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HYDROMAGNETIC FLOW AND HEAT TRANSFER OF A HEAT-GENERATING FLUID OVER A SURFACE EMBEDDED IN A POROUS MEDIUM

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ABSTRACT

Similarity solutions for the laminar boundary-layer equations describing steady hydromagnetic two-dimensional flow and heat transfer in a stationary electrically-conducting and heat-generating fluid driven by a continuously moving porous surface immersed in a fluid-saturated porous medium are obtained. The flow is exposed to a transverse magnetic field and the surface is moving with a constant velocity. Fluid suction or injection is imposed at the wall. The dimensionless similar equations are solved numerically by an implicit finite-difference method. The numerical flow and heat transfer results are illustrated graphically for various parametric conditions to show special features of the solutions. Favorable comparisons with previously published work confirm the correctness of the numerical results. © 1997 Elsevier Science Ltd

Introduction

There has been considerable renewed interest in studying flows of electrically-conducting fluids over surfaces in the presence of a magnetic field in recent years. This renewed interest stems from the fact that these fluids have been used extensively in many geophysical and industrial applications. Flow and heat transfer over a continuously moving surface with a constant speed was initially studied by Sakiadis [1,2]. Extensions of this problem were reported previously by many investigators (see, for instance, Erickson et al. [3], Tsuo et al. [4], Jacobi [5], Chen and Strobel [6], and Ali [7]. Cobble [8] obtained similarity solutions for hydromagnetic flow over semi-infinite porous plate with suction or injection being imposed at the plate surface and . Later, the heat transfer aspects of the problem of Cobble [8] was discussed by Soundalgekar and Murthy [9] and Singh [10]. Recently, Chandran et al. [11] have made extensions to the above works and considered electrically-conducting fluids in the presence of a transverse magnetic field and allowed for possible fluid suction or blowing at the surface boundary. Other earlier works dealing with magnetic field effects on this type of problem can be found in the papers by Gupta [12], Soundalgekar [13] and Pop [14].

The possible use of porous media adjacent to surfaces to enhance heat transfer characteristics have lead to extensive research in heat transfer and flows over bodies embedded in porous media. Most early works on flow and heat transfer in porous media adjacent to surfaces have used Darcy's law which neglects both boundary and inertia effects which become significant in the presence of a solid boundary and when the flow in the porous medium is considered fast. Vafai and Tien [15] and Hong et al. [16] discuss the importance of these effects in flows over surfaces embedded in porous media.

In certain applications dealing with chemical reactions and dissociating fluids, heat generation effects become significant (see, for instance, Vajravelu and Nayfeh [17]). Moalem [18] has studied the effect of temperature-dependent heat sources occurring in electrical heating on the steady-state heat transfer in a porous medium. Chamkha [19] obtained a similarity solution for non-Darcy hydromagnetic free convection from a cone and a wedge in porous media with heat generation effects. Other works dealing with heat generation effects can be found in the earlier papers of Sparrow and Cess [20], Topper [21], and Foraboschi and Federico [22].

Motivated by the works referenced above, it is of interest in this paper to consider steady, two-dimensional, hydromagnetic flow over a continuously moving porous surface embedded in a fluid-saturated porous medium with heat generation effects. The applied magnetic field is assumed to be non-uniform and the magnetic Reynolds number is assumed to be small so that the induced magnetic field can be neglected. In addition, it is assumed that there is no external electric field and that the electric field due to polarization of charges is negligible.

Problem Formulation

Consider steady, laminar, incompressible two-dimensional, boundary-layer flow of an electrically-conducting and heat-generating fluid over a semi-infinite porous flat surface immersed in a porous medium in the presence of a transverse magnetic field. Let the x-axis representing the axial or tangential distance be placed along the horizontal plate and the y-axis representing the normal distance be perpendicular to it. Let the plate be moving with a constant speed U and is maintained at a uniform temperature T_w. Far above the plate, the fluid is stationary and is kept at a temperature T_∞. The porous medium is assumed to be uniform (that is, it has a constant porosity and permeability). All thermophysical properties are assumed constant. The magnetic Reynolds number is assumed small so that the induced magnetic field is neglected. In addition, the external electric field, the electric field due to polarization of charges, the Hall effect of magnetohydrodynamics, and viscous and magnetic dissipations are all assumed negligible. The governing boundary-layer equations which are based on the balance laws of mass, linear momentum and energy for this investigation can be written as:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B^2(x)}{\rho}u-\frac{\nu}{K}u-Cu^2 \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k_e}{\rho\, c_p}\frac{\partial^2 T}{\partial y^2}+\frac{Q}{\rho\, c_p}\left(T-T_\infty\right) \tag{3}$$

