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Research Article

Entropy Generation in a Rotating Couette Flow with Suction/Injection

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Abstract
The present paper is concerned with an analytical study of entropy generation in viscous incompressible Couette flow with suction/injection in a rotating frame of reference. One of the plate is held at rest and the other one moves with an uniform velocity.The flow induced by the moving plate. An exact solution of governing equations has been obtained in closed form. The entropy generation number and the Bejan number are also obtained. The influences of each of the governing parameters on velocity, temperature, entropy generation and Bejan number are discussed with the help of graphs.

Keywords: Couette flow, rotation, suction/injection, entropy generation number and Bejan number.

1 Introduction

Within the past 50 years our view of nature has changed drastically. Classical science emphasized equilibrium and stability. Now we see fluctuations, instability, evolutionary processes on all levels from chemistry and biology to cosmology. Everywhere we observe irreversible processes in which time symmetry is broken. The distinction between reversible and irreversible processes was first introduced in thermodynamics through the concept of entropy. In the modern context the formulation of entropy is for the understanding the thermodynamics aspects of self-organization, evolution of order and life that we see in nature. Entropy generation minimization studies are vital for ensuring optimal thermal systems in contemporary industrial and technological fields like geothermal systems, electronic cooling, heat exchangers to name a few. All thermal systems confront with entropy generation. Entropy generation is squarely associated with thermodynamic irreversibility.
In fluid dynamics, Couette flow refers to the laminar flow of a viscous fluid in the space between two parallel plates, one of which is moving relative to the other. This type of flow is named in honor of Maurice Marie Alfred Couette, a Professor of Physics at the French university of Angers in the late 19th

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century [1]. Couette flow occurs in fluid machinery involving moving parts and is especially important for hydrodynamic lubrication [2]. Couette parallel plate channel flow is a classical problem in fluid mechanics which offers analytical solution to highly nonlinear Navier-Stokes equations for constant fluid property fluids and undoubtedly serves as an important idealized configuration to peep into rather more complex real world systems. Though, in the present scenario of digital world where one has advantage of state of art numerical techniques and softwares, the *analytical solutions* may not sound that much *lucratively big thing* but one has to acknowledge the fact that there are very few flow problems in fluid mechanics which are amenable to closed form solution. These idealized flow studies have contributed a lot in the development of fluid mechanics as pertinent discipline. Analysis of flow formation in Couette motion as predicted by classical fluid mechanics was presented by Schlichting and Gersten [3]. Muhuri [4] studied the Couette flow between two porous plates when one of the plates moves with a uniform acceleration with uniform suction/injection. Several other works on fluid flow induced by moving boundaries or in channels are Bruin [5], Papathanasiou [6], Papathanasiou et al. [7], Katagiri [8], Balaram and Govindarajulu [9], Cramer [10], Ramamoorty [11]. Umavathi et al. [12], Kumar et al. [13] and Keimanesh et al. [14]. The significance of suction/injection on the boundary layer control in the field of aerodynamics and space science is well recognized [15]. Suction or injection of a fluid through the bounding surface can significantly change the flow field and, as a consequence, affect the heat transfer rate from the porous plate [16].

In general, suction tends to increase the skin friction and heat transfer coefficient whereas injection act in the opposite manner [17]. The study of fluid flow and heat transfer in a porous channel finds many applications in engineering such as ground water hydrology, irrigation and drainage problems and also in absorption and filtration processes in chemical engineering. The scientific treatment of the problems of irrigation, soil erosion and tile drainage are the present focus of the development of porous channel flow. One of the methods used for predicting the performance of the engineering processes is the second law analysis. The second law of thermodynamics is applied to investigate the irreversibilities in terms of the entropy generation rate. Since entropy generation is the measure of the destruction of the available work of the system, the determination of the factors responsible for the entropy generation is also important in upgrading the system performances. The method is introduced by Bejan [18, 19]. The entropy generation is encountered in many of energy-related applications, such as solar power collectors, geothermal energy systems and the cooling of modern electronic systems. Efficient utilization of energy is the primary objective in the design of any thermodynamic system. This can be achieved by minimizing entropy generation in processes. The theoretical method of entropy generation has been used in the specialized literature to treat external and internal irreversibilities. The irreversibility phenomena, which are expressed by entropy generation in a given system, are related to heat and mass transfers, viscous dissipation, magnetic field etc. Several researchers have discussed the irreversibility in a system under various flow configurations [20 -32]. They showed that the pertinent flow parameters might be chosen in order to minimize entropy generation inside the system. Flow, thermal and entropy generation characteristic inside a porous channel with viscous dissipation have been investigated by Mahmud and Fraser [33]. The heat transfer and entropy generation during compressible fluid flow in a channel partially filled with porous medium have been analyzed by Chauhan and Kumar [34]. Eegunjobi and Makinde [35] have studied the combined effect of buoyancy force and Navier slip on entropy generation in a vertical porous channel. The second law analysis of laminar flow in a channel filled with saturated porous media has been studied by Makinde and Osalusi [36]. Makinde and Maserumule [37] has presented the thermal criticality and entropy analysis for a variable viscosity Couette flow. Makinde and Osalusi [38] have investigated the entropy generation in a liquid film falling along an inclined porous heated plate. Cimpean and Pop [39, 40] have presented the parametric analysis of entropy generation in a channel. Dwivedi et al. [41] have made an analysis on the incompressible viscous laminar flow through a channel filled with porous media. Chinyoka and Makinde [42] have presented a numerical investigation of entropy generation in unsteady MHD generalized Couette flow with variable electrical conductivity.

The present paper is an analytical study of entropy generation in a Couette flow with suction/injection in rotating frame of reference. The flow is caused due to the movement of the upper channel wall. The entire system rotates about an axis perpendicular to the planes of the plates. Both the channel walls are maintained at constant but different temperatures. Closed form solution has been obtained for the fluid velocity and the fluid temperature. Effects of various physical parameters on the fluid velocity, temperature, the rate of heat transfer, entropy generation number and the Bejan number are presented graphically and discussed quantitatively.

2 Problem Formulation and Solution

Consider the viscous incompressible fluid bounded by two infinite horizontal parallel porous plates separated by a distance h. Choose a cartesian co-ordinate system with x-axis along the lower stationary plate in the direction of the flow, the y-axis is perpendicular to the plates and the z-axis normal to xy-plane. The upper plate moves with a uniform velocity u_0 in the x direction. The plates and the fluid are in a state of rigid body rotation with a uniform angular velocity Ω^* about the y-axis (see Fig.1). The velocity components are (u, v, w) relative to a frame of reference rotating with the fluid. The flow is driven by the shear generated due to the motion of the upper wall. The flow is fully developed hydrodynamically and thermally, hence the fluid velocity and the temperature in the channel are functions of y only. The plates' surface temperatures are non-uniform with temperature T_0 at the lower plate and T_w at the upper moving plate such that $T_w > T_0$. The equation of continuity then gives $v = -v_0$ everywhere in the fluid where v_0 is the suction velocity at the plates.

