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Technical Notes

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Singular Behavior in Boundary-Layer Flow of a Dusty Gas

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Introduction

NUMERICAL solutions for steady boundary-layer flow of a dusty gas past an impermeable semi-infinite flat plate that exhibit singular behavior in the particle phase density at the plate surface have been reported by Prabha and Jain[1] and Osiptsov.[2] In neither of these papers is an opinion given regarding whether the singularity is a feature of the dusty gas model or a numerical artifact. Certain approximate local closed-form solutions suggest the former. To gain further insight into this matter, fluid phase suction is employed herein to remove the singularity. Solutions are then computed for decreasing amounts of suction, and it is shown that the results obtained in this way appear to be approaching the behavior reported by Prabha and Jain[1] and Osiptsov.[2] It is concluded, therefore, that the singularity associated with an impermeable plate is a property of the dusty gas equations.

Governing Equations

The boundary-layer form of the governing equations for the problem under consideration can be written as (assuming an incompressible fluid phase)

$$\partial_x u + \partial_y v = 0$$
$$u\partial_x u + v\partial_y u - \nu\partial_{yy}^2 u + (\rho_p/\rho)(u - u_p)/\tau = 0$$
$$\partial_x(\rho_p u_p) + \partial_y(\rho_p v_p) = 0 \tag{1}$$
$$u_p\partial_x u_p + v_p\partial_y u_p + (u_p - u)/\tau = 0$$
$$u_p\partial_x v_p + v_p\partial_y v_p + (v_p - v)/\tau = 0$$

where x is the tangential coordinate, y the normal coordinate, u the fluid phase tangential velocity, v the fluid phase normal velocity, u_p the particle phase tangential velocity, v_p the particle phase normal velocity, ρ the fluid phase in-suspension density, ρ_p the particle phase in-suspension density, ν the fluid phase kinematic viscosity, and τ the momentum relaxation time (see Marble[3]).

It is convenient in the following to substitute the transformations

$$x = V_\infty\tau\xi/(1-\xi), \qquad y = [2\nu\tau\xi/(1-\xi)]^{1/2}\eta$$
$$\rho_p = \rho_{p\infty}Q_p(\xi,\eta), \qquad u = V_\infty F(\xi,\eta)$$
$$v = [\nu(1-\xi)/(2\tau\xi)]^{1/2}[G(\xi,\eta) + \eta F(\xi,\eta)] \tag{2}$$
$$u_p = V_\infty F_p(\xi,\eta)$$
$$v_p = [\nu(1-\xi)/(2\tau\xi)]^{1/2}[G_p(\xi,\eta) + \eta F_p(\xi,\eta)]$$

(where V_∞ and $\rho_{p\infty}$ are the freestream velocity and particle-phase density, respectively) into Eq. (1) to yield

$$\partial_\eta G + F + 2\xi(1-\xi)\partial_\xi F = 0$$
$$\partial_{\eta\eta}F - G\,\partial_\eta F - 2\xi(1-\xi)F\partial_\xi F + 2\xi\kappa Q_p(F_p - F)/(1-\xi) = 0$$
$$\partial_\eta(Q_pG_p) + Q_pF_p + 2\xi(1-\xi)\partial_\xi(Q_pF_p) = 0 \tag{3}$$
$$G_p\partial_\eta F_p + 2\xi(1-\xi)F_p\partial_\xi F_p + 2\xi(F_p - F)/(1-\xi) = 0$$
$$G_p\,\partial_\eta G_p - \eta F_p^2 + 2\xi(1-\xi)F_p\,\partial_\xi G_p + 2\xi(G_p - G)/(1-\xi) = 0$$

where $\kappa = \rho_{p\infty}/\rho$ is the particle loading.

The corresponding boundary and matching conditions are

$$F(\xi,0) = 0, \qquad G(\xi,0) = -G_w$$
$$F \sim 1, \quad F_p \sim 1, \quad G_p \sim G, \quad Q_p \sim 1 \quad \text{as} \quad \eta \to \infty \tag{4}$$

In the present work G_w will be taken to be constant. This leads to an unrealistic suction distribution but is sufficient to produce nonsingular behavior.