where x and y are the tangential and normal distances, respectively. u, v, and T are the fluid x-component of velocity, y-component of velocity, and temperature, respectively. ρ, ν, and c_p are the fluid density, kinematics viscosity, and specific heat at constant pressure, respectively. K, C, and k_e are the porous medium permeability, Forcheimer inertia coefficient and effective thermal conductivity, respectively. σ, $B(x)$, and Q are the fluid electrical conductivity, magnetic induction, and heat generation/absorption coefficient, respectively.

The appropriate boundary conditions for the velocity and temperature of this problem are:

$$u(x,0)=U,\quad v(x,0)=-v_w(x),\quad T(x,0)=T_w,\quad u(x,\infty)=0,\quad T(x,\infty)=T_\infty \tag{4}$$

where U is a constant and $v_w(x)$ is the suction or blowing velocity. Positive values of $v_w(x)$ correspond to fluid suction at the plate surface while negative values of $v_w(x)$ indicate fluid blowing or injection at the wall.

A useful similarity transformation is possible for this problem. Substituting the following dimensionless similarity variables

$$y=\eta\sqrt{\frac{2\nu x}{U}},\quad u=Uf'(\eta),\quad v=\sqrt{\frac{\nu U}{2x}}(\eta f'-f),\quad T=(T_w-T_\infty)\theta+T_\infty \tag{5}$$

into Eqs. (1)-(4) and rearranging yields

$$f'''+ff''-(M+Da^{-1})f'-\Gamma_x(f')^2=0 \tag{6}$$

$$\theta''+\Pr f\,\theta'+\Pr\phi_x\theta=0 \tag{7}$$

$$f(0)=f_w,\quad f'(0)=1,\quad \theta(0)=1,\quad f'(\infty)=0,\quad \theta(\infty)=0 \tag{8}$$

where $M = \frac{2\sigma x \; B(x)^2}{\rho \, U^2}$, $Da^{-1} = \frac{2 \nu x}{KU}$, $\Gamma_x = 2Cx$, $Pr = \frac{\rho \nu c_p}{k_e}$, $\phi_x = \frac{2Qx}{U \rho c_p}$, and $f_w = -v_w(x)\sqrt{\frac{2x}{\nu U}}$ are the square of the Hartmann number, the inverse Darcy number, the dimensionless porous medium inertia coefficient, the fluid Prandtl number, the dimensionless heat generation/absorption coefficient, and the dimensionless suction/blowing coefficient, respectively. It should be mentioned that positive values of ϕ_X represent heat generation (source) while negative values of ϕ_X indicate heat absorption (sink). If Da^{-1}, Γ_X, and ϕ_X are formally set to zero, Eqs. (6)-(8) reduce to those reported by Chandran et al. [11].

Of special significance for this type of flow and heat transfer situation are the skin-friction coefficient and the Nusselt number. These physical quantities can be defined, respectively, as follows:

$$C_f = \frac{-\mu \frac{\partial u}{\partial y}(x,0)}{\rho U^2} = -Re^{-1/2} \; f''(0) \tag{9}$$

$$Nu = \frac{-k_e \frac{\partial T}{\partial y}(x,0)}{k_e / \nu \, (T_w - T_\infty)} = -Re^{1/2} \; \theta'(0) \tag{10}$$

where $\mu = \frac{\nu}{\rho}$ is the dynamic viscosity of the fluid and $Re = \frac{Ux}{2\nu}$ is the Reynolds number.

Results and Discussion

The two-point boundary-value problem represented by Eqs. (6)-(8) is nonlinear and exhibits no closed-form solution. Therefore, it must be solved numerically. The fourth-order Runge Kutta method and the implicit-finite difference method are appropriate for the solution of this problem. In the present work, numerical results to be presented subsequently are obtained using the latter method. The numerical method is a fully implicit, iterative, tri-diagonal method in which the differential equations are converted into linear algebraic equations. The resulting equations are then solved by the Thomas algorithm. For more details on the numerical method the reader is advised to see the paper by Blottner [23]. It is obvious from Eqs. (6) and (7) that Eq. (6) governing the hydrodynamic flow problem is independent of the thermal problem governed by Eq. (7) and, therefore,can first be solved for f. The solution for f is then substituted into Eq. (7) and then solved for θ. The number of nodal points employed is 196. It is expected that the changes in the dependent variables will be great in the immediate vicinity of the plate surface and small far above it. For this reason, variable step sizes with an initial step size of 0.001 and a growth factor of 1.03 are used. The choice of these numbers is arrived at after many numerical experimentation which were performed to assess grid independence. The convergence criterion is based on the relative difference between the current and the previous iterations and is set to 10^{-5} in the present work. Many numerical results were obtained throughout the course of this work. A representative set of graphical results is illustrated in Figs. 1 through 14.