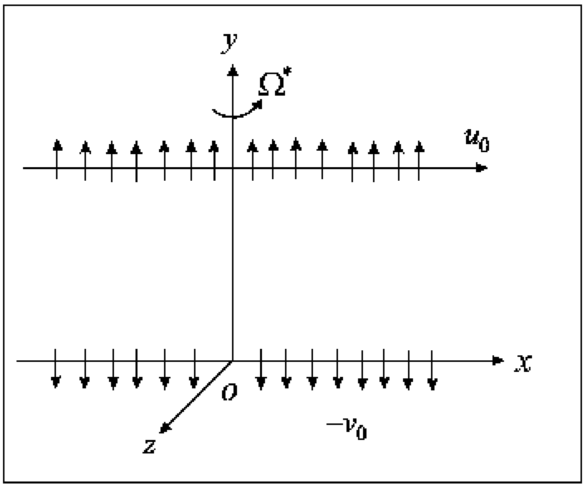

Figure 1: Geometry of the problem

The Navier-Stokes equations of motion along x-and z-directions in a rotating frame of reference are

$$2\Omega^* w - v_0 \frac{du}{dy} = \nu \frac{d^2u}{dy^2}, \tag{2.1}$$

$$-2\Omega^* u - v_0 \frac{dw}{dy} = \nu \frac{d^2w}{dy^2}, \tag{2.2}$$

where u and w are the velocity components in the x and z-directions respectively, ρ is the fluid density and ν the kinematic viscosity.

The energy equation is

$$-v_0 \frac{dT}{dy} = \frac{k}{\rho c_p}\frac{d^2T}{dy^2} + \frac{\mu}{\rho c_p}\left[\left(\frac{du}{dy}\right)^2 + \left(\frac{dw}{dy}\right)^2\right], \tag{2.3}$$

where μ is the coefficient of viscosity, k the thermal conductivity, c_p the specific heat at constant pressure

The boundary conditions are

$$u = w = 0,\ T = T_0 \ \text{at}\ \ y = 0,$$

$$u = u_0,\ w = 0,\ T = T_w \ \text{at}\ \ y = h. \tag{2.4}$$

Introducing the non-dimensional variables

$$\eta = \frac{y}{h},\ u_1 = \frac{u}{u_0},\ w_1 = \frac{w}{u_0},\ \theta = \frac{T - T_0}{T_w - T_0}, \tag{2.5}$$

equations (2.2), (2.3) and (2.4) become

$$2\mathrm{K}^2 w_1 - \mathrm{S}\frac{du_1}{d\eta} = \frac{d^2u_1}{d\eta^2}, \tag{2.6}$$

$$-2\mathrm{K}^2 u_1 - \mathrm{S}\frac{dw_1}{d\eta} = \frac{d^2w_1}{d\eta^2}, \tag{2.7}$$

$$\mathrm{S}\,\mathrm{Pr}\frac{d\theta}{d\eta} = \frac{d^2\theta}{d\eta^2} + \mathrm{Br}\left[\left(\frac{du_1}{d\eta}\right)^2 + \left(\frac{dw_1}{d\eta}\right)^2\right], \tag{2.8}$$

where $\mathrm{Br} = \dfrac{\mu u_0^2}{k(T_w - T_0)}$ is the Brinkmann number which is a measure of magnitude of viscous heating, $\mathrm{K}^2 = \dfrac{\Omega^* h^2}{\nu}$ the rotation parameter which is the reciprocal of the Ekman number, $\mathrm{S} = \dfrac{v_0 h}{\nu}$ the suction parameter and $\mathrm{Pr} = \dfrac{\mu c_p}{k}$ the Prandtl number which is inversely proportional to the thermal diffusivity of the working fluid.

Equations (2.7) and (2.8) can be combined into the following equation

$$\frac{d^2F}{d\eta^2} + \mathrm{S}\frac{dF}{d\eta} + 2i\,\mathrm{K}^2 F = 0, \tag{2.9}$$

where

$$F = u_1 + i w_1 \ \text{ and }\ i = \sqrt{-1}. \tag{2.10}$$

The boundary conditions for $F(\eta)$ and $\theta(\eta)$ are

$$F = 0,\ \theta = 0 \ \text{at}\ \eta = 0 \ \text{ and }\ F = 1,\ \theta = 1 \ \text{at}\ \eta = 1. \tag{2.11}$$

The solution of the equation (2.9) subject to the boundary conditions (2.11) is

$$F(\eta) = \frac{\sinh(\alpha - i\beta)\eta}{\sinh(\alpha - i\beta)}\, e^{\frac{S}{2}(1-\eta)}. \tag{2.12}$$

On the use of equation (2.10) and separating into a real and imaginary parts, we get

$$u_1 = \frac{2e^{\frac{S}{2}(1-\eta)}}{\cosh 2\alpha - \cos 2\beta}[\sinh\alpha\eta\cos\beta\eta\sinh\alpha\cos\beta + \cosh\alpha\eta\sin\beta\eta\cosh\alpha\sin\beta], \tag{2.13}$$

$$w_1 = \frac{2e^{\frac{S}{2}(1-\eta)}}{\cosh 2\alpha - \cos 2\beta}[\sinh\alpha\eta\cos\beta\eta\cosh\alpha\sin\beta - \cosh\alpha\eta\sin\beta\eta\sinh\alpha\cos\beta], \tag{2.14}$$

where

$$\alpha, \beta = \frac{1}{\sqrt{2}}\left[\left(\frac{S^4}{16} + 4K^4\right)^{\frac{1}{2}} \pm \frac{S^2}{4}\right]^{\frac{1}{2}}. \tag{2.15}$$

The solution given by equations (2.13) and (2.14) exists for both $S < 0$ (corresponding to $v_0 < 0$ for the blowing at the plates) and $S > 0$ (corresponding to $v_0 > 0$ for the suction at the plates).
On the use of (2.13) and (2.14), the equation (2.8) becomes

$$\frac{d^2\theta}{d\eta^2} + \mathrm{SPr}\frac{d\theta}{d\eta} = -\frac{Br e^{S(1-\eta)}}{\cosh 2\alpha - \cos 2\beta}\left[\left\{(\alpha^2+\beta^2) - \frac{S^2}{4}\right\}\cosh 2\alpha\eta \right.$$
$$\left. + \left\{(\alpha^2+\beta^2) - \frac{S^2}{4}\right\}\cosh 2\beta\eta - S(\alpha\sinh 2\alpha\eta + \beta\sin 2\beta\eta)\right]. \tag{2.16}$$

