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SIMILARITY SOLUTION FOR THERMAL BOUNDARY LAYER ON A STRETCHED SURFACE OF A NON-NEWTONIAN FLUID

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ABSTRACT

Continuum equations governing steady, laminar, boundary-layer flow and heat transfer in a quiescent non-Newtonian, power-law fluid driven by a stretched porous surface are developed. The resulting partial differential equations are nondimensionalized and transformed into ordinary differential equations using similarity transformations for both variable power-law surface temperature and constant wall heat flux cases. The surface is assumed to be moving with a constant velocity. The transformed equations are solved numerically using a standard implicit finite-difference method. Graphical results for the tangential velocity and temperature profiles as well as tabulated results for the skin-friction coefficient and the Nusselt number for various parametric conditions are presented and discussed.

Introduction

The study of flow and heat transfer aspects of a continuously moving surface in a quiescent fluid has attracted many investigators in recent years. This is due to the application of such a problem in many manufacturing processes such as metal extrusion, glass fiber production, continuous casting, hot rolling, manufacturing of plastic and rubber sheets, and crystal growing (see, for instance Altan et al. [1], Fisher [2], Tadmor and Klein [3], and Ali [4]). Boundary-layer flow over a stretched surface moving with a constant speed was initially studied by Sakiadis [5,6] and obtained a numerical solution using a similarity transformation. Later, Erickson et al. [7] extended the works of Sakiadis [5,6] to account of possible suction or blowing at the surface and investigation its effect on the heat and mass transfer in the boundary layer. Tsuo et al. [8] confirmed the results of Sakiadis [5,6] experimentally. Both Chen and Strobel [9] and Jacobi [10] have also reported some results on the uniform motion of the extended surface. As mentioned by Vajravelu and Hadjinicolaou [11], the above investigations have a definite bearing on the problem of continuous extrusion of a polymer sheet from a dye. Most investigations have concentrated on the problem of a stretching sheet moving with a linear velocity and different thermal boundary conditions (see, for instance, Crane [12], Vleggaar [13], Soundalgekar and Murty [14], Gupta and Gupta [15], Grubka and Bobba [16]). Other works have studied the flow and heat transfer characteristics due to a stretched impermeable and permeable surface subject to a power-law velocity and various thermal conditions (see, for example, Banks [17], Ali [4,18,19], and Danberg and Fansler [20]). Recently, Chen and Char [21] studied the effects of power-law surface temperature and power-law wall heat flux on the heat transfer aspects of a linearly stretched permeable plate with suction and injection.

All of the above referenced studies have dealt with flows of Newtonian fluids. In recent years, non-Newtonian liquids have been appearing in increasing numbers. Examples of non-Newtonian fluids abound in every day life. Molten plastics, polymer solutions, dyes, varnishes, industrial suspensions, multi-grade oils, paints and printing ink all fall within this category. Although non-Newtonian fluids have been used extensively in many industries such as polymer melts and polymer solutions used in the plastic processing industries there have been little work done on flow and heat transfer aspects of these fluids over moving surfaces. Troy et al. [22] have obtained an exact solution for the problem of viscoelastic fluid flow over a stretched surface. Vajravelu and Rollins [23] considered the problem of Chen and Char [21] with a power-law wall heat flux distribution of viscoelastic fluids. Recently, Seshadri et al. [24] have also considered viscoelastic fluids and mass transfer effects.

The well-known non-Newtonian power-law model have been employed on the problem of a continuous stretching surface by Fox et al. [25]. Motivated by the works mentioned above, it is of interest in the present paper to investigate the steady thermal boundary layer on a stretched surface moving with a constant velocity of a power-law fluid for both power-law surface temperature distribution and constant wall heat flux cases. All thermophysical properties of the power-law fluid are assumed constant.