Figures 1 and 2 present typical fluid velocity f' and temperature θ profiles for various values of the Hartmann number M, respectively. Application of a transverse magnetic field normal to the flow direction have a tendency to create a drag-like Lorentz force which tends to resist the flow and increase the temperature of the fluid. This is depicted by the decreases in f' and increases in θ as M increases in Figs. 1 and 2, respectively.

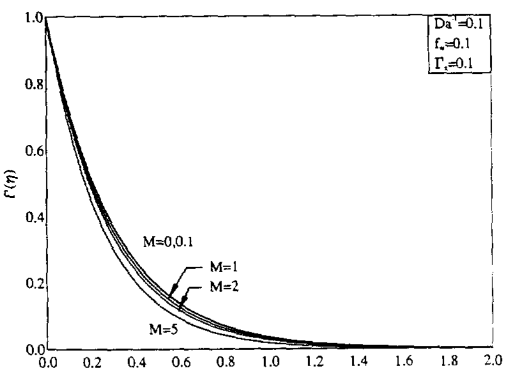

FIG. 1
Effects of M on Fluid Velocity Profiles

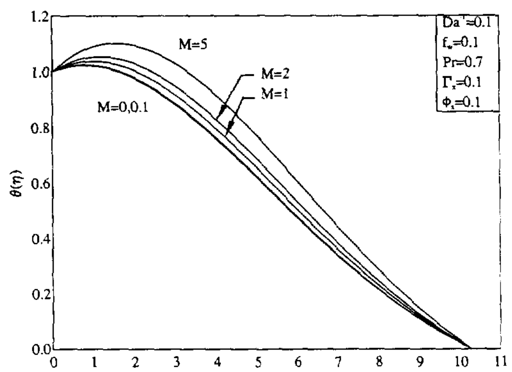

FIG. 2
Effects of M on Fluid Temperature Profiles

Figures 3 and 4 show representative profiles for f' and θ for different fluid wall suction and injection conditions, respectively. Imposition of fluid suction at the surface has a tendency to reduce both the hydrodynamic and thermal thicknesses of the boundary layer where viscous effects dominate. This has the effect of reducing both the fluid velocity and temperature above the plate. This is evident from the decreases in f' and θ as the suction/injection parameter f_w increases shown in Fig. 3 and 4. It should be mentioned herein that the negative values of f_w correspond to blowing or injection at the plate while the positive values of f_w indicate suction at the wall.

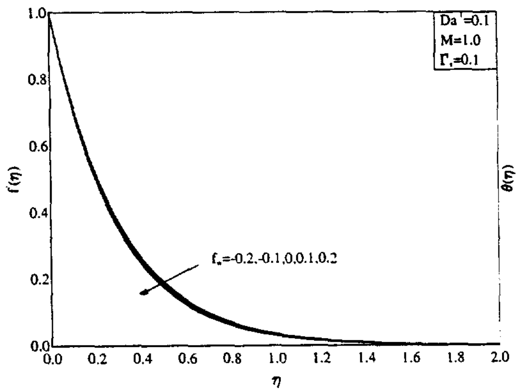

FIG. 3
Effects of f_w on Fluid Velocity Profiles

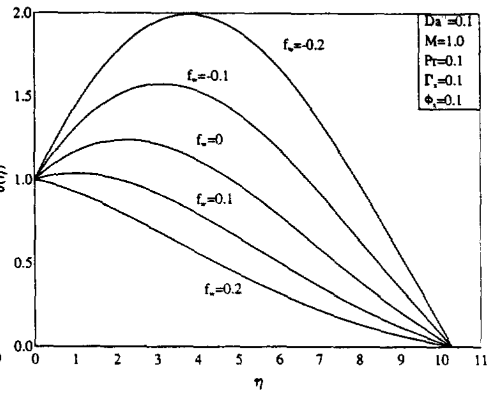

FIG. 4
Effects of f_w on Fluid Temperature Profiles

Figures 5 through 7 illustrate the influence of the porous medium inverse Darcy number Da^{-1} and the inertia coefficient Γ_x on the velocity and temperature profiles. Both the parameters, Da^{-1} and Γ_x, represent resistance to flow since they restrict the motion of the fluid along the plate. Therefore, they have the same effect as the Hartmann number M, namely, decreasing the fluid velocity and increasing its temperature as shown in Figs. 5 through 7. It should be noted that for the parametric values used to obtain the results of Fig. 7, f' seems to be affected slightly by increases in Γ_x.

