International Journal of Numerical Methods for Heat & Fluid Flow

Lie group analysis of chemical reaction effects on MHD free convection dissipative fluid flow past an inclined porous surface

G. Venkata Ramana Reddy Ali J Chamkha

Lie group analysis of chemical reaction effects on MHD free convection dissipative fluid flow past an inclined porous surface

G. Venkata Ramana Reddy
Department of Mathematics, KL University, Guntur, India, and
Ali J. Chamkha
Manufacturing Engineering Department,
College of Technological Studies, Shuwaikh, Kuwait

Received 30 August 2014
Revised 27 October 2014
Accepted 27 October 2014

Abstract

Purpose – The purpose of this paper is to study chemical reaction and heat and mass transfer effects on steady free convection flow in an inclined porous plate in the presence of MHD and viscous dissipation through the application of scaling group of transformation and numerical method.
Design/methodology/approach – The fourth-order Runge-Kutta along with the shooting method is employed in the numerical solution of the governing equations.
Findings – The magnetic field parameter, the permeability of porous medium and the viscous dissipation are demonstrated to exert a more significant effect on the flow field and, thus, on the heat transfer from the plate to the fluid.
Originality/value – The problem is relatively original.

Keywords MHD, Chemical reaction, Viscous dissipation, Free convection, Inclined plate, Lie group analysis, Radiation, Inclined porous surface
Paper type Research paper

Nomenclature

B_0	applied magnetic field
C	species concentration in the boundary layer
C_∞	the species concentration in the fluid far away from the plate
Cp	specific heat at constant pressure
D	mass diffusivity
Ec	Eckert number
f	dimensionless stream function
g	acceleration due to gravity
Gr	local temperature Grashof number
Gc	local mass Grashof number
K'	the permeability of the porous medium
K	permeability parameter
K'_{r-}	chemical reaction parameter
k	thermal conductivity of the fluid
M	magnetic field parameter
Pr	Prandtl number
Sc	Schmidt number
T	the temperature of the fluid in the boundary layer
T_∞	the temperature of the fluid far away from the plate
u, v	velocity components in x, y directions

International Journal of Numerical Methods for Heat & Fluid Flow
Vol. 25 No. 7, 2015
pp. 1557-1573

0961-5539
DOI 10.1108/HFF-08-2014-0270

Greek Symbols			
η	similarity variable	θ	dimensionless temperature
α	angle of inclination	φ	dimensionless concentration
β	coefficient of thermal expansion	*Subscripts*	
β^*	coefficient of concentration expansion	w	condition at wall
σ	electrical conductivity	∞	condition at infinity
ρ	density of the fluid	*Superscript*	
ν	kinematic viscosity	()'	differentiation with respect to η

Introduction

The study of free convection flow for an incompressible viscous fluid past an inclined porous surface has attracted the interest of many researchers in view of its important applications to many engineering problems such as cooling of nuclear reactors, the boundary layer control in aerodynamics, crystal growth, food processing and cooling towers. Effect of porosity on the free convection flow along a vertical plate embedded in a porous medium was investigated by Beithou *et al.* (1998). Their results show that as the porosity is increased the temperature variation becomes steeper, that is, the heat transfer rate is increased. Chen (2004) studied the natural convection flow over a permeable inclined surface with variable wall temperature and concentration. The results show that the velocity is decreased in the presence of a magnetic field. Increasing the angle of inclination decreases the effect of buoyancy force. Heat transfer rate is increased when the Prandtl number is increased. Duwairi (2005), who investigated the effect of viscous and Joule heating on forced convection flow from radiative isothermal surfaces found that the heat transfer rate is decreased as the radiation parameter is increased. Radiative and magnetic effects on free convection and mass transfer flow past a flat plate were studied by Ibrahim *et al.* (2005). They obtained similarity reductions and found analytical and numerical solutions using scaling symmetry. Furthermore, the study of MHD flow of an electrically conducting fluid is of considerable interest in modern metallurgical and metal-working processes. There has been a great interest in the study of magnetohydrodynamic flow and heat transfer in any medium due to the effect of a magnetic field on the boundary layer flow control and on the performance of many systems using electrically conducting fluids. This type of flow has attracted the interest of many researchers due to its application in many engineering problems such as MHD generators, plasma studies, nuclear reactors and geothermal energy extractions. By the application of a magnetic field, hydromagnetic techniques are used for the purification of molten metals from non-metallic inclusions. Therefore, the type of problem that we are dealing with is very useful to polymer technology and metallurgy. Rashad *et al.* (2011) studied the problem of MHD free convective heat and mass transfer of a chemically reacting fluid from radiate stretching surface embedded in a saturated porous medium. Chamkha *et al.* (2013) considered the coupled heat and mass transfer by MHD free convection flow along a vertical plate with streamwise temperature and species concentration variations. Abolbashari *et al.* (2014) studied the entropy analysis for an unsteady MHD flow along a stretching surface in nanofluid. Rashad and Chamkha (2014) have analyzed heat and mass transfer by MHD natural convection flow about a truncated cone in porous media with Soret and Dufour effects. Rashidi and Mehr (2014) obtained series solutions for the flow in the vicinity of the equator of an MHD boundary layer over a porous rotating sphere with heat transfer. Sheikholeslami *et al.* (2014) presented numerical simulation of MHD nanofluid flow and heat transfer. Rashidi *et al.* (2014) studied the problem of heat and mass transfer by MHD free convection over a vertical stretching sheet with radiation and buoyancy effects.

Lie group analysis is a classical method discovered by Norwegian mathematician Sophus Lie for finding invariant and similarity solutions (Bluman and Kumei, 1989; Ibragimov, 1995; Olver, 1986; Ovsiannikov, 1982; Yurusoy and Pakdemirli, 1999). Yurusoy and Pakdemirli (Yurusoy *et al.*, 2001) presented exact solution of boundary layer equations of a special non-Newtonian fluid over a stretching sheet by the method of Lie group analysis. They extended their work to creeping flow of second-grade fluid (Sivasankaran *et al.*, 2006a). Sivasankaran *et al.* (2006b) studied the problem of natural convection heat and mass transfer flow past an inclined plate for various parameters using Lie group analysis without and with heat generation. Yurusoy and Pakdemirli (1997) studied the boundary layer equations for Newtonian/non-Newtonian fluids by using Lie group method. EL-Kabeir *et al.* (2008a) used group method analysis to study of combined heat and mass transfer by MHD non-Darcy non-Newtonian natural convection adjacent to horizontal cylinder in a saturated porous medium. They (EL-Kabeir *et al.*, 2008b) have also analyzed unsteady MHD three dimensional by natural convection from an inclined stretching surface saturated porous medium using Lie group analysis. So far no attempt has been made to study the heat and mass transfer in a porous medium using Lie groups, and hence we study the problem of MHD free convection dissipative fluid flow past an inclined porous surface with chemical reaction for various parameters using Lie groups.