Solution of equation (2.16) subject to boundary condition (2.11) is given by

$$\theta(\eta) = c_1 + c_2 e^{-\mathrm{SPr}\eta} - \frac{\mathrm{Br}e^{\mathrm{S}(1-\eta)}}{8(\cosh 2\alpha - \cos 2\beta)}\big(A_1\cosh 2\alpha\eta + A_2\sinh 2\alpha\eta$$
$$+ 2A_3\cos 2\beta\eta + 2A_4\sin 2\beta\eta\big), \tag{2.17}$$

where

$$X_1 = (2\alpha - \mathrm{S})(2\alpha - \mathrm{S} + \mathrm{SPr}),\ X_2 = (2\alpha + \mathrm{S})(2\alpha + \mathrm{S} - \mathrm{SPr}),$$
$$X_3 = -4\beta^2 + \mathrm{S}^2(1-\mathrm{Pr}),\ X_4 = 2\beta\mathrm{S}(2-\mathrm{Pr}),$$
$$A_1 = (4\alpha^2 + 4\beta^2 + \mathrm{S}^2)\left(\frac{1}{X_1} + \frac{1}{X_2}\right) - 4\alpha\mathrm{S}\left(\frac{1}{X_1} - \frac{1}{X_2}\right),$$
$$A_2 = (4\alpha^2 + 4\beta^2 + \mathrm{S}^2)\left(\frac{1}{X_1} - \frac{1}{X_2}\right) - 4\alpha\mathrm{S}\left(\frac{1}{X_1} + \frac{1}{X_2}\right),$$
$$A_3 = \frac{4(\alpha^2 + 4\beta^2 - \mathrm{S}^2)X_3 - 4\mathrm{S}\beta X_4}{X_3^2 + X_4^2},\ A_4 = \frac{4(\alpha^2 + 4\beta^2 - \mathrm{S}^2)X_4 + 4\mathrm{S}\beta X_3}{X_3^2 + X_4^2}.$$
$$A_4 = \frac{\mathrm{Br}e^{\mathrm{S}}}{8(\cosh 2\alpha - \cos 2\beta)}\big(A_1 + 2A_3\big), \tag{2.18}$$
$$A_5 = \frac{\mathrm{Br}}{8(\cosh 2\alpha - \cos 2\beta)}\big(A_1\cosh 2\alpha + A_2\sinh 2\alpha + 2A_3\cos 2\beta + 2A_4\sin 2\beta\big),$$
$$c_1 = -\frac{A_4 e^{-\mathrm{SPr}} - A_5 - 1}{1 - e^{-\mathrm{SPr}}},\ c_2 = \frac{A_4 - A_5 - 1}{1 - e^{-\mathrm{SPr}}}.$$

3 Results and Discussion

To study the effects of rotation and suction/blowing on the velocity field we have presented the non-dimensional velocity components u_1 and w_1 against η in Figs.2-3 for several values of rotation parameter K^2 and suction parameter S. It is seen from Fig.2 that the primary velocity u_1 decreases whereas the secondary velocity w_1 increases with an increase in rotation parameter K^2. Rotation is known to induce

the secondary fluid velocity in the flow-field by suppressing the primary fluid velocity. This is due the reason that Coriolis force is dominant in the region near to the axis of rotation. Significant influence on the fluid velocity components is prevalent with rotation up till around the middle of the channel. Fig.3 shows that both the primary velocity u_1 and the secondary velocity w_1 increase with an increase in suction parameter S. As S increases, fluid injection into the channel increases. This leads to an increase in the fluid flow in the channel. It is also asymmetrically distributed because of the impulsive motion of one of the bounding plates. We have presented the temperature distribution θ against η in Figs.4-7 for several values of rotation parameter K^2, suction parameter S, Brinkmann number Br and Prandlt number Pr. It is seen from Figs.4-7 that the fluid temperature θ increases with an increase in either rotation parameter K^2 or suction parameter S or Brinkmann number Br or Prandlt number Pr. The terms linked to the Brinkmann number act as strong heat sources in the energy equation. An increases in the Brinkman number leads to increase the fluid temperature significantly shown in Fig.6. Larger values of the Prandtl number correspondingly increase the strength of the heat sources in the energy equation and hence this strength in turn increases the overall fluid temperature as clearly illustrated in Fig. 7.
The rate of heat transfer at the plates $\eta = 0$ and $\eta = 1$ can be obtained from (40) as

$$\theta'(0) = -\mathrm{S}\,\mathrm{Pr}\, c_2 + \frac{\mathrm{Br}e^{\mathrm{S}}}{16(\cosh 2\alpha - \cos(2\beta))}\left[(A_1 + 2A_3)\mathrm{S} - 2(\alpha A_2 + 2\beta A_4)\right], \tag{3.19}$$

$$\begin{aligned}\theta'(1) = &-\mathrm{S}\,\mathrm{Pr}\, c_2 e^{-\mathrm{S}\mathrm{Pr}} + \frac{\mathrm{Br}\, e^{-\mathrm{S}\mathrm{Pr}}}{16(\cosh 2\alpha - \cos 2\beta)}\\ &\times\big[(A_1\cosh 2\alpha + A_2\sinh 2\alpha + 2A_3\cos 2\beta + 2A_4\sin 2\beta)S\\ &-2(A_1\alpha\cosh 2\alpha + A_2\alpha\sinh 2\alpha - 2A_3\beta\sin 2\beta + 2A_4\beta\cos 2\beta)\big],\end{aligned} \tag{3.20}$$

where α, β, A_1, A_2, A_3 and A_4 are given by (2.18).

The numerical values of the rate of heat transfers $\theta'(0)$ and $-\theta'(1)$ are entered in Tables 1 and 2 for several values of K^2, S, Br and Pr. It is seen from Table 1 that the rate of heat transfer $\theta'(0)$ at the stationary plate $\eta = 0$ increases with an increase in either rotation parameter or suction parameter S. On the other hand, the rate of heat transfer $-\theta'(1)$ at the moving plate $\eta = 1$ increases with an increase in rotation parameter K^2 while it decreases with an increase in suction parameter S. The influence of increasing the suction velocity is to increase the rate of heat transfer at the stationary plate, but increasing the suction velocity is to further decrease the rate of heat transfer at the moving plate. This is expected from the physical point of view. The viscous fluid is more heated with increasing values of rotation parameter and hence the rate of heat transfer at the plates is enhanced. Table 2 that shows the rate of heat transfers $\theta'(0)$ and $-\theta'(1)$ increase with an increase in either Brinkmann number Br or Prandtl number Pr. It is expected because increase in the values of Pr actually leads to decrease in thermal conductivity of the fluid, which reduces the frictional forces between the viscous layers. Consequently, rate of heat transfer increases in the vicinity of the plates. The negative value of $\theta'(1)$ signifies that the fluid flows to the plate $\eta = 1$.