Problem Formulation

Consider steady, laminar incompressible, two-dimensional, boundary-layer flow and heat transfer of a horizontal stretched flat permeable surface with a constant velocity in a non-Newtonian power-law fluid at rest. Let the surface be maintained at a power-law temperature or a constant heat flux and allowing for fluid suction or blowing. The governing equations for this investigation are based on the balance laws of mass, linear momentum, and energy modified to account for non-Newtonian effects. These equations can be written as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\frac{K}{\rho}\frac{\partial}{\partial y}\left(\left|\frac{\partial u}{\partial y}\right|^{n-1}\frac{\partial u}{\partial y}\right)-u\frac{\partial u}{\partial x}\,v\frac{\partial u}{\partial y}=0 \tag{2}$$

$$\alpha\frac{\partial}{\partial y}\left(\left|\frac{\partial u}{\partial y}\right|^{n-1}\frac{\partial T}{\partial y}\right)-u\frac{\partial T}{\partial x}-v\frac{\partial T}{\partial y}=0 \tag{3}$$

where x and y are the horizontal and vertical distances, respectively. u, v, and T are the fluid x-component and y-components of velocity and temperature, respectively. ρ and α are the fluid density and modified thermal diffusivity, respectively. K and n are the power-law coefficient and index, respectively.

The appropriate boundary conditions for this problem are

$$u(x,0)=U_0,\ v(x,0)=v_w,\ T(x,0)=T_\infty+Ax^\gamma \text{ or } q_w=-k\frac{\partial T}{\partial y}(x,0)=\text{constant} \tag{4}$$
$$u(x,\infty)=0,\quad T(x,\infty)=T_\infty$$

where k and T_∞ are the fluid thermal conductivity and ambient temperature and U_0, A and γ are constants. It should be noted that positive values of v_w indicate fluid blowing at the plate surface while negative values of v_w correspond to fluid suction at the wall.

Equations (1) through (3) are non-dimensionalized by using

$$X=x\left(\frac{U_0^{2-n}}{K/\rho}\right)^{1/n},\quad Y=y\left(\frac{U_0^{2-n}}{K/\rho}\right)^{1/n},\quad U=\frac{u}{U_0},\quad V=\frac{v}{U_0},\quad H=\frac{T-T_\infty}{Ax^\gamma} \tag{5}$$

$$A = \begin{cases} \dfrac{T(x,0) - T_\infty}{x^\gamma} & \text{variable wall temperature} \\ \dfrac{q_w}{k}\left(\dfrac{U_o^{2-n}}{K/\rho}\right)^{1/n}, & \text{constant wall heat flux } (\gamma = 0) \end{cases}$$

Introducing the dimensionless stream function such that

$$U = \frac{\partial \psi}{\partial Y}, \qquad V = -\frac{\partial \psi}{\partial X} \tag{6}$$

and substituting

$$\eta = X^a Y, \qquad \psi = X^b f(\eta), \qquad H = X^c \theta(\eta) \tag{7}$$

(where η is the similarity space variable, $f(\eta)$ is the transformed stream function and $\theta(\eta)$ is the dimensionless fluid temperature) into the resulting dimensionless equations and boundary conditions produces the following dimensionless transformed equations and boundary conditions:

$$(|f''|^{n-1} f'')' - (a+b)(f')^2 + bff'' = 0 \tag{8}$$

$$(|f''|^{n-1} \theta')' + b\,\mathrm{Pr}\, f\theta' - c\,\mathrm{Pr}\, f'\theta = 0 \tag{9}$$

$$f'(0) = 1\ ,\ f(0) = g_o,\ \ \theta(0) = 1 \ \text{ or } \ \theta'(0) = -1,\ \ f'(\infty) = 0,\ \ \theta(\infty) = 0 \tag{10}$$

where a prime denotes ordinary differentiation with respect to η and the constants a, b, and c and the suction/blowing parameter g_o and the modified Prandtl number Pr are given by

$$a = \frac{-1}{n+1}, \qquad b = \frac{1}{n+1}, \qquad c = \begin{cases} \gamma & \text{for variable wall temperature} \\ \dfrac{1}{n+1} & \text{for constant wall heat flux} \end{cases}$$

$$g_o = -\frac{v_w}{bU_o} X^{1-b}, \quad \mathrm{Pr} = \frac{K/\rho}{\alpha} \tag{11}$$