In this paper, application of scaling group of transformation for chemical reaction and heat and mass transfer effects on steady free convection flow in an inclined porous plate in the presence of MHD and viscous dissipation has been employed. This reduces the system of nonlinear coupled partial differential equations governing the motion of fluid into a system of coupled ordinary differential equations by reducing the number of independent variables. The system remains invariant due to some relations among the parameters of the transformations. Three absolute invariants are obtained and used to derive a third-order ordinary differential equation corresponding to momentum equation and two second-order ordinary differential equations corresponding to energy and diffusion equations. With the use of the fourth-order Runge-Kutta along with the shooting method, the equations are solved. Finally, analysis has been made to investigate the effects of thermal and solutal Grashof numbers, magnetic field parameter, Prandtl number, Viscous dissipation parameter, Schmidt number and chemical reaction on the motion of fluid using scaling group of transformations, namely, Lie group transformations.

Mathematical analysis

Consider the heat and mass transfer of a steady two-dimensional hydromagnetic flow of a viscous, incompressible, electrically conducting and dissipating-fluid past a semi-infinite inclined plate with an acute angle α to the vertical. The physical schematic diagram of the problem is shown in Figure 1. The flow is assumed to be in the x-direction, which is taken along the semi-infinite inclined porous plate and y-axis normal to it. A magnetic field of uniform strength B_0 is introduced normal to the direction of the flow. In the analysis, we assume that the magnetic Reynolds number is much less than unity so that the induced magnetic field is neglected in comparison

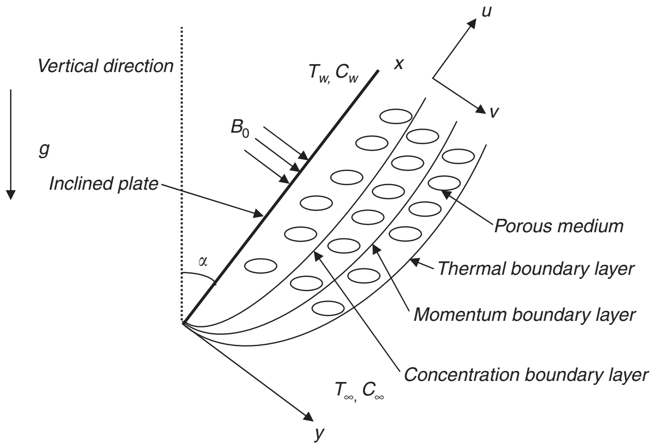

Figure 1.
Physical schematic diagram of the problem

to the applied magnetic field. It is also assumed that all fluid properties are constant except that of the influence of the density variation with temperature and concentration in the body force term. The surface is maintained at a constant temperature T_w, which is higher than the constant temperature T_∞ of the surrounding fluid and the concentration C_w is greater than the constant concentration C_∞. The level of concentration of foreign mass is assumed to be low, so that the Soret and Dufour effects are negligible. Then, under the usual Boussinesq's and boundary layer approximations, the governing equations are:

Continuity equation:

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=0 \tag{1}$$

Momentum equation:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=v\frac{\partial^2 u}{\partial y^2}+g\beta(T-T_\infty)\cos\alpha+g\beta^*(C-C_\infty)\cos\alpha-\frac{\sigma B_0^2}{\rho}u-\frac{v}{K'}u \tag{2}$$

Energy equation:

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}+\frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2 \tag{3}$$

Species equation:

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}-K_r'(C-C_\infty) \tag{4}$$

The boundary conditions for the velocity, temperature and concentration fields are:

$$\begin{aligned} &u=v=0, \quad T=T_w, \quad C=C_w \quad \text{at } y=0 \\ &u\to 0, \quad T\to T_\infty, \quad C\to C_\infty \quad \text{as } y\to\infty \end{aligned} \tag{5}$$

On introducing the following non-dimensional quantities:

$$x^* = \frac{xU_\infty}{v}, \quad y^* = \frac{yU_\infty}{v}, \quad u^* = \frac{u}{U_\infty}, \quad v^* = \frac{v}{U_\infty}, \quad M = \frac{\sigma B_0^2 v}{U_\infty^3}, \quad Kr = \frac{K_r' v}{V_0^2},$$
$$Gr = \frac{vg\beta(T_w - T_\infty)}{U_\infty^3}, \quad Gm = \frac{vg\beta^*(C_w - C_\infty)}{U_\infty^3}, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty}, \tag{6}$$
$$\phi = \frac{C - C_\infty}{C_w - C_\infty}, \quad \Pr = \frac{v}{\alpha}, \quad Ec = \frac{U_\infty^2}{c_p(T_w - T_\infty)}, \quad Sc = \frac{v}{D},$$

and substituting Equation (6) into Equations (1)-(4) and dropping bars, we obtain:

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0 \tag{7}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + Gr\theta Cos\alpha + Gm\phi Cos\alpha - \left(M + \frac{1}{K}\right)u \tag{8}$$

$$u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial y^2} + Ec\left(\frac{\partial u}{\partial y}\right)^2 \tag{9}$$

$$u\frac{\partial \phi}{\partial x} + v\frac{\partial \phi}{\partial y} = \frac{1}{Sc}\frac{\partial^2 \phi}{\partial y^2} - K_r\phi \tag{10}$$

The corresponding boundary conditions take the form:

$$\begin{aligned} &u = v = 0, \quad \theta = 1, \quad \phi = 1 \quad \text{at} \quad y = 0 \\ &u \to 0, \quad \theta \to 0, \quad \phi \to 0 \quad \text{as} \quad y \to \infty \end{aligned} \tag{11}$$