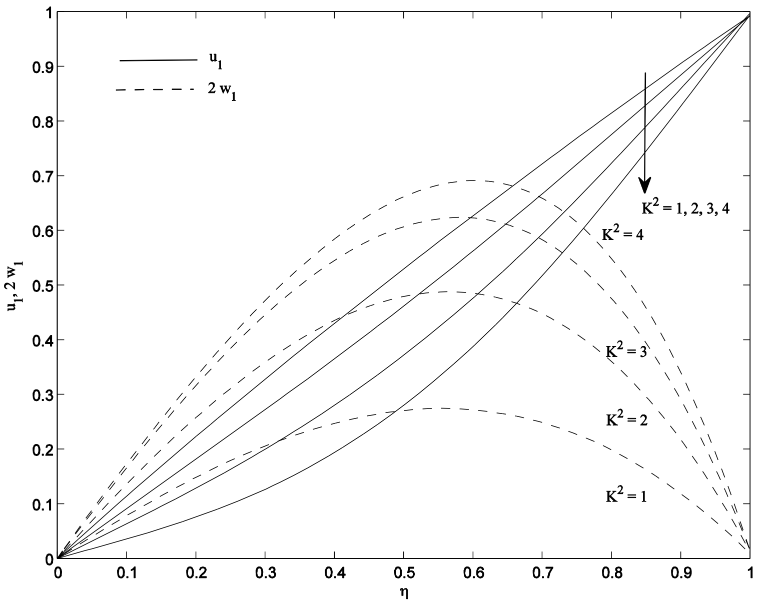

Figure 2: Velocity profiles for different K^2 when $S = 0.5$

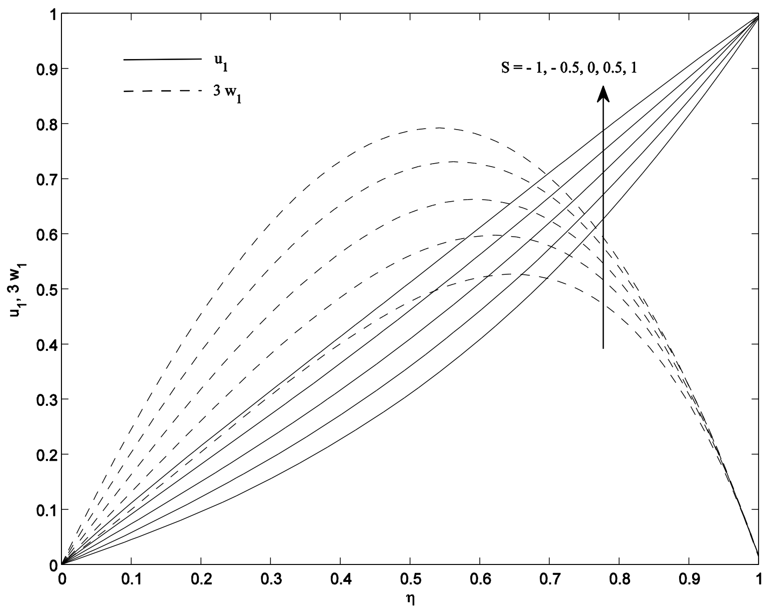

Figure 3: Velocity profiles for different S when $K^2 = 2$

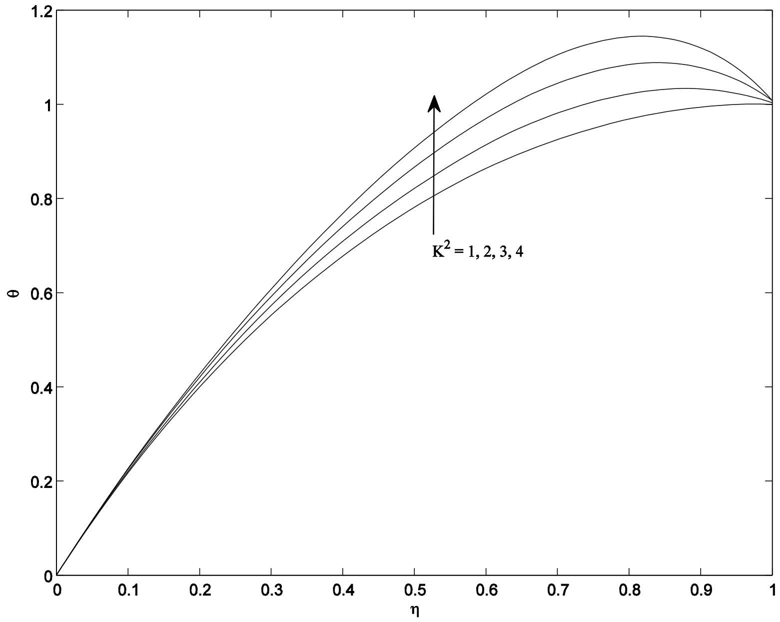

Figure 4: Temperature profiles for different K^2 when $S = 0.5$, $Pr = 0.71$ and $Br = 0.5$

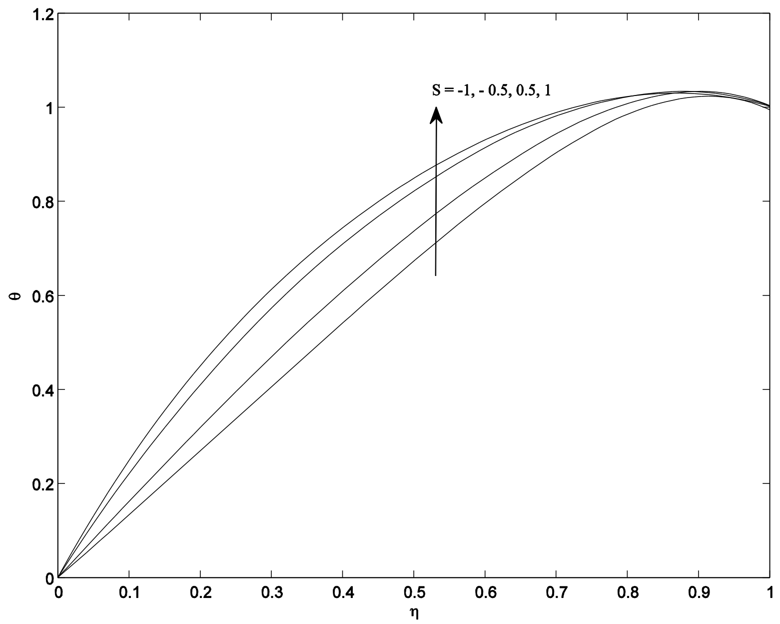

Figure 5: Temperature profiles for different S when $K^2 = 2$, $Pr = 0.71$ and $Br = 0.5$

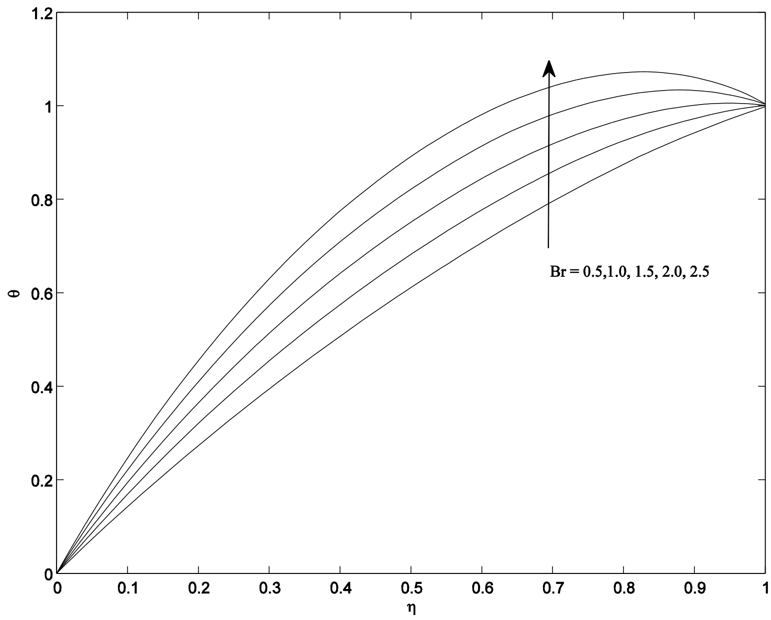

Figure 6: Temperature profiles for different Br when $K^2 = 2$, $Pr = 0.71$ and $S = 0.5$