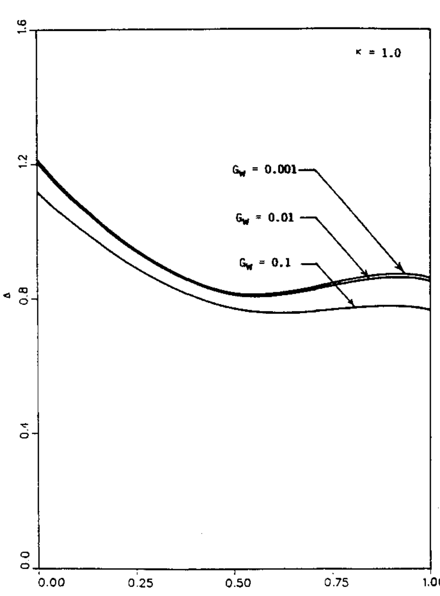

Fig. 1 Fluid-phase displacement thickness vs position.

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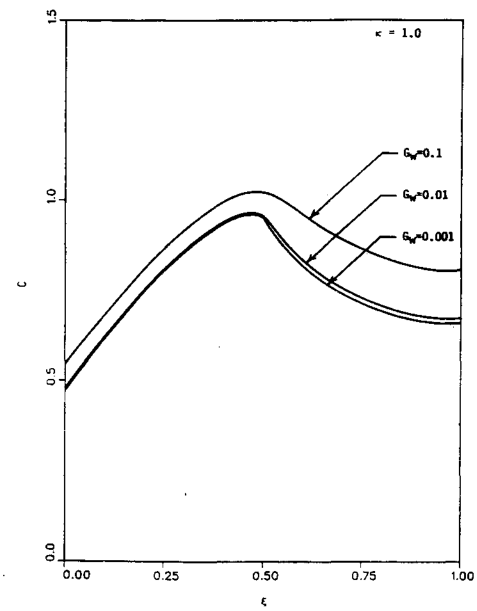

Fig. 2 Fluid-phase skin-friction coefficient vs position.

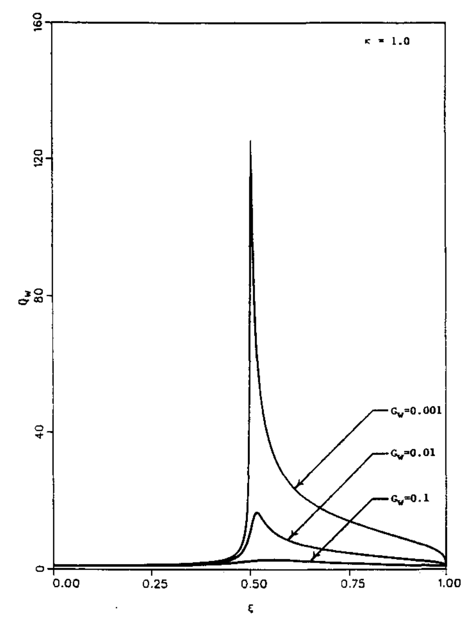

Fig. 4 Particle-phase wall density vs position.

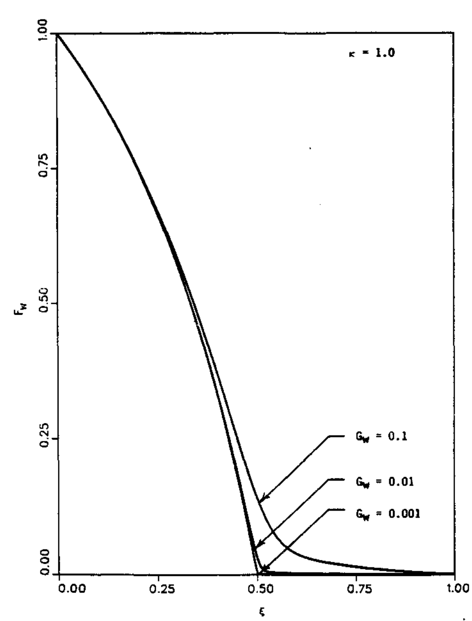

Fig. 3 Particle-phase wall tangential velocity vs position.