By using the stream function $u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}$ we have:

$$\left(\frac{\partial \psi}{\partial y}\frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial \psi}{\partial x}\frac{\partial^2 \psi}{\partial y^2}\right) = \frac{\partial^3 \psi}{\partial y^3} + Gr\theta Cos\alpha + Gm\phi Cos\alpha - \left(M + \frac{1}{K}\right)\frac{\partial \psi}{\partial y} \tag{12}$$

$$\frac{\partial \psi}{\partial y}\frac{\partial \theta}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial \theta}{\partial y} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial y^2} + Ec\left(\frac{\partial^2 \psi}{\partial y^2}\right)^2 \tag{13}$$

$$\frac{\partial \psi}{\partial y}\frac{\partial \phi}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial \phi}{\partial y} = \frac{1}{Sc}\frac{\partial^2 \phi}{\partial y^2} - K_r\phi \tag{14}$$

Finding the similarity solutions of Equations (12)-(14) is equivalent to determining the invariant solutions of these equations under a particular continuous one parameter group. One of the methods is to search for a transformation group from the elementary set of one parameter scaling transformation. We now introduce the simplified form of

Lie group transformations namely, the scaling group of transformations (Mukhopadhyay *et al.*, 2005):

$$\begin{aligned}\Gamma : x^* = xe^{\varepsilon\alpha_1}, \quad y^* = ye^{\varepsilon\alpha_2}, \quad \psi^* = \psi e^{\varepsilon\alpha_3}, \\ u^* = ue^{\varepsilon\alpha_4}, \quad v^* = ve^{\varepsilon\alpha_5}, \quad \theta^* = \theta e^{\varepsilon\alpha_6}, \quad \phi^* = \phi e^{\varepsilon\alpha_7},\end{aligned} \tag{15}$$

where α_1, α_2, α_3, α_4, α_5, α_6 and α_7 are transformation parameters and ε is a small parameter whose interrelationship will be determined by our analysis. Equation (15) may be considered as a point-transformation which transforms coordinates ($x, y, \psi, u, v, \theta, \phi$) to the coordinates ($x^*, y^*, \psi^*, u^*, v^*, \theta^*, \phi^*$). Substituting transformations Equation (15) in Equations (12), (13) and (14), we get:

$$\begin{aligned}&e^{\varepsilon(\alpha_1+2\alpha_2-2\alpha_3)}\left(\frac{\partial\psi^*}{\partial y^*}\frac{\partial^2\psi^*}{\partial x^*\partial y^*}-\frac{\partial\psi^*}{\partial x^*}\frac{\partial^2\psi^*}{\partial y^{2*}}\right)\\ &= \begin{bmatrix} e^{\varepsilon(3\alpha_2-\alpha_3)}\frac{\partial^3\psi}{\partial y^3}+e^{-\varepsilon\alpha_6}Gr\theta Cos\alpha \\ +e^{-\varepsilon\alpha_7}Gm\phi Cos\alpha - e^{\varepsilon(\alpha_2-\alpha_3)}\left(M+\frac{1}{K}\right)\frac{\partial\psi^*}{\partial y^*}\end{bmatrix}\end{aligned} \tag{16}$$

$$\begin{aligned}&e^{\varepsilon(\alpha_1+\alpha_2-\alpha_3-\alpha_6)}\left(\frac{\partial\psi^*}{\partial y^*}\frac{\partial\theta^*}{\partial x^*}-\frac{\partial\psi^*}{\partial x^*}\frac{\partial\theta^*}{\partial y^*}\right)\\ &= e^{\varepsilon(2\alpha_2-\alpha_6)}\frac{1}{\Pr}\frac{\partial^2\theta^*}{\partial y^{*2}}+e^{\varepsilon(4\alpha_2-2\alpha_3)}Ec\left(\frac{\partial^2\psi}{\partial y^2}\right)^2\end{aligned} \tag{17}$$

$$\begin{aligned}&e^{\varepsilon(\alpha_1+\alpha_2-\alpha_3-\alpha_6)}\left(\frac{\partial\psi^*}{\partial y^*}\frac{\partial\phi^*}{\partial x^*}-\frac{\partial\psi^*}{\partial x^*}\frac{\partial\phi^*}{\partial y^*}\right)\\ &= \frac{1}{Sc}e^{\varepsilon(2\alpha_2-\alpha_7)}\frac{\partial^2\phi^*}{\partial y^{2*}}-K_r e^{-\varepsilon\alpha_7}\phi^*\end{aligned} \tag{18}$$

The system will remain invariant under the group of transformations Γ, and we would have the following relations among the parameters, namely:

$$\begin{aligned}&\alpha_1+2\alpha_2-2\alpha_3 = 3\alpha_2-\alpha_3 = -\alpha_6 = -\alpha_7 = \alpha_2-\alpha_3\\ &\alpha_1+\alpha_2-\alpha_3-\alpha_6 = 2\alpha_2-\alpha_6 = 4\alpha_2-2\alpha_3\\ &\alpha_1+\alpha_2-\alpha_3-\alpha_7 = 2\alpha_2-\alpha_7 = -\alpha_7\end{aligned}$$

These relations gives:

$$\alpha_2 = \frac{1}{4}\alpha_1 = \frac{1}{3}\alpha_3, \quad \alpha_4 = \frac{1}{2}\alpha_1, \quad \alpha_2 = -\frac{1}{4}\alpha_1, \quad \alpha_6 = \alpha_7 = 0$$

Thus, the set of transformations Γ reduces to one parameter group of transformations as:

$$x^* = xe^{\varepsilon\alpha_1}, \quad y^* = ye^{\varepsilon\frac{\alpha_1}{4}}, \quad \psi^* = \psi e^{\varepsilon\frac{3\alpha_1}{4}}, \quad u^* = ue^{\varepsilon\frac{\alpha_1}{2}}, \quad v^* = ve^{-\varepsilon\frac{\alpha_1}{4}}, \quad \theta^* = \theta, \quad \phi^* = \phi,$$

Expanding by Taylors method in powers of ε and keeping terms up to the order ε we get:

$$\begin{aligned}&x^*-x = x\varepsilon\alpha_1, \quad y^*-y = y\varepsilon\frac{\alpha_1}{4}, \quad \psi^*-\psi = \psi\frac{3\alpha_1}{4},\\ &u^*-u = u\varepsilon\frac{\alpha_1}{2}, \quad v^*-v = -v\varepsilon\frac{\alpha_1}{4}, \quad \theta^*-\theta = 0, \quad \phi^*-\phi = 0\end{aligned}$$

$$\frac{dx}{x\alpha_1} = \frac{dy}{y\frac{\alpha_1}{4}} = \frac{d\psi}{\psi\frac{3\alpha_1}{4}} = \frac{du}{u\frac{\alpha_1}{4}} = \frac{dv}{-v\frac{\alpha_1}{4}} = \frac{d\theta}{0} = \frac{d\phi}{0} \tag{19}$$