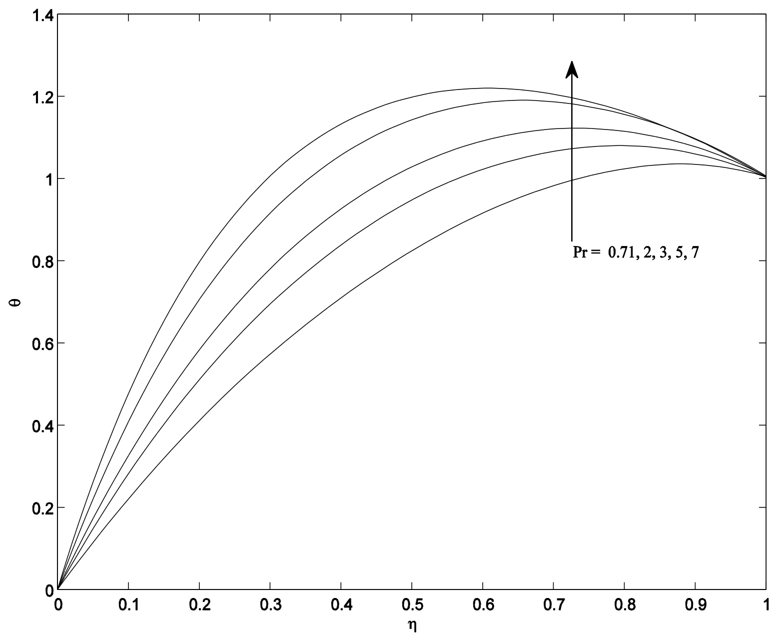

Figure 7: Temperature profiles for different Pr when $K^2 = 2$, $S = 0.5$ and $Br = 0.5$

Table 1: Rate of heat transfer at the plates $\eta = 0$ and $\eta = 1$ when $Br = 0.5$ and $Pr = 0.71$

	$\theta'(0)$				$-\theta'(1)$			
$K^2 \backslash S$	-1	-0.5	0.5	1	-1	-0.5	0.5	1
2	1.33341	1.64601	2.38449	2.78027	0.77066	0.69476	0.54172	0.46671
3	1.33595	1.64513	2.39568	2.82146	1.29341	1.24957	1.13910	1.06867
4	1.33674	1.64452	2.40128	2.84215	1.84566	1.82805	1.74472	1.67339
5	1.33763	1.64452	2.40380	2.85233	2.36967	2.36934	2.30216	2.22950

Table 2. Rate of heat transfer at the plates $\eta = 0$ and $\eta = 1$ with $S = 0.5$ and $K^2 = 2$

	$\theta'(0)$				$-\theta'(1)$			
$Br \setminus Pr$	0.71	2	3	5	0.71	2	3	5
2.0	2.38449	3.09849	3.65725	4.72743	0.54172	0.80646	0.94903	1.07248
2.5	2.68362	3.47762	4.08886	5.22839	0.88539	1.15356	1.29400	1.39648
3.0	2.98274	3.85675	4.52046	5.72936	1.22906	1.50067	1.63896	1.72049
3.5	3.28187	4.23588	4.95207	6.23032	1.57274	1.84778	1.98393	2.04450

4 Entropy Generation

Entropy analysis is a medium to quantified the thermodynamics irreversibility in any fluid flow process. The first law of thermodynamics is simply an expression of the conservation of energy principle. The second law of thermodynamics states that all real processes are irreversible. Entropy generation is a measure of the account of irreversibility associated to the real processes. As entropy generation takes place, the quality of energy decreases. In order to preserve the quality of energy in a fluid flow process or at least to reduce the entropy generation, it is important to study the distribution of the entropy generation within the fluid volume. The optimal design for any thermal system can be achieved by minimizing entropy generation in the systems. Entropy generation in thermal engineering systems destroys available work and thus reduces its efficiency. Many studies have been published to assess the sources of irreversibility in components and systems. The entropy generation rate (see Baytas [43] and Bejan [44]) is define as

$$E_G = \frac{k}{T_0^2}\left(\frac{dT}{dy}\right)^2 + \frac{\mu}{T_0}\left[\left(\frac{du}{dy}\right)^2 + \left(\frac{dw}{dy}\right)^2\right]. \tag{4.21}$$

The entropy generation equation consists of two parts, the first part is the irreversibility due to finite temperature gradient and generally termed as heat transfer irreversibility, this part is due to conduction and the second is due to viscous dissipation which termed as fluid friction irreversibility .

The dimensionless entropy generation number may be defined by the following relationship:

$$N_S = \frac{T_0^2 h^2 E_G}{k(T_w - T_0)^2}. \tag{4.22}$$

In terms of the dimensionless velocity and temperature, the entropy generation number becomes

$$N_S = \left(\frac{d\theta}{d\eta}\right)^2 + \frac{Br}{\Omega}\left[\left(\frac{du_1}{d\eta}\right)^2 + \left(\frac{dw_1}{d\eta}\right)^2\right], \tag{4.23}$$

where $\Omega = \dfrac{T_w - T_0}{T_0}$ is the non-dimensional temperature difference.

The entropy generation number N_S can be written as a summation of the local entropy generation due to heat transfer denoted by N_h and the local entropy generation due to fluid friction, given as N_f , as follows

$$N_h = \left(\frac{d\theta}{d\eta}\right)^2, \ N_f = \frac{Br}{\Omega}\left[\left(\frac{du_1}{d\eta}\right)^2 + \left(\frac{dw_1}{d\eta}\right)^2\right]. \tag{4.24}$$

The Bejan number Be is an alternative irreversibility distribution parameter and it represents the ratio between the heat transfer irreversibility N_h and the total irreversibility due to heat transfer and fluid friction N_S . It is defined by

$$\mathrm{Be} = \frac{\mathrm{N_h}}{\mathrm{N_S}} = \frac{1}{1+\Phi}, \tag{4.25}$$

where $\Phi = \frac{\mathrm{N_f}}{\mathrm{N_h}}$ is the irreversibility ratio.

The Bejan number takes the values between 0 and 1, see Cimpean et al. [45]. The value of $\mathrm{Be}=1$ is the limit at which the heat transfer irreversibility dominates, $\mathrm{Be}=0$ is the opposite limit at which the irreversibility is dominated by fluid friction effects and $Be = 0.5$ is the case in which the heat transfer and fluid friction entropy production rates are equal (see Varol et al. [46, 47]).