Important physical parameters for this problem are the skin-friction coefficient and the Nusselt number. These flow and heat transfer properties can be defined in dimensionless form, respectively, as

$$C_f = -(f''(0))^n , \qquad Nu = -\theta'(0) \qquad (12)$$

Results and Discussion

Equations (6) through (8) are nonlinear and exhibit no closed-form solution and, therefore, they must be solved numerically. The Runge Kutta method and the standard implicit finite-difference method have proven to be suitable and successful for the solution of such equations. It is obvious that the flow problem is uncoupled from the problem. Therefore, Equation (6) is solved first for f and the obtained solution is then substituted into Equation (7) which is solved for θ subject to the boundary conditions (8c) or (8d) and (8f). Since the implicit, tri-diagonal, iterative finite-difference method discussed by Blottner [26] has proven to be accurate and simple to use, it has been chosen for the solution of the present problem. 196 nodal points in the η direction are employed. Since it is expected that the changes of f and θ in the region close to the surface are great due to the boundary layer, small step sizes are needed to approximate the derivatives of f and θ correctly. However, far above the surface where the ambient conditions exist, small changes in the variables are expected and, thus, larger step sizes can be utilized there. For these reasons, variable step sizes in η with the initial step size $\Delta\eta_1 = 0.001$ and a growth factor equals to 1.03 are employed in the present problem. These numbers were chosen after performing many numerical experimentations to assess grid independence and to insure accuracy of the numerical results. A convergence criterion based on the difference between the current and the previous iterations was employed. When this difference reached 10^{-5}, the solution was assumed converged and the iteration process was terminated. For more details of the numerical procedure, the reader is advised to see the paper by Blottner [26]. No numerical difficulties were encountered in producing the results for this problem and a representative set of numerical results is illustrated in figures 1 through 8.

Figures 1 and 2 illustrate the effect of imposition of fluid suction or blowing at the plate surface on the tangential velocity f' and temperature θ profiles, respectively. Imposition of suction at the wall have the tendency to lower or reduce the boundary-layer thickness. This causes the fluid motion to slow down and the fluid temperature to increase slightly. This is depicted in the decreases and increases in the values of f' and θ as g_0 increases, shown in figures 1 and 2, respectively.

Figures 3 and 4 present typical tangential velocity and temperature profiles for various values of the power-law fluid index coefficient n, respectively. Increases in the values of n tend to reduce both the hydrodynamic and thermal boundary layers. This has