Solving the above equations, we find the similarity transformations:

$$\eta = x^{-\frac{1}{4}}y, \quad \psi^* = x^{\frac{3}{4}}f(\eta), \quad \theta^* = \theta(\eta), \quad \phi^* = \phi(\eta) \tag{20}$$

Substituting these values in Equations (16)-(18), we finally obtain the system of nonlinear ordinary differential equations:

$$f''' + \frac{3}{4}ff'' - \frac{1}{2}f'^2 + Gr\theta Cos\alpha + Gm\phi Cos\alpha - \left(M + \frac{1}{K}\right)f' = 0 \tag{21}$$

$$\theta'' + \frac{3}{4}\Pr f\theta' + \Pr Ecf''^2 = 0 \tag{22}$$

$$\phi'' + \frac{3}{4}Scf\phi' - kr\phi = 0 \tag{23}$$

The corresponding boundary conditions are:

$$\begin{aligned} &f = 0, \quad f' = 0, \quad \theta = 1, \quad \phi = 1 \quad at\ \eta = 0 \\ &f' \to 0, \quad \theta \to 0, \quad \phi \to 0 \quad as\ \eta \to \infty \end{aligned} \tag{24}$$

Numerical solution

The set of nonlinear ordinary differential Equations (21)-(23) with boundary conditions Equation (24) have been solved by using the Runge-Kutta fourth-order along with shooting technique. First of all, higher order nonlinear differential Equations (21)-(23) are converted into simultaneous linear differential equations of first order and they are further transformed into initial value problem by applying the shooting technique (Jain *et al.*, 1985). The resultant initial value problem is solved by employing Runge-Kutta fourth-order technique. The step size $\Delta\eta = 0.01$ is used to obtain the numerical solution with five decimal place accuracy as the criterion of convergence. It is possible to compare the results obtained by this numerical method with the previously published work of Sivasankaran *et al.* (2006a). Figure 2 shows the comparison of the velocity profiles for different values of Gr and 1/K = 0 and $K_r = 0$. As seen in this figure, excellent agreement between the results exists. This lends confidence in the numerical results to be reported subsequently. In the next section, the results are discussed in detail.

Results and discussion

To analyze the results, numerical computation has been carried out using the method described in the previous paragraph for various in governing parameters, namely, thermal Grashof number *Gr*, mass Grashof number *Gc,* magnetic field parameter *M*, permeability parameter *K*, Prandlt number *Pr*, Eckert number *Ec*, inclination angles α, Schmidt number *Sc*, chemical reaction parameter *Kr*. In the present study following default parameter values are adopted for computations: $Gr = 2.0$, $Gc = 2.0$, $M = 1.0$, $K = 1.0$, $Pr = 0.71$, $Ec = 0.01$, $\alpha = 30^\circ$, $Sc = 0.6$, $Kr = 0.5$. All graphs therefore correspond to these values unless specifically indicated on the appropriate graph.

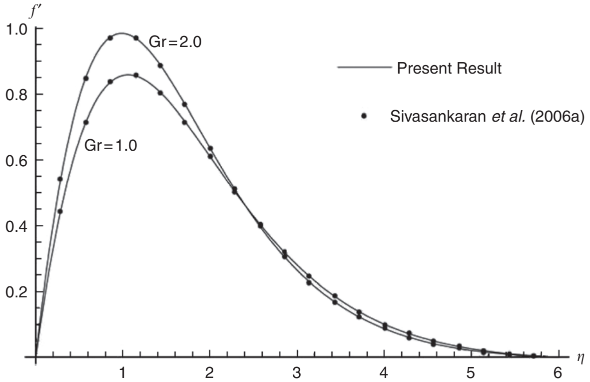

Figure 2. Comparison of velocity profiles with Sivasankaran *et al.* (2006a)

The influence of the thermal Grashof number on the velocity profiles are presented in Figure 3. The thermal Grashof number signifies the relative effect of the thermal buoyancy force to the viscous hydrodynamic force in the boundary layer. As expected, it is observed that there is a rise in the velocity due to the enhancement of thermo buoyancy force. Here, the positive values of *Gr* correspond to cooling of the plate. Also, as *Gr* increases, the peak values of the velocity increases rapidly near the porous plate and then decays smoothly to the free stream velocity.

Figure 4 presents typical velocity profiles in the boundary layer for various values of the solutal Grashof number *Gc,* while all other parameters are kept at some fixed values. The solutal Grashof number *Gc* defines the ratio of the species buoyancy force to the viscous hydrodynamic force. As expected, the fluid velocity increases and the peak value is more distinctive due to increase in the species buoyancy force. The velocity distribution attains a distinctive maximum value in the vicinity of the plate and then decreases properly to approach the free stream value.

For various values of the magnetic parameter *M* and permeability parameter *K,* the velocity profiles are plotted in Figures 5 and 6, respectively. It can be seen that as *M* and *K* are increases, the velocity profiles are decreases. This result qualitatively agrees