Effects of the suction parameter S, rotation parameter K^2, Brinkmann number Br, Prandtl number Pr and group parameter $\mathrm{Br}\,\Omega^{-1}$ on the entropy generation number and Bejan number have been shown in Figs.8-17. It is seen from Fig.8 that the entropy generation number increases near the stationary plate $\eta=0$ while it decreases near the moving plate $\eta=1$ with increase in suction parameter S. It is observed from Fig.9 that the entropy generation number increases near the stationary plate $\eta=0$ while it decreases near the moving plate $\eta=1$ with an increase in rotation parameter K^2. Fig.10 reveals that the entropy generation number increases for $0 \le \eta \le 0.52$ and it decreases for $0.52 < \eta \le 1$ with an increase in the Brinkmann number Br. It is emphasized that Brinkman number Br is a measure of fluid friction in the dissipative flow system. Larger vales of Br are indicative of larger frictional heating in the system. Thus Br contributes significantly in entropy generation as one of the pertinent fluid friction irreversibility parameter. It is observed from Fig.11 that the entropy generation number increases near the stationary plate $\eta=0$ while it decreases near the moving plate $\eta=1$ with an increase in Prandlt number Pr. The figure reveals an increase in entropy generation as Pr increases. This is due to increase in temperature gradient as Pr increases. Fig.12 shows that the entropy generation number increases with an increase in group parameter $\mathrm{Br}\,\Omega^{-1}$. Since the group parameter determines the relative importance of viscous effects, therefore the magnitude of entropy generation takes higher value for higher group parameter. In Figs.8-12, the entropy generation rate is expectedly maximum at the plate where velocity and temperature gradients as well as fluid viscosity are highest and minimum at the channel centerline. Fig.13 reveals that the Bejan number decreases with an increase in suction parameter S. It is observed from Fig.14 that the Bejan number increases for $0 \le \eta \le 0.58$ and it decreases for $0.58 < \eta \le 1$ with an increase in the rotation parameter K^2. Fig.15 reveals that the Bejan number increases for $0 \le \eta \le 0.58$ and it decreases for $0.58 < \eta \le 1$ with an increase in the Brinkmann number Br. It is observed from Fig.16 that the Bejan number increases near the stationary plate $\eta=0$ while it decreases near the moving plate $\eta=1$ with an increase in Prandlt number Pr. Fig.17 shows that the Bejan number decreases for $0 \le \eta \le 0.88$ and it increases for $0.88 < \eta \le 1$ with an increase in group parameter $\mathrm{Br}\,\Omega^{-1}$. The group parameter is an important dimensionless number for irreversibility analysis. It determines the relative importance of viscous effects to that of temperature gradient entropy generation. In these figures, we see that in the lower part of the channel, the contribution in the entropy generation is due to heat transfer irreversibility. Near walls, the contribution to entropy production is mainly due to the rate of heat transfer and at some distance above the mid region of the channel, main contribution to the entropy production is due to fluid friction.

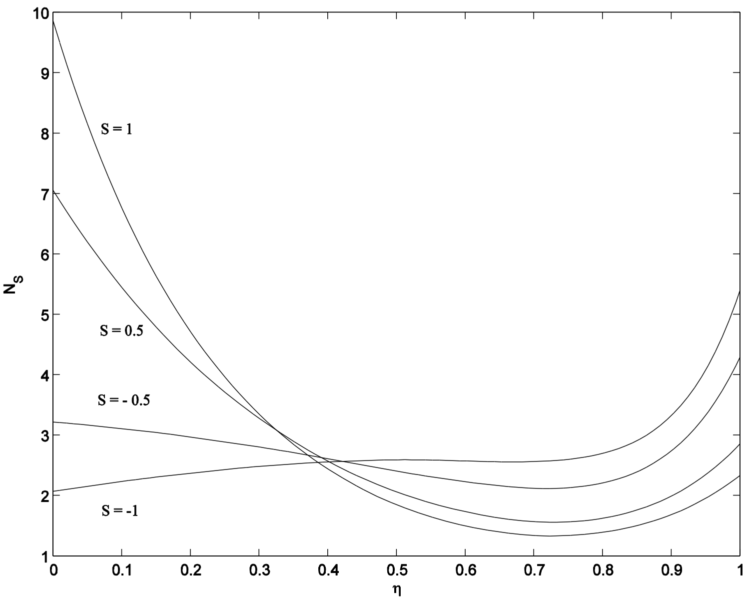

Figure 8: N_S for different S when $K^2 = 2$, $Pr = 0.71$, $Br\Omega^{-1} = 1$ and $Br = 0.5$

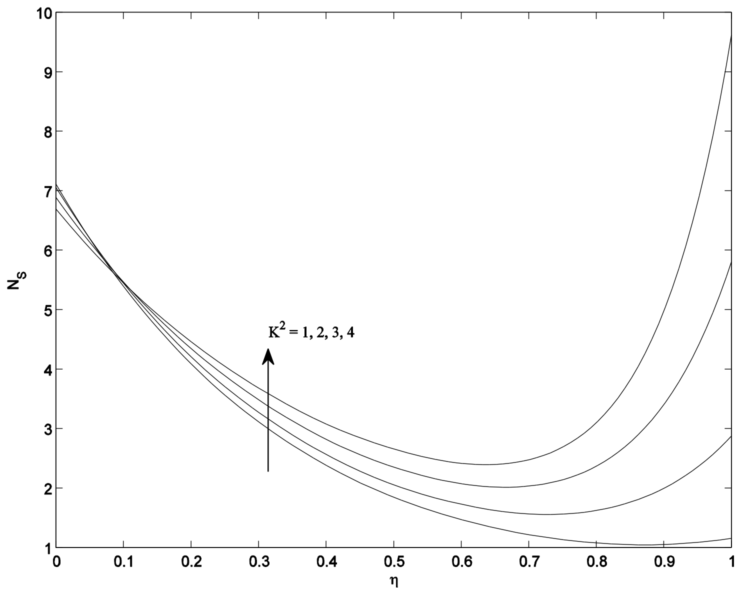

Figure 9: N_S for different K^2 when $S = 0.5$, $Pr = 0.71$, $Br\Omega^{-1} = 1$ and $Br = 0.5$

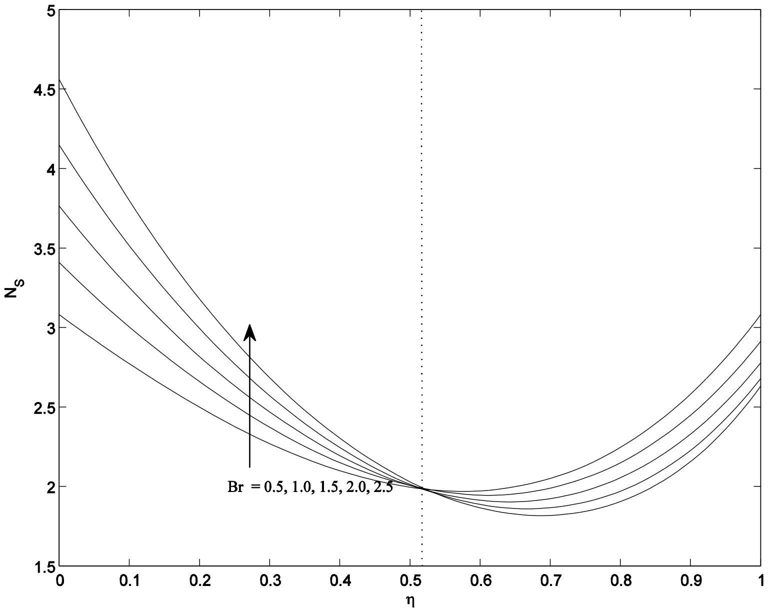

Figure 10: N_S for different Br when $K^2 = 2$, $Pr = 0.71$, $Br\Omega^{-1} = 1$ and $S = 0.5$