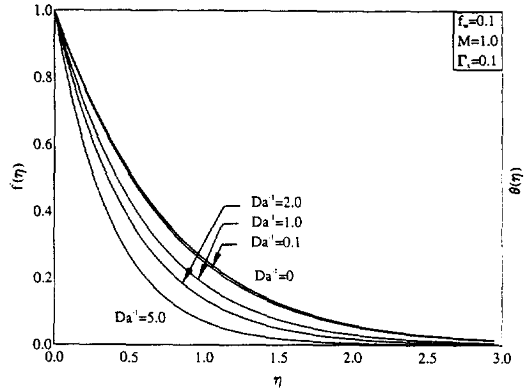

FIG. 5
Effects of Da^{-1} on Fluid Velocity Profiles

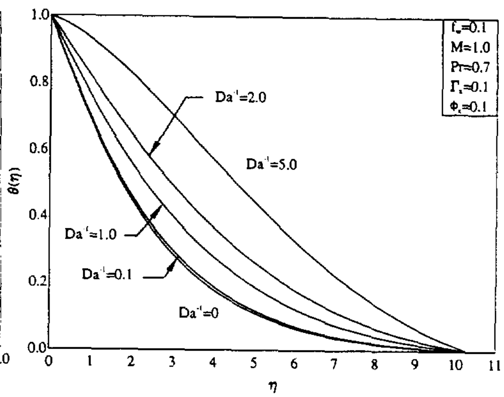

FIG. 6
Effects of Da^{-1} on Fluid Temperature Profiles

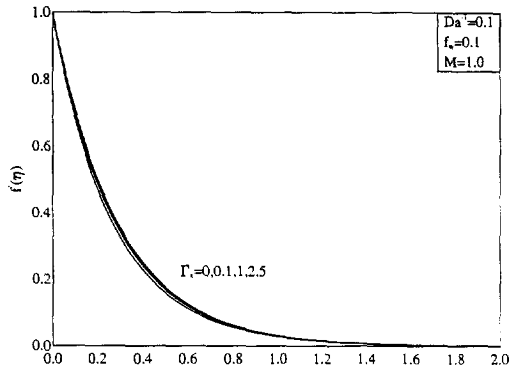

FIG. 7
Effects of Γ_x on Fluid Velocity Profiles

Figures 8 and 9 present the influence of the Prandtl number Pr and the heat generation/absorption coefficient ϕ_x on the temperature profiles, respectively. Since the flow problem is uncoupled from the thermal problem, changes in the values of Pr or ϕ_x will not affect the fluid velocity. For this reason, the velocity profiles for this case are not shown. Increasing the Prandtl number tends to reduce the thermal boundary layer along the plate. This yields a reduction in the fluid temperature. For small values of Pr the temperature profile is essentially linear. This is clearly seen from Eq. (7) and the curve corresponding to $Pr = 0.1$ in Fig. 8. However, for moderate values of Pr ($Pr = 0.7, 1$) and finite values of ϕ_x, the temperature profile is nonlinear. For $\phi_x = 0.1$ and $Pr = 0.7$ or 1, the heat generation effect overcomes the convective thermal effects (second term of Eq. (7)). For this reason, the fluid temperature increases as Pr increases. However, for large values of Pr, the convective thermal effects become more pronounced causing θ to decrease. These behaviors are clearly observed in Fig. 8. Increasing the heat generation/absorption coefficient ϕ_x for $Pr = 0.7$ tends to increase the temperature of the fluid at every point above the plate as is evident from Fig. 9.