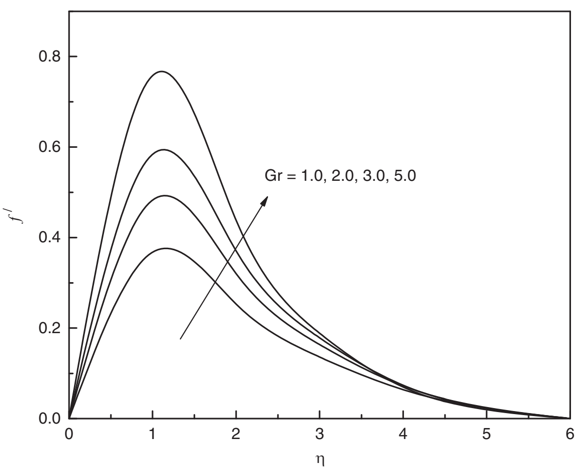

Figure 3. Velocity profiles for different values of Gr

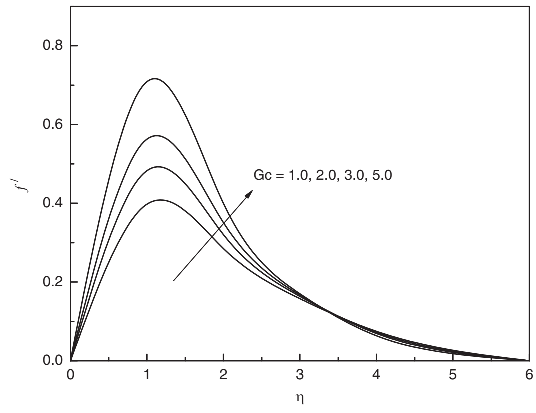

Figure 4.
Velocity profiles for different values of Gc

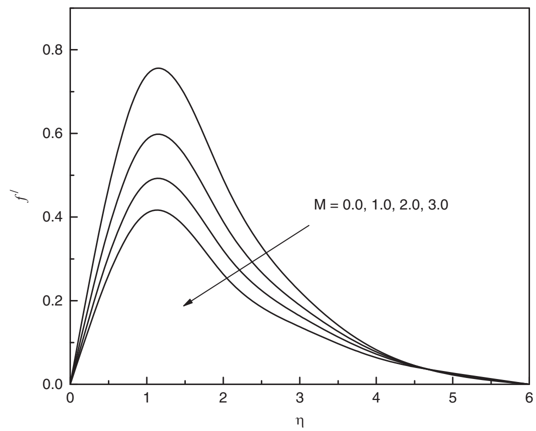

Figure 5.
Velocity profiles for different values of M

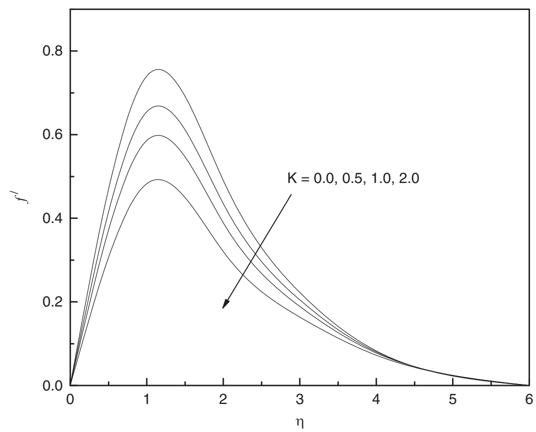

Figure 6.
Velocity profiles for different values of K

with the expectations, since the magnetic field exerts a retarding force on the free convection flow. Figures 7 and 8 illustrate the velocity and temperature profiles for different values of the Prandtl number *Pr*. The Prandtl number defines the ratio of momentum *Pr* diffusivity to thermal diffusivity. The numerical results show that the effect of increasing values of Prandtl number results in a decreasing velocity (Figure 7). From Figure 8, it is observed that an increase in the Prandtl number results a decrease of the thermal boundary layer thickness and in general lower average temperature within the boundary layer. The reason is that smaller values of Pr are equivalent to increasing the thermal conductivities, and therefore heat is able to diffuse away from the heated plate more rapidly than for higher values of Pr. Hence, in the case of smaller Prandtl numbers as the boundary layer is thicker and the rate of heat transfer is reduced.

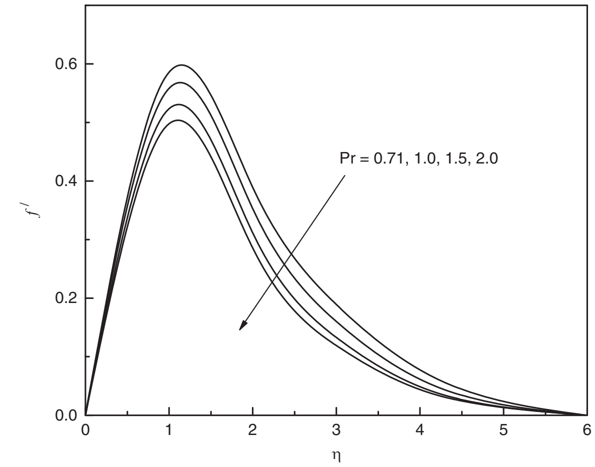

Figure 7.
Velocity profiles for different values of Pr

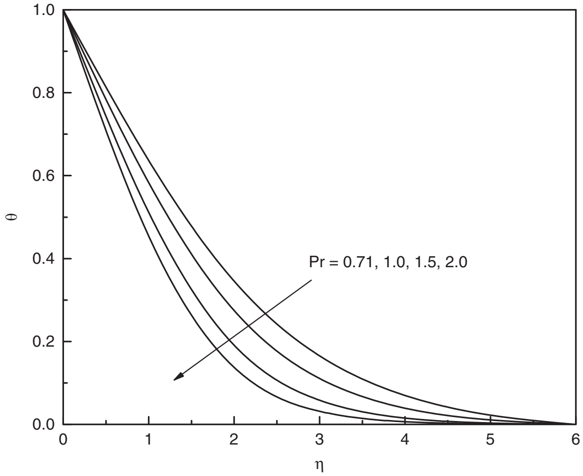

Figure 8.
Temperature profiles for different values of Pr

The effect of the viscous dissipation parameter, i.e., the Eckert number *Ec* on the velocity and temperature are shown in Figures 9 and 10, respectively. The Eckert number expresses the relationship between the kinetic energy in the flow and the enthalpy. It embodies the conversion of kinetic energy into internal energy by work done against the viscous fluid stresses. The positive Eckert number implies cooling of the plate, i.e., loss of heat from the plate to the fluid. Hence, greater viscous dissipative heat causes a rise in the temperature as well as the velocity, which is evident from Figures 9 and 10.

The influences of the Schmidt number *Sc* on the velocity and concentration profiles are plotted in Figures 11 and 12, respectively. The Schmidt number embodies the ratio of the momentum to the mass diffusivity. The Schmidt number therefore quantifies the relative effectiveness of momentum and mass transport by diffusion in the