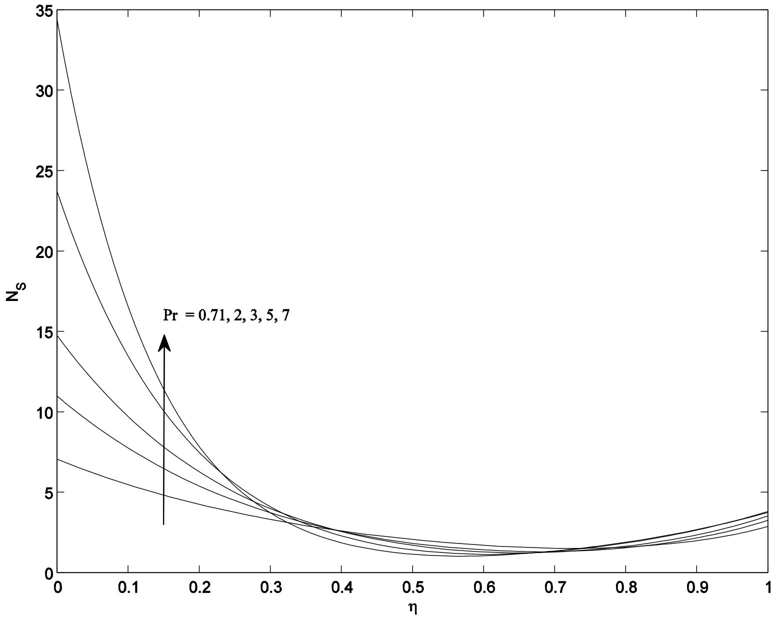

Figure 11: N_S for different Pr when $K^2 = 2$, $S = 0.5$, $Br\Omega^{-1} = 1$ and $Br = 0.5$

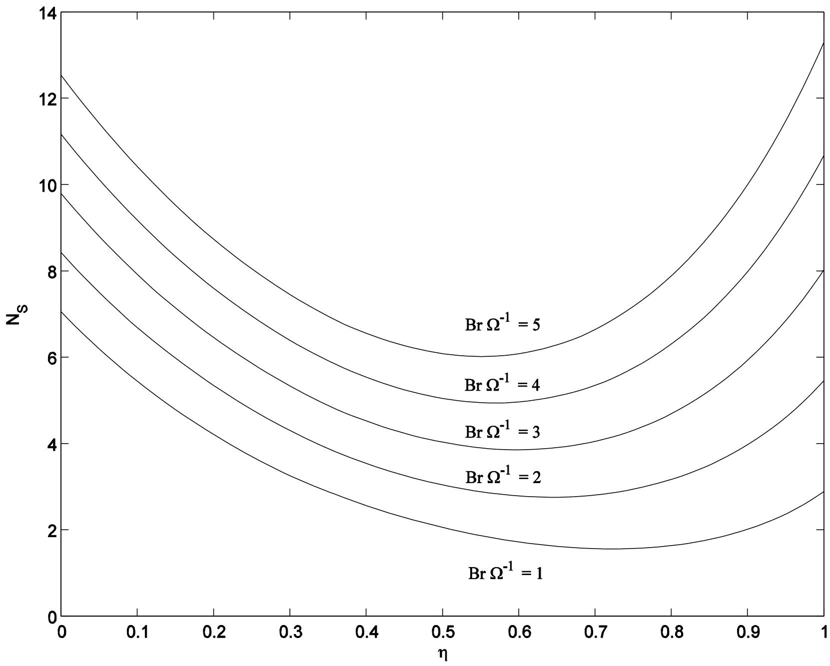

Figure 12: N_S for different $\mathrm{Br}\,\Omega^{-1}$ when $\mathrm{K}^2 = 2$, $\mathrm{Pr} = 0.71$, $\mathrm{S} = 0.5$ and $\mathrm{Br} = 0.5$

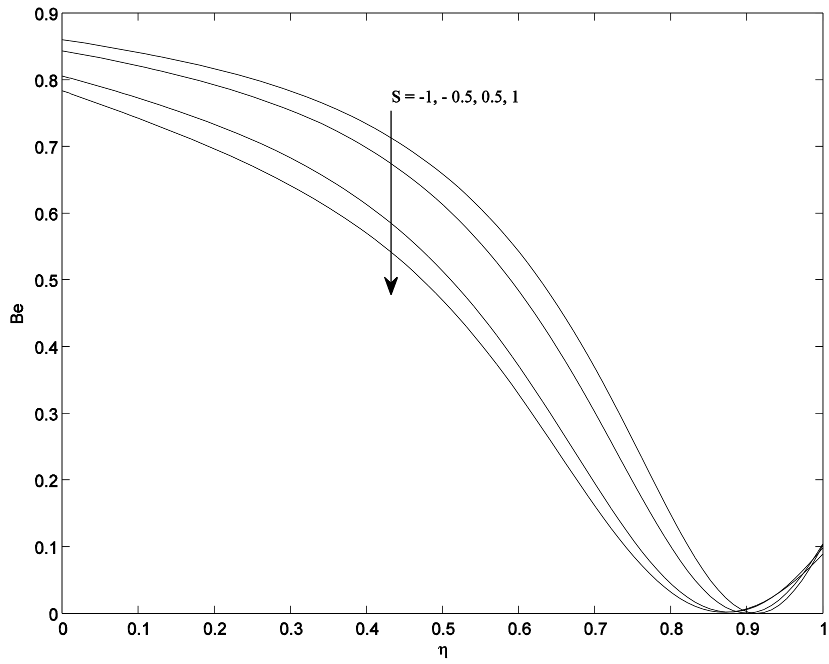

Figure 13: Bejan number for different S when $\mathrm{K}^2 = 2$, $\mathrm{Pr} = 0.71$, $\mathrm{Br}\,\Omega^{-1} = 1$ and $\mathrm{Br} = 0.5$

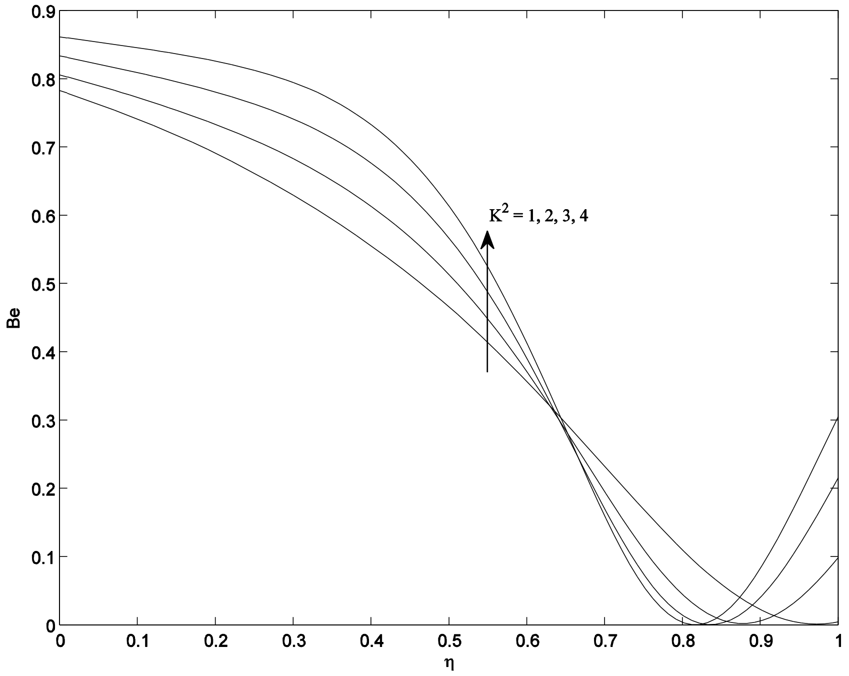

Figure 14: Bejan number for different K^2 when $S = 0.5$, $Pr = 0.71$, $Br\Omega^{-1} = 1$ and $Br = 0.5$

