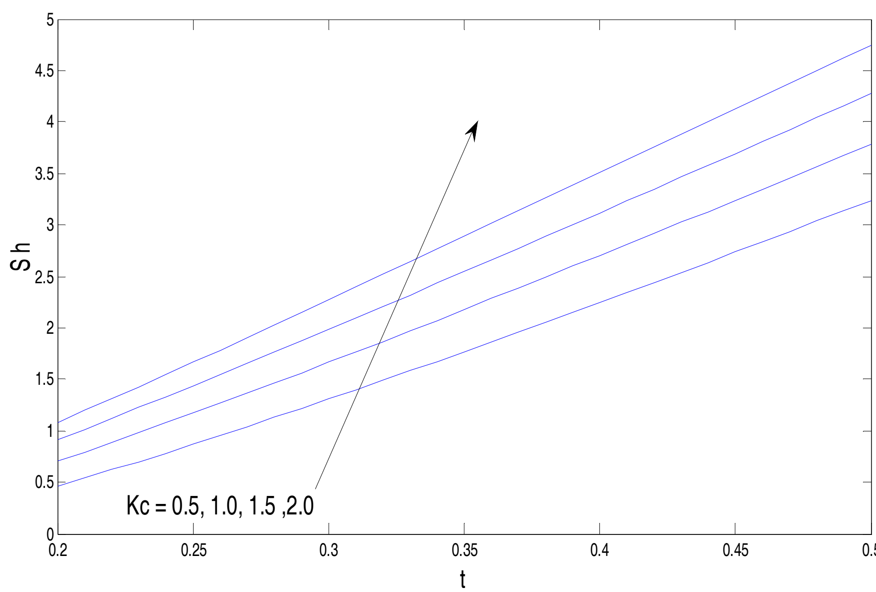

Figure 11. Effects of K_c on Sherwood number profiles.

CONCLUSION

1. The velocity decreased with the increase in either the radiation parameter F or the heat absorption parameter H.
2. The velocity increased with the increase in either the acceleration coefficient ‘a’ or the Soret number S_0.

3. The skin-friction coefficient increased due to increases in the concentration buoyancy effects while it decreased due to increased in the magnetic parameter M.
4. The Nusselt number and the Sherwood number increased with increasing values of the radiation parameter F, the heat absorption parameter H or the chemical reaction parameter K_c.

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