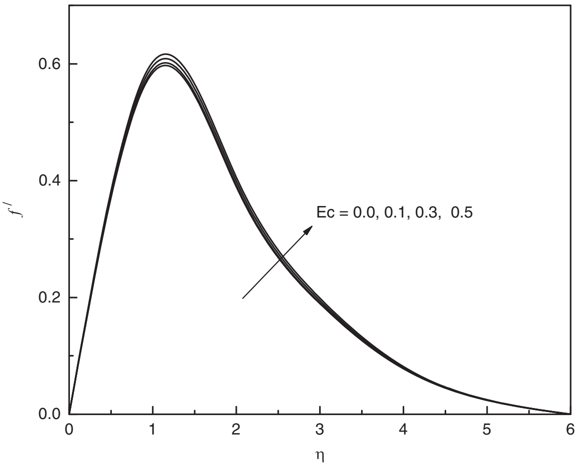

Figure 9.
Velocity profiles for different values of Ec

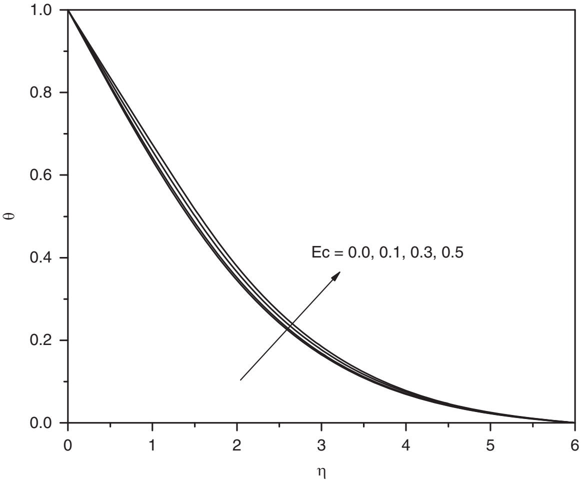

Figure 10.
Temperature profiles for different values of Ec

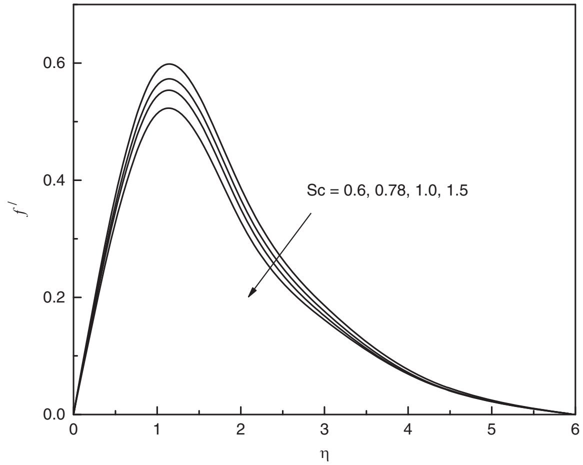

Figure 11.
Velocity profiles for different values of Sc

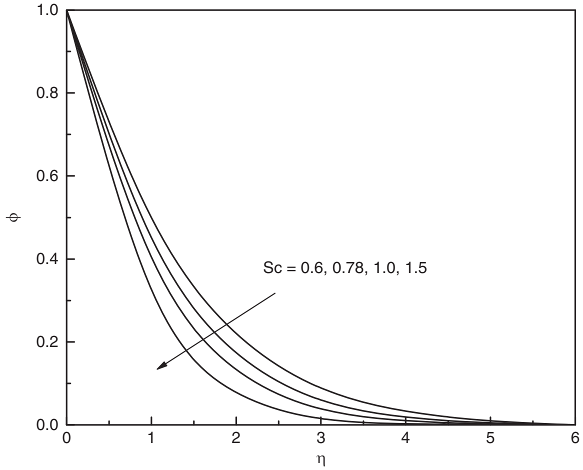

Figure 12.
Concentration profiles for different values of Sc

hydrodynamic (velocity) and concentration (species) boundary layers. As the Schmidt number increases the concentration decreases. This causes the concentration buoyancy effects to decrease yielding a reduction in the fluid velocity. The reductions in the velocity and concentration profiles are accompanied by simultaneous reductions in the velocity and concentration boundary layers. These behaviors are clear from Figures 11 and 12.

The influences of chemical reaction parameter Kr on the velocity and concentration profiles across the boundary layer are presented in Figures 13 and 14, respectively. We see that the velocity as well as concentration distribution across the boundary layer decreases with increasing of Kr.

The effect of inclination of the surface for different parameters is depicted in Figure 15. For fixed values of the all parameter the velocity is decreased with inclination angles as shown in Figure 15. The fluid has higher velocity when the surface is vertical than when inclined because the buoyancy effect decreases due to gravity components $\cos\alpha$ as the plate is inclined. The fact is that as the angle of

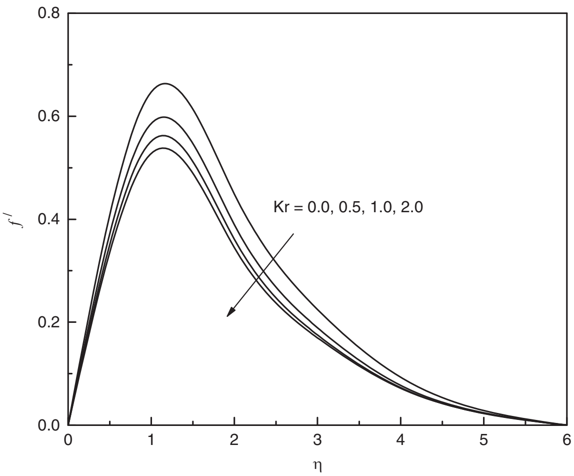

Figure 13. Velocity profiles for different values of Kr

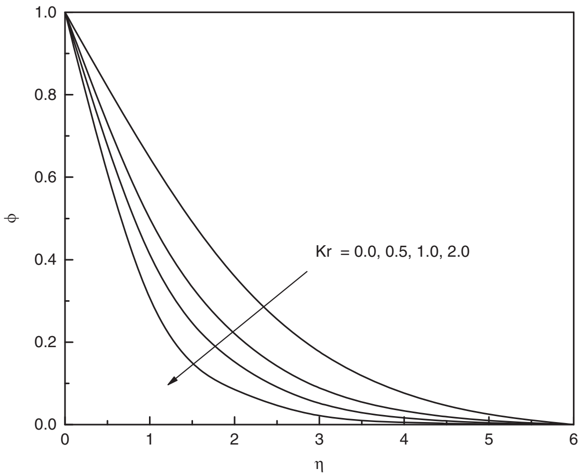

Figure 14. Concentration profiles for different values of Kr

inclination increases the effect of the buoyancy force due to thermal diffusion decreases by a factor of cosα. Consequently, the driving force to the fluid decreases as a result velocity profiles decreases.

For different values of the magnetic field parameter M on the temperature profiles are plotted in Figure 16. It is observed that the magnetic parameter increases, the temperature also increases. Figure 17 presents the effect of the porosity parameter K on the temperature profiles. There are very small changes that occur in both momentum and thermal boundary layers when changes are made in the porosity parameter. The temperature is increased with increase in the porosity parameter.