In the subsequent presentation of numerical results, reference will be made to the fluid phase displacement thickness and skin-friction coefficients and the particle phase wall velocity and density. These are, respectively,

$$\Delta(\xi) = \int_0^\infty [1 - F(\xi, \eta)]\mathrm{d}\eta, \qquad C(\xi) = \partial_\eta F(\xi, 0)$$
$$F_w(\xi) = F_p(\xi, 0), \qquad Q_w(\xi) = Q_p(\xi, 0) \tag{5}$$

Results and Discussion

Equations (3) were solved subject to Eqs. (4) by an extension of a standard implicit finite difference method for single-phase boundary-layer flow (see, e.q., Blottner[4]) to the dusty gas. The results associated with a solid wall ($G_w = 0$) were found to exhibit the same catastrophic growth in the particle phase density at the wall reported by Prabha and Jain[1] and Osiptsov.[2] For the sake of brevity, only results associated with a porous wall ($G_w \neq 0$) will be presented herein. For such cases finite continuous solutions exist in the entire range $0 \leq \xi \leq 1$.

Figures 1–4 illustrate the respective influence of reductions in the suction parameter G_w on the quantities Δ, C, F_w, and Q_w. It can be seen that, even for very small values of G_w, singularity free solutions exist. It is expected that the solution for an impermeable wall ($G_w = 0$) will be approached by successively reducing the amount of suction, and this appears to be confirmed by Figs. 1–4. In particular, Fig. 4 suggests an approach to the singular behavior reported by Prabha and Jain[1] and Osiptsov.[2] These figures are representative of the results of a large number of computations involving various combinations of parameters.

Conclusion

The problem of steady laminar boundary-layer flow of a dusty gas over a semi-infinite flat plate was solved numerically using an implicit finite difference method. Fluid phase suction was employed to create a limiting process in which the solution for an impermeable plate was approached by a gradual reduction in the amount of suction. The results obtained in this way provided evidence that the singular behavior in numerical solutions for an impermeable plate reported by previous investigators is a property of the dusty gas equations.

The work reported in the present paper, together with that contained in Refs. 1 and 2, indicates that the dusty gas model, by itself, is inadequate for the solution of the problem of steady boundary-layer flow past a semi-infinite flat plate. Many previous investigators have pointed out that the particle phase of a dusty gas is a pressureless, inviscid, compressible material that may exhibit discontinuities. It is possible, therefore, that supplementing the dusty gas model with an appropriate theory of discontinuities would remove the singular behavior discussed herein. It is also possible (as suggested by Soo[5]) that the singularity in the particle phase density indicates that a packed bed of particles will form on the plate surface. Modeling this phenomenon is clearly beyond the capacity of the dusty gas model, which is based on the assumptions of an infinitesimal particle phase volume fraction and the absence of particle phase stresses. It is clear that in a packed bed the volume fraction would be finite and significant particle phase stresses (due to particle-particle interactions) would exist.

Finally, it should be mentioned that experimental results would be most helpful in resolving the issues raised earlier. The present authors have been unable to locate any data for laminar flow of a suspension past a flat plate in the published literature.

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Curvature Corrections to Reynolds Stress Model for Computation of Turbulent Recirculating Flows

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Introduction

THE effect of streamline curvature on third-order velocity correlation has been experimentally investigated by Chung et al.[1] They found that the third-order correlation $\overline{u_i u_i u_j}$ in a curved-streamline field can be effectively represented by the simple gradient transport model with a model coefficient as a function of the ratio between the velocity time scale $\tau_v = k/\epsilon$ and a curvature time scale $\tau_c = \epsilon/(N^2 k)$, where $N^2 = 2(U/R)/(U/R + \partial U/\partial n)$ is the frequency squared of small oscillations of a fluid element displaced radially in a flow with a radius of curvature R. Park and Chung[2] adopted such a curvature correction to the third-order terms $\overline{kv}$ and $\overline{\epsilon v}$ and to the isotropic decay constant $C_{\epsilon 2}$ in the standard k-ϵ equations. Their curvature-dependent k-ϵ model was found satisfactory for predictions of various kinds of separated recirculating turbulent flows. More recently, Park and Chung[3] extended the curvature corrections to the Reynolds stress model for the computation of a turbulent flow over a mildly curved axisymmetric body. During the review process of the paper, one of the reviewers raised a serious question about the necessity of curvature correction to the Reynolds stress model (RSM). In fact, the RSM has been frequently applied to recirculating flows of high streamline curvature without any curvature correction. But it is noted that most of the numerical solutions by the conventional RSM show poor predictions with severe zonal dependence.[4,5] Since the streamlines are mildly curved in the test flow of Park and Chung,[3] the computational improvement by the curvature corrections is not sufficiently demonstrated.