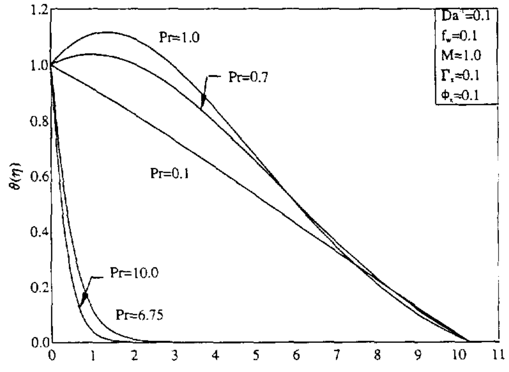

FIG. 8
Effects of Pr on Fluid Temperature Profiles

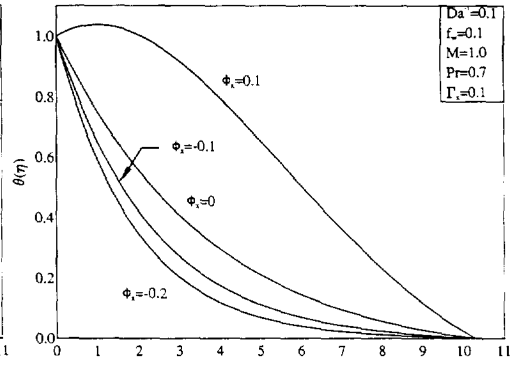

FIG. 9
Effects of ϕ_x on Fluid Temperature Profiles

Figures 10 and 11 depict the effects of both M and Da^{-1} on the skin-friction coefficient C_f and the Nusselt number Nu, respectively. The reductions in the velocity profiles and the increases in the temperature profiles as either of M or Da^{-1} increase cause the slopes of the velocity and temperature profiles to increase and decrease, respectively. This has the direct effect of increasing C_f and decreasing Nu as clearly shown in Figs. 10 and 11, respectively.

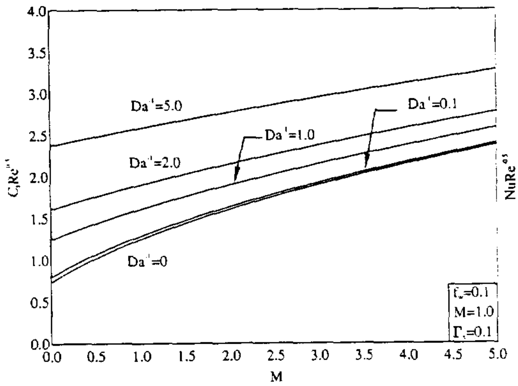

FIG. 10
Effects of Da' on Skin Friction Coefficient

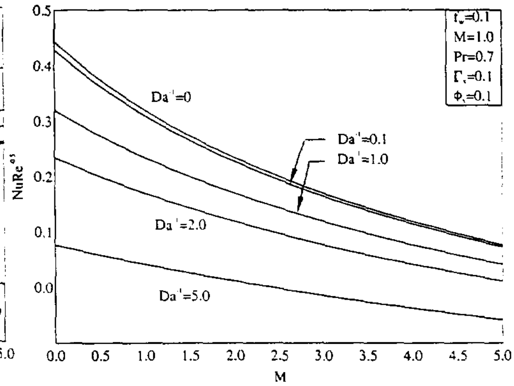

FIG. 11
Effects of Da' on Wall Heat Transfer Coefficient

Figures 12 and 13 illustrate the change in the values of C_f and Nu as a result of changing either of M or f_w. Again, the decreases in the hydrodynamic and thermal boundary layers and, therefore, the values of f' and θ as f_w increases cause increases in the velocity and temperature gradients at the wall. This results in increasing both C_f and Nu as f_w increases.