Tables I-III indicate the values of skin-friction coefficient, the wall temperature gradient and the wall concentration gradient in terms of $f''(0)$, $-\theta'(0)$ and $-\phi'(0)$, respectively for various values embedded flow parameter. From Tables I-III, we have

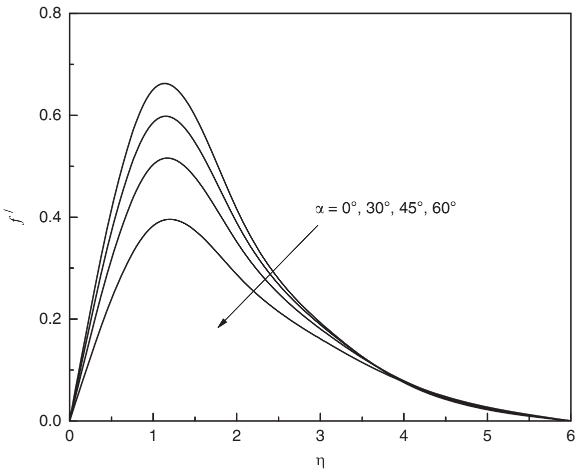

Figure 15.
Velocity profiles for different values of α

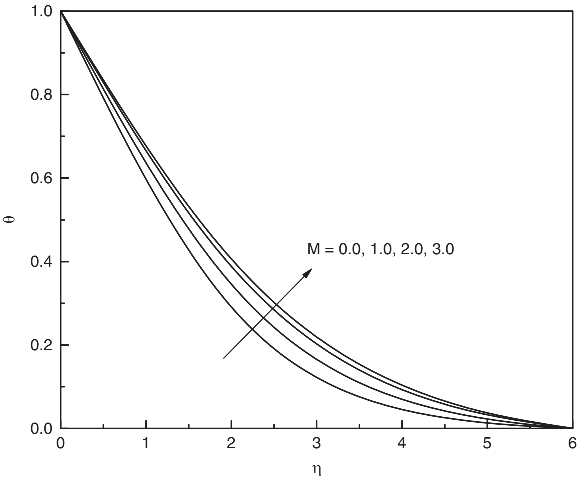

Figure 16.
Temperature profiles for different values of M

noticed that skin-friction coefficient, Nusselt number and Sherwood number are increases with an increasing of Grashof number or mass Grashof number, as increasing values of magnetic field parameter (M) or porosity parameter a reduces in the skin-friction, Nusselt number and Sherwood. The Nusselt number reduces as increase the values of dissipation Ec or inclination angle, while it is increases for increasing value of Prandtl number Pr. It is also observed that the increase in Schmidt number Sc or chemical reaction parameter Kr parameter lead to the increase in the Sherwood number.

Conclusions

By using the Lie group analysis, we first find the symmetries of the partial differential equations and then reduce the equations to ordinary differential equations by using scaling and translational symmetries. Exact solutions for translational symmetry and a

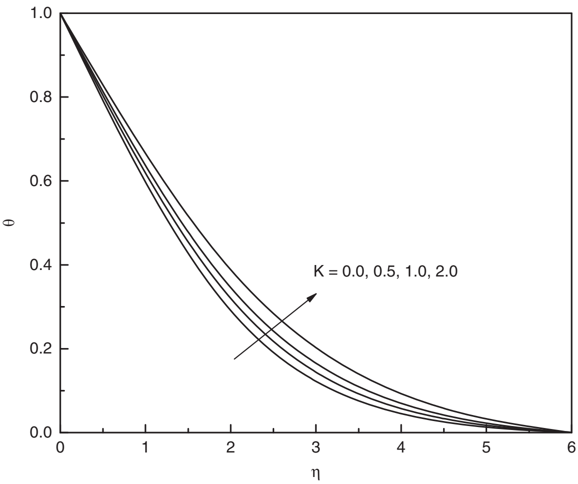

Figure 17. Temperature profiles for different values of K

Table I. Values of $f''(0)$, $-\theta'(0)$ and $-\phi'(0)$ for Pr = 0.71, Ec = 0.01, $\alpha = 30°$, Sc = 0.6, Kr = 0.5

Gr	*Gc*	*M*	*K*	$f''(0)$	$-\theta'(0)$	$-\phi'(0)$
2.0	2.0	1.0	1.0	1.67872	0.37536	0.632846
3.0	2.0	1.0	1.0	2.06767	0.404755	0.648809
4.0	2.0	1.0	1.0	2.44141	0.429506	0.66312
2.0	3.0	1.0	1.0	2.03459	0.399334	0.646135
2.0	4.0	1.0	1.0	2.38237	0.420826	0.658602
2.0	2.0	2.0	1.0	1.49761	0.348147	0.619813
2.0	2.0	3.0	1.0	1.36399	0.327933	0.610413
2.0	2.0	1.0	2.0	1.49761	0.348174	0.619813
2.0	2.0	1.0	3.0	1.36399	0.327933	0.610413

Table II. Values of $f''(0)$, $-\theta'(0)$ and $-\phi'(0)$ for Gr = 2.0, Gc = 2.0, M = 1.0, K = 1.0, Sc = 0.6, Kr = 0.5

Pr	*Ec*	α	$f''(0)$	$-\theta'(0)$	$-\phi'(0)$
0.71	0.01	30°	1.67872	0.37536	0.632846
1.0	0.01	30°	1.64902	0.429375	0.629509
2.0	0.01	30°	1.57721	0.570278	0.622038
0.71	0.1	30°	1.68425	0.337882	0.633256
0.71	0.2	30°	1.69051	0.29549	0.633719
0.71	0.01	45°	1.39653	0.352877	0.621139
0.71	0.01	60°	1.01526	0.318826	0.60434

Table III. Values of $f''(0)$, $-\theta'(0)$ and $-\phi'(0)$ for Gr = 2.0, Gc = 2.0, M = 1.0, K = 1.0, Pr = 0.71, Ec = 0.01, $\alpha = 30°$

Sc	*Kr*	$f''(0)$	$-\theta'(0)$	$-\phi'(0)$
0.6	0.5	1.67872	0.37536	0.632846
0.78	0.5	1.65106	0.369773	0.716312
1.0	0.5	1.62341	0.364526	0.805778
0.6	1.0	1.6253	0.36603	0.828406
0.6	2.0	1.55591	0.355195	1.12767

numerical solution for scaling symmetry are obtained. From the numerical results, the magnetic field parameter, the permeability of porous medium and the viscous dissipation are demonstrated to exert a more significant effect on the flow field and, thus, on the heat transfer from the plate to the fluid. It is observed that the effect of increasing thermal Grashof number and modified Grashof number are exhibited as an increase in flow velocity. It is worth noting that the temperature decreases much faster than the air temperature. The velocity and concentration is found to decrease gradually as the Schmidt number and chemical reaction parameter are increased. The results of the study are of great interest because flows on a vertical stretching surface play an important role in nature in connection with the applications of science, engineering and technology.