The purpose of the present study is to examine more clearly the necessity of the curvature corrections to the RSM. Test flows selected here for comparisons are the backward-facing step flows of Pronchick[6] and Driver and Seegmiller.[7]

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Turbulence Models

The turbulent transport $(\overline{u_i u_j u_k})$ in the Reynolds stress equation is approximated by the gradient transport model of Hanjalic and Launder[8] as follows:

$$\overline{u_i u_j u_k} = -C_s \frac{k}{\epsilon}\left(\overline{u_i u_\ell}\frac{\partial \overline{u_j u_k}}{\partial x_\ell} + \overline{u_j u_\ell}\frac{\partial \overline{u_k u_i}}{\partial x_\ell} + \overline{u_\ell u_k}\frac{\partial \overline{u_i u_j}}{\partial x_\ell}\right)$$
$$C_s = 0.11 \quad (1)$$

The pressure-strain correlation term π_{ij} can be decomposed into a slow term $\pi_{ij,1}$, a rapid term $\pi_{ij,2}$, and a near-wall term $\pi_{ij,w}$. Incorporating the nonlinear effect, Sarkar and Speziale[9] developed a quadratic nonlinear model for the slow pressure-strain correlation term as follows:

$$\pi_{ij,1} = -\epsilon\{C_1' b_{ij} - C_2' \,[b_{ik}b_{kj} - (II_b/3)\delta_{ij}]\}$$
$$II_b = b_{ik}b_{ki}, \qquad C_1' = 3.4, \qquad C_2' = 4.2 \quad (2)$$

where b_{ij} is the anisotropy tensor defined by $b_{ij} = 0.5\overline{u_i u_j}/k - \delta_{ij}/3$.

The rapid term is represented by the model of Launder et al.[10] The near-wall term $\pi_{ij,w}$ is further decomposed into $\pi_{ij,w1}$ and $\pi_{ij,w2}$, which are approximated by the models of Shir[11] and Gibson and Launder,[12] respectively.

Finally, the dissipation rate equation is taken as

$$\frac{D\epsilon}{Dt} = \frac{\partial}{\partial x_\ell}\left(C_\epsilon \frac{k}{\epsilon}\overline{u_\ell u_m}\frac{\partial \epsilon}{\partial x_m}\right) + \frac{\epsilon}{k}\left(C_{\epsilon 1}P - C_{\epsilon 2}\,\epsilon\right)$$
$$C_\epsilon = 0.15, \qquad C_{\epsilon 1} = 1.44, \qquad C_{\epsilon 2} = 1.92 \quad (3)$$

In the present study, adopting the same corrections as in Park and Chung,[3] the diffusive coefficients C_s in Eq. (1) and C_ϵ in Eq. (3) are replaced by modified coefficients C_s' and C_ϵ':

$$C_s' = C_s\frac{1}{1 + aH(N^2)\tau_v/\tau_c}, \qquad C_\epsilon' = C_\epsilon\frac{1}{1 + aH(N^2)\tau_v/\tau_c} \quad (4)$$

Here, $H(N^2)$ is the Heaviside step function ($H = 1$ when $N^2 \geq 0$, and $H = 0$ when $N^2 < 0$).

In addition, the isotropic decay rate constant $C_{\epsilon 2}$ in Eq. (3) is replaced by

$$C_{\epsilon 2}' = C_{\epsilon 2}\frac{1}{1 + b\tau_v/\tau_c} \quad (5)$$

Here, the model constants a and b were proposed to be 0.12 and 0.5, respectively, by Park and Chung.[2] In the present study, however, it was found that $b = 0.15$ gives better predictions. Note that theoretically $C_{\epsilon 2}'$ is a bounded value in a range of $1.4 < C_{\epsilon 2}' < 2.0$.[13]

Computations and Discussion of the Results

The governing equations are solved using a variant of the line-by-line SIMPLE procedure, in which the velocity components are stored midway between the pressure storage locations. All of the Reynolds stresses are evaluated at the scalar node points. The hybrid differencing scheme is used with 75 × 78 fine grids to reduce false diffusion. At the inlet plane, the streamwise mean velocity profile was given by the experimental data. At the outlet, gradients of flow properties in the flow direction are zero, i.e., $d\phi/dx = 0$, where ϕ is the flow property in question. This outlet is located at 60 times the step height downstream from the backward-facing step. At the wall boundaries an improved wall treatment proposed by Ciofalo and Collins[14] was used to calculate local sublayer thickness y_v^+ and friction velocity u_τ^2.