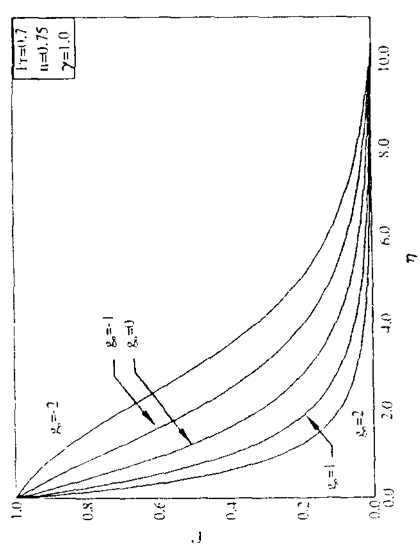

FIG. 1
Effects of g_∞ on Tangential Velocity Profiles

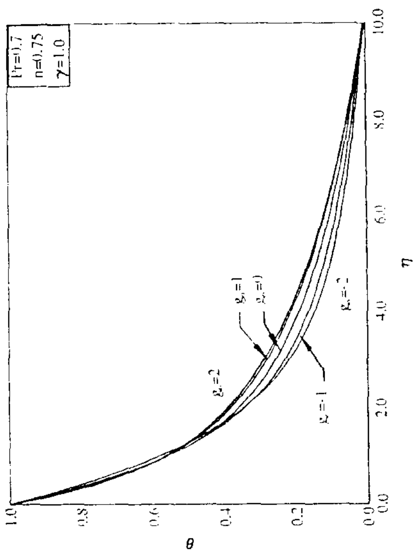

FIG. 2
Effects of g_∞ on Temperature Profiles

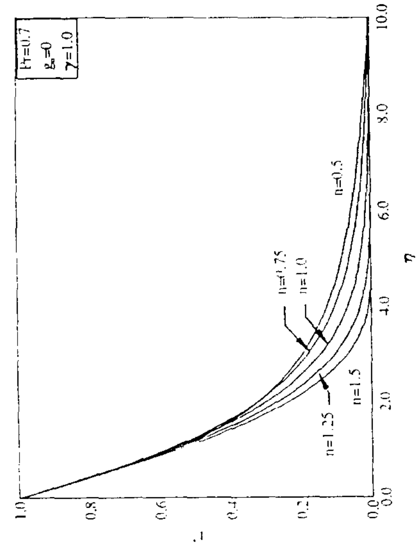

FIG. 3
Effects of n on Tangential Velocity Profiles

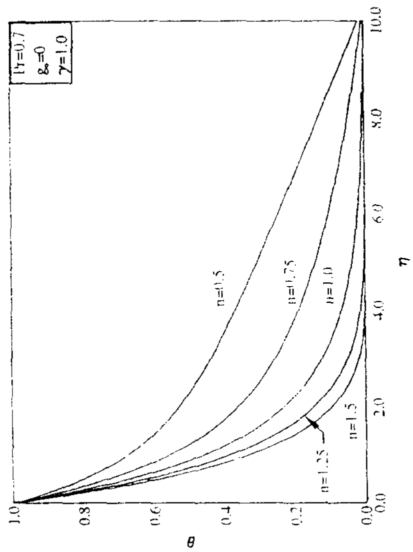

FIG. 4
Effects of n on Temperature Profiles

the effect of reducing the values of both f' and θ as clearly shown in figures 3 and 4, respectively.

Figures 5 and 6 elucidate the influence of both the fluid Prandtl number Pr and the wall temperature power coefficient γ on the temperature profiles, respectively. It is evident from these figures that increases in either of these physical parameters result in decreases in the fluid temperature. It should be mentioned herein that the tangential velocity profiles were found to be insensible to changes in both Pr and γ and, therefore, not shown herein for brevity.

Table 1 shows a comparison of Nusselt numbers of Newtonian fluids for isothermal wall conditions and various Prandtl numbers of the present work with those reported by previous investigators. This comparison shows excellent agreement between the results.

Table 2 presents representative numerical results for the skin-friction coefficient C_f and the Nusselt number Nu of power-law fluids for non-isothermal wall conditions and various parametric values. It can be seen from this table that the values of C_f are insensible to changes in Pr and γ (since f' does not change with Pr and γ) while they increase as g_0 increases. It is also observed (from results not all shown in this table) that as n is increased from $n = 0.25$ to $n = 1.5$, C_f decreased in the psuedo-plastic range ($n < 1$) and increased in the dilitant range ($n > 1$). In addition, the Nusselt number Nu increased as either of g_0, n, Pr and γ is increased.

Figures 7 and 8 as well as Table 3 are obtained for the case of constant wall heat flux. Figures 7 and 8 show the influence of the power-law fluid index coefficient n and the Prandtl number Pr on the temperature profiles, respectively. Again, it is seen that both the thermal boundary layer as well as θ decrease as either of n or Pr increases. Tables 3 gives some values for the skin-friction coefficient C_f for various values of n. It is clearly seen that increases in the values of n results in decreases in the values of C_f.

Conclusion

The problem of steady, laminar, thermal boundary layer on a stretched porous surface of non-Newtonian power-law fluid was formulated. The surface was assumed to be moving with a constant velocity. Similarity transformations were used to convert the governing partial differential equations into ordinary ones for the cases of variables power-law surface temperature and constant wall heat flux. The transformed equations were solved numerically by a standard implicit finite-difference method. Numerical results for the tangential velocity and temperature profiles were illustrated graphically and result for