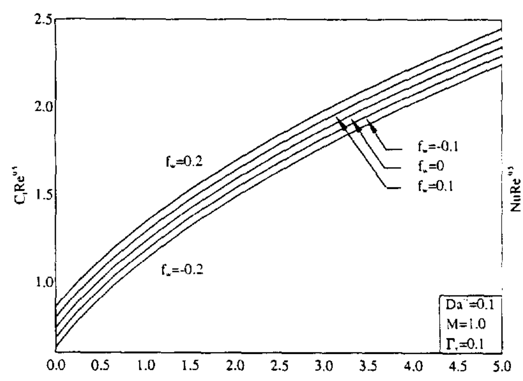

FIG. 12
Effects of f_w on Skin Friction Coefficient

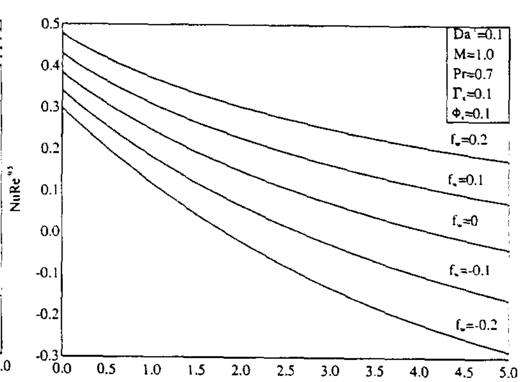

FIG. 13
Effects of f_w on Wall Heat Transfer Coefficient

Figure 14 presents the influence of both Pr and ϕ_x on the values of Nu. For the same reasons mentioned above, increasing ϕ_x tends to decrease Nu. In addition, it is observed from Fig. 14 that Nu increases as Pr increases.

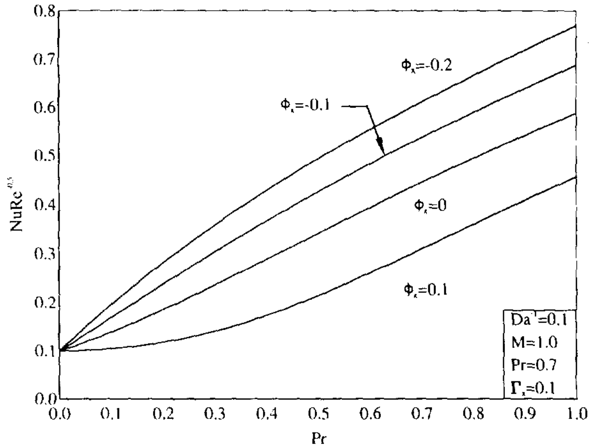

FIG. 14
Effects of ϕ_x on Wall Heat Transfer Coefficient

Finally, Tables 1 and 2 show a comparison between the results for the skin-friction coefficient and the Nusselt number reported herein and those reported earlier by previous authors. It is clearly seen from these tables that these results are in excellent agreement except the Nusselt number results of Chandran et al. [11] as these results seem to be a little off. For consistent comparisons with other published results, the values of the Nusselt numbers reported by Chandran et al. [11] should be divided by a factor of $\sqrt{2}$. There appears to be a mistake in the calculation of their results for the Nusselt number. It should be mentioned that the results reported herein were also confirmed by separate calculations using the fourth-order Runge Kutta method. These comparisons and confirmations ensure the accuracy of the numerical results.

TABLE 1

Comparison of Nusselt numbers $Nu\, Re^{-1/2}$ for various Pr values

Pr	[5]	[9]	[6]	[4]	[11]	[7]	Present results
0.7	0.3492	0.3508	0.34924	0.3492	0.3369	0.3476	0.3524
1.0	0.4438	--	--	0.4438	--	0.4416	0.4453
10	1.6790	--	--	1.6804	--	1.6713	1.6830

TABLE 2

Comparison of skin-friction coefficients $C_f\ Re^{1/2}$

M	f_w	Chandran et al. [11]	Present results
0.0	-0.2	0.5155	0.5174
0.0	-0.1	0.5700	0.5714
0.0	0.0	0.6275	0.6288
0.0	0.1	0.6881	0.6894
0.0	0.2	0.7515	0.7530
0.1	-0.2	0.5856	0.5873
0.1	-0.1	0.6393	0.6417
0.1	0.0	0.6960	0.6979
0.1	0.1	0.7554	0.7573
0.1	0.2	0.8176	0.8193

Conclusion

The problem of steady, laminar, hydromagnetic flow and heat transfer caused by the uniform movement of a semi-infinite porous flat plate immersed in a fluid-saturated porous medium in the presence of a magnetic field was solved numerically. The fluid was assumed to be electrically conducting and heat generating or absorbing. The possibility of fluid suction or blowing at the surface was also considered. Similarity transformation for this problem was possible provided some parameters were allowed to be functions of the axial distance x. The resulting similar equations were solved numerically by a standard fully implicit finite-difference method. No numerical difficulties were encountered and many results were obtained. It was found that the application of a transverse magnetic field normal to the flow direction or the presence of a porous medium increased the skin-friction coefficient and decreased the Nusselt number. However, suction of fluid at the wall increases both the skin-friction coefficient and the Nusselt number. Furthermore, the Nusselt number was decreased as the heat generation/absorption parameter was increased. Comparisons with previously published work were performed and favourable agreements were obtained. It is hoped that the results reported in this paper find use in industrial applications dealing with the flow and heat transfer situation considered in this paper.

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