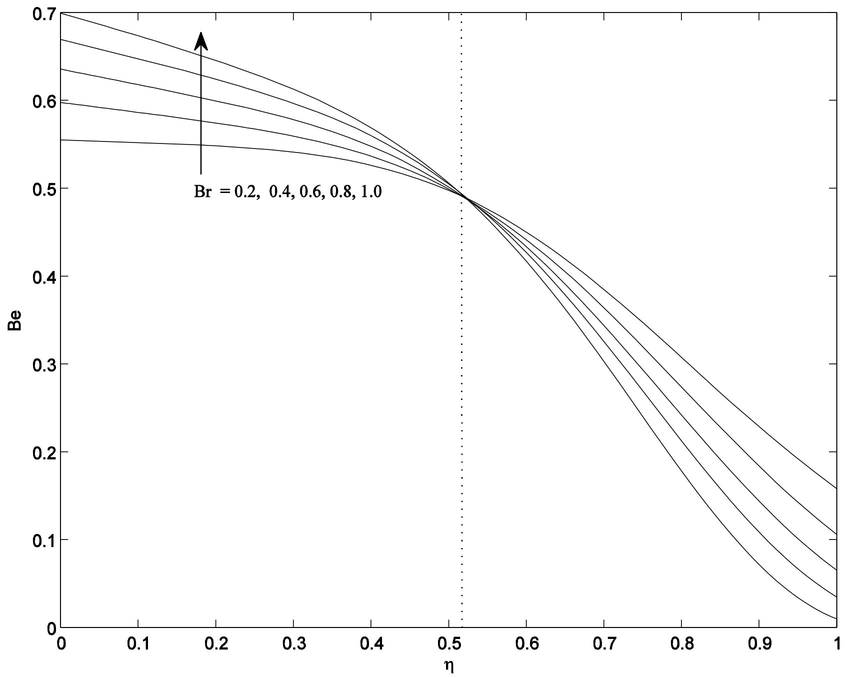

Figure 15: Bejan number for different Br when $K^2 = 2$, $Pr = 0.71$, $Br\Omega^{-1} = 1$ and $S = 0.5$

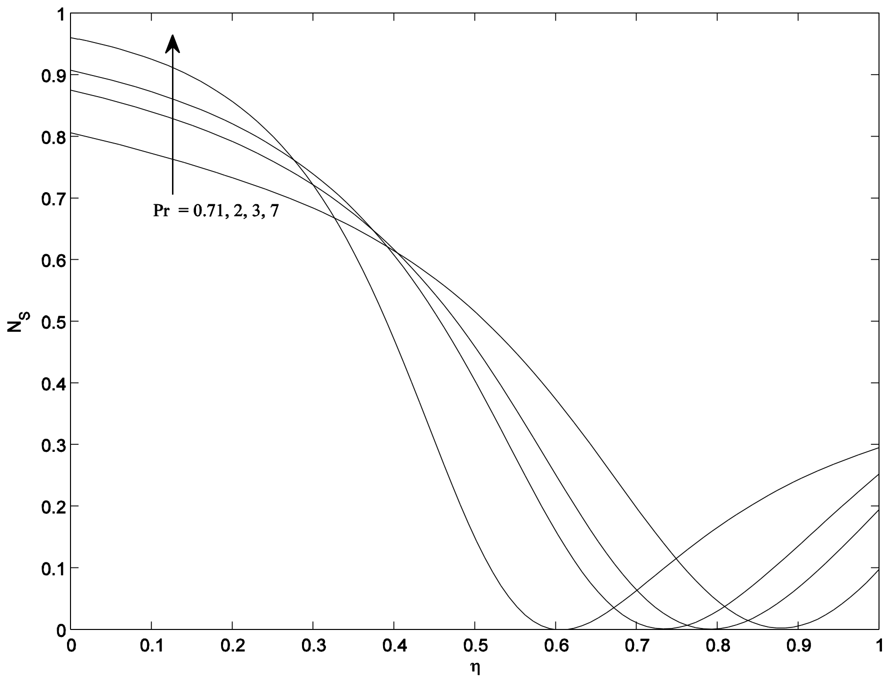

Figure 16: Bejan number for different Pr when $K^2 = 2$, $S = 0.5$, $Br\Omega^{-1} = 1$ and $Br = 0.5$

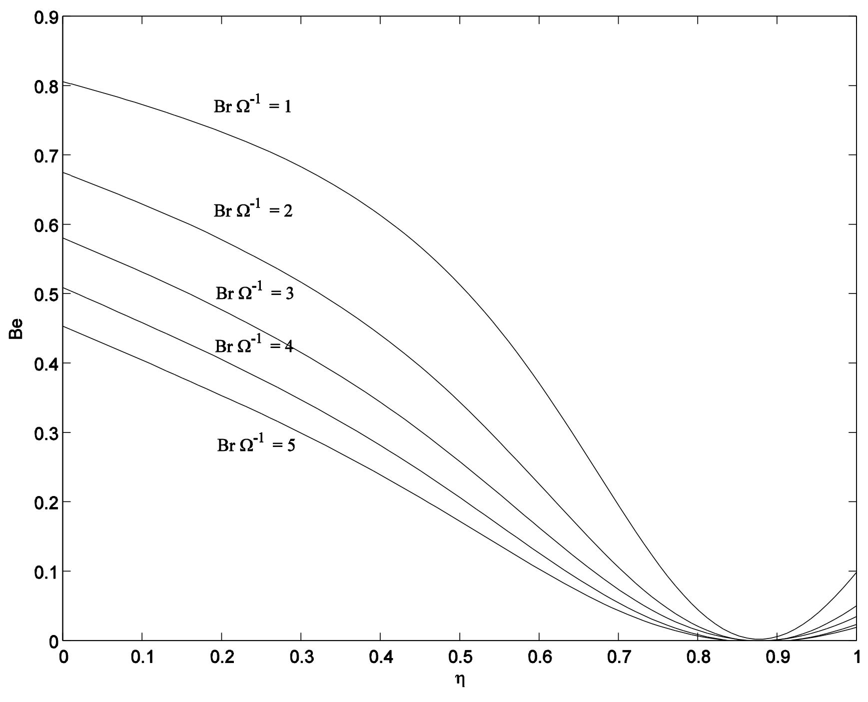

Figure 17: Bejan number for different $Br\Omega^{-1}$ when $K^2 = 2$, $Pr = 0.71$, $S = 0.5$ and $Br = 0.5$

5 Conclusion

In the present paper, we have studied the entropy generation in a Couette flow in a rotating system. The velocity and temperature distributions are evaluated to compute the entropy production. The non-dimensional entropy generation number and the Bejan number are calculated for the problem involved. It is found that the entropy generation increases with an increase either rotation parameter or suction parameter or Brinkmann number or Prandtl number or group parameter. Rotation as well as suction/injection exert a significant influence on the velocity and temperature distributions, which transitively affects the entropy generation within the channel. It is also found that magnitude of entropy generation due to fluid friction takes higher value for higher group parameter. Entropy minimization takes place when both plates are almost at the same temperature. When this same temperature value is higher, minimum value of entropy tends to take place at higher temperature. It is expected that the present model would serve as an important study and a designing tool.

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