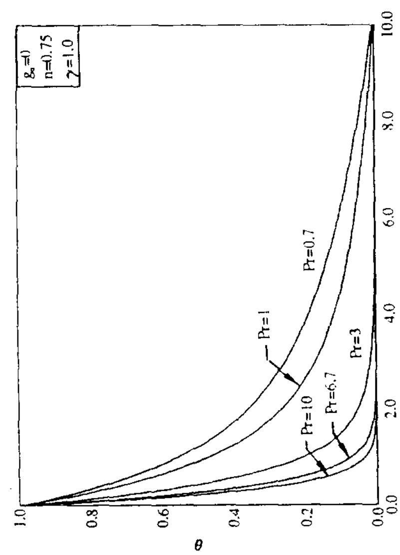

FIG. 5
Effects of Pr on Temperature profiles

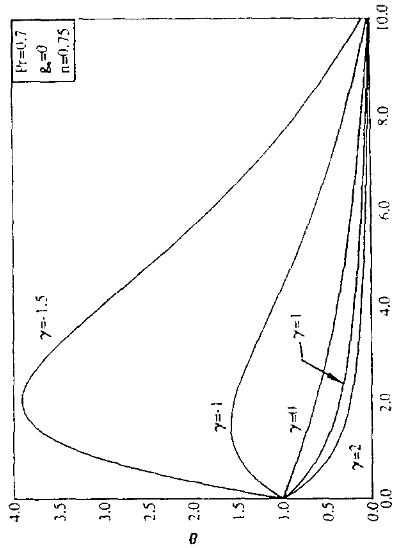

FIG. 6
Effects of γ on Temperature Profiles

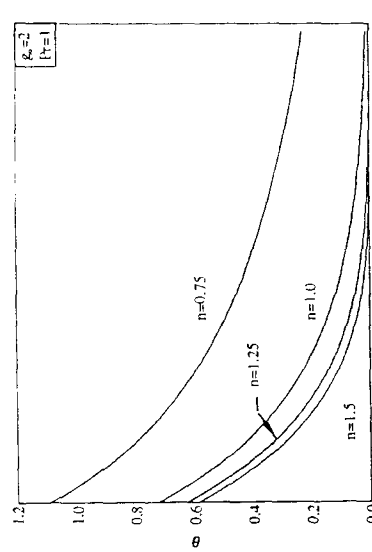

FIG. 7
Isoflux Temperature Profiles for Various n

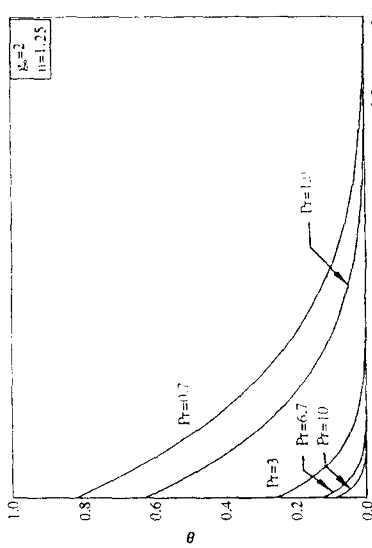

FIG. 8
Isoflux Temperature Profiles for Various Pr

the skin-friction coefficient and the Nusselt number were presented in tabulated form. It was found that the Nusselt number increased as either of the wall suction, the power-law fluid index coefficient, the Prandtl number, and the wall temperature power coefficient was increased. Comparisons with previously published results were performed and found to be in excellent agreement. It is hoped that these results be of use for industries dealing with non-Newtonian fluids.

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TABLE 1
Comparison of Nusselt numbers for n = 1 and various Pr values for constant wall temperatures

Pr	Jacobi [10]	Soundagekar and Murty [14]	Chen and Strobel [9]	Tsou et al. [8]	Ali [4]	Present results
0.7	0.3492	0.3508	0.34924	0.3492	0.3476	0.3524
1.0	0.4438	-	-	0.4438	0.4416	0.4453
10	1.6790	-	-	1.6804	1.6713	1.6830

TABLE 2
Representative results for C_f and Nu for variable wall temperatures
Reference values : $g_0 = 0$, $n = 0.75$, $Pr = 0.7$, $\gamma = 1.0$

	C_f	Nu
Reference	0.44711	0.65802
$g_0 = -1$	0.19754	0.59134
$g_0 = 1$	0.87976	0.71616
$n = 0.5$	0.48480	0.50204
$n = 1.5$	0.44756	1.04462
$Pr = 3$	0.44681	1.57697
$Pr = 10$	0.44650	3.01929
$\gamma = -1$	0.44699	-0.82803
$\gamma = 0$	0.44681	0.20801

TABLE 3
Isoflux C_f values ($g_0 = 2$, $Pr = 10$)

n	C_f
0.5	2.51465
0.75	1.50473
1.0	1.20017
1.25	1.05908
1.5	0.98096

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