References

Abolbashari, M.H., Freidoonimehr, N., Nazari, N.F.F. and Rashidi, M.M. (2014), "Entropy analysis for an unsteady MHD flow past a stretching permeable surface in nano-fluid", *Powder Technology*, Vol. 267, November, pp. 256-267.

Beithou, N., Albayrak, K. and Abdulmajeed, A. (1998), "Effects of porosity on the free convection flow of non-Newtonian fluids along a vertical plate embedded in a porous medium", *Turkish Journal of Engineering and Environmental Sciences*, Vol. 22, pp. 203-209.

Bluman, G.W. and Kumei, S. (1989), *Symmetries and Differential Equations, Vol. 81 of Applied Mathematical Sciences*, Springer, New York, NY.

Chamkha, A.J., EL-Kabeir, S.M.M. and Rashad, A.M. (2013), "Coupled heat and mass transfer by MHD free convection flow along a vertical plate with streamwise temperature and species concentration variations", *Heat Transfer-Asian Research*, Vol. 42 No. 2, pp. 100-110.

Chen, C.H. (2004), "Heat and mass transfer in MHD flow by natural convection from a permeable inclined surface with variable wall temperature and concentration", *Acta Mechanica*, Vol. 172 Nos 3-4, pp. 219-235.

Duwairi, H.M. (2005), "Viscous and Joule heating effects on forced convection flow from radiate isothermal porous surfaces", *Int. J. Num. Methods Heat Fluid Flow*, Vol. 15 No. 5, pp. 429-440.

EL-Kabeir, S.M.M., EL-Hakiem, M.A. and Rashad, A.M. (2008a), "Group method analysis of combined heat and mass transfer by MHD non-Darcy non-Newtonian natural convection adjacent to horizontal cylinder in a saturated porous medium", *Applied Mathematical Modelling*, Vol. 32 No. 11, pp. 2378-2395.

EL-Kabeir, S.M.M., EL-Hakiem, M.A. and Rashad, A.M. (2008b), "Lie group analysis of unsteady MHD three dimensional by natural convection from an inclined stretching surface saturated porous medium", *J. Computational Applied Mathematics*, Vol. 213 No. 2, pp. 582-603.

Ibragimov, N.H. (Ed.) (1995), *Volume 2: Applications in Engineering and Physical Sciences, CRC Handbook of Lie Group Analysis of Differential Equations*, CRC Press, Boca Raton, FL.

Ibrahim, F.S., Mansour, M.A. and Hamad, M.A.A. (2005), "Lie-Group analysis of radiative and magnetic field effects on free convection and mass transfer flow past a semi-infinite vertical flat plate", *Electronic J. Differential Equations (2005)*, Vol. 39 No. 39, pp. 1-17.

Jain, M.K., Iyengar, S.R.K. and Jain, R.K. (1985), *Numerical Methods for Scientific and Engineering Computation*, Wiley Eastern Ltd, New Delhi.

Mukhopadhyay, S., Layek, G.C. and Samad, S.A. (2005), "Study of MHD boundary layer flow over a heated stretching sheet with variable viscosity", *Int. J. Heat Mass Transfer*, Vol. 48 Nos 21-22, pp. 4460-4466.

Olver, P.J. (1986), *Application of Lie Groups to Differential Equations*, Springer, New York, NY.

Ovsiannikov, L.V. (1982), *Group Analysis of Differential Equations*, Academic Press, New York, NY.

Rashad, A.M. and Chamkha, A.J. (2014), "Heat and mass transfer by natural convection flow about a truncated cone in porous media with Soret and Dufour effects", *International Journal of Numerical Methods for Heat and Fluid Flow*, Vol. 24 No. 3, pp. 595-612.

Rashad, A.M., Modather, M. and Chamkha, A.J. (2011), "MHD free convective heat and mass transfer of a chemically-reacting fluid from radiate stretching surface embedded in a saturated porous medium", *International Journal of Chemical Reactor Engineering*, Vol. 9, Article A66, pp. 1-15.

Rashidi, M.M. and Mehr, N.F. (2014), "Series solutions for the flow in the vicinity of the equator of an MHD boundary-layer over a porous rotating sphere with heat transfer", *Thermal Science*, Vol. 18 No. S2, pp. S527-S537.

Rashidi, M.M., Rostami, B., Freidoonimehr, N. and Abbasbandy, S. (2014), "Free convective heat and mass transfer for MHD fluid flow over a permeable vertical stretching sheet in the presence of the radiation and buoyancy effects", *Ain Shams Engineering Journal*, Vol. 5 No. 3, pp. 901-912.

Sheikholeslami, M., Abelman, S. and Ganji, D.D. (2014), "Numerical simulation of MHD nanofluid flow and heat transfer considering viscous dissipation", *International Journal of Heat and Mass Transfer*, Vol. 79, December, pp. 212-222.

Sivasankaran, S., Bhuvaneswar, M., Kandaswamy, P. and Ramasami, E.K. (2006a), "Lie group analysis of natural convection heat and mass transfer in an inclined surface", *Nonlinear Analysis. Modelling and Control*, Vol. 11 No. 1, pp. 201-212.

Sivasankaran, S., Bhuvaneswar, M., Kandaswamy, P. and Ramasami, E.K. (2006b), "Lie group analysis of natural convection heat and mass transfer in an inclined porous surface with heat generation", *International Journal Applied Mathematics and Mechanics*, Vol. 2, pp. 34-40.

Yurusoy, M. and Pakdemirli, M. (1997), "Symmetry reductions of unsteady three-dimensional boundary layers of some non-Newtonian fluids", *Int. J. Engg. Sci.*, Vol. 35 No. 8, pp. 731-740.

Yurusoy, M. and Pakdemirli, M. (1999), "Exact solutions of boundary layer equations of a special non-Newtonian fluid over a stretching sheet", *Mechanics Research Communications*, Vol. 26 No. 2, pp. 171-175.

Yurusoy, M., Pakdemirli, M. and Noyan, O.F. (2001), "Lie Group analysis of creeping flow of a second grade fluid", *International Journal of Non-Linear Mechanics*, Vol. 36 No. 6, pp. 955-960.

Corresponding author
Professor Ali J. Chamkha can be contacted at: achamkha@yahoo.com