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UNSTEADY MAGNETOHYDRODYNAMIC FREE CONVECTIVE DOUBLE-DIFFUSIVE VISCOELASTIC FLUID FLOW PAST AN INCLINED PERMEABLE PLATE IN THE PRESENCE OF VISCOUS DISSIPATION AND HEAT ABSORPTION

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In this paper, we have investigated an unsteady, magnetohydrodynamic convection flow of a double-diffusive viscoelastic fluid past an inclined permeable plate in the presence of viscous dissipation and heat absorption. A transverse magnetic field of uniform strengths is applied perpendicular to the plate along the direction of the flow. The nondimensional governing equations have been solved by using a multiple perturbation method, subject to the corresponding boundary conditions. The effects of various physical parameters on flow quantities such as velocity, temperature, and concentration are studied through graphs. The expressions for local skin friction, Nusselt number, and Sherwood number are derived and discussed with the help of tables.

KEY WORDS: *MHD, viscoelastic fluid, free convective, inclined plate, perturbation method*

1. INTRODUCTION

Magnetohydrodynamics (MHD) is the branch of fluid dynamics where magnetic fields are important in the flow. Magnetic fields play an important role in determining the structure of the corona and triggering solar flares and mass ejections. Afify (2009) discussed the MHD free convective heat- and mass-transfer flow over a stretching sheet in the presence of suction/injection with thermal diffusion and diffusion thermo effects. In this present paper, we have made an attempt to study an unsteady MHD free convective double-diffusive fluid flow past an inclined permeable plate. Ahmed et al. (2010) have studied MHD free convective Poiseuille flow and mass transfer through a porous medium bounded by two infinite vertical porous plates. Anjali Devi1and Vasantha Kumari (2014) employed numerical investigation of slip flow effects on unsteady hydromagnetic flow over a stretching surface with thermal radiation. Bakr (2011) considered the steady and unsteady MHD micropolar flow and mass-transfer flow with constant heat source in a rotating frame of reference, taking an oscillatory plate velocity and a constant suction velocity at the plate. Bhattacharyya et al. (2013) analyzed the similarity solution of mixed convective boundary layer slip flow over a vertical plate. On the diffusion of a chemically reactive species in a laminar boundary layer, flow has been examined by Chambre and Young (1958). Chamkha et al. (2004) investi-

NOMENCLATURE

B_0	magnetic field of uniform strength	Q	sink strength
B_1	kinematical viscoelasticity	R_m	magnetic Reynolds number
T	fluid temperature	T_∞	temperature far away from the plate
T_w	temperature at the plate	t	time
C_P	specific heat at constant temperature	u	fluid velocity component along x axis
C	concentration	v_0	constant velocity
D	mass diffusion coefficient	w	frequency parameter
D_1	thermal diffusion coefficient	x, y	Cartesian coordinates along the plate and normal to it
Ec	Eckert number		
Gr	Grashof number		
g	acceleration due to gravity	**Greek Symbols**	
K	permeability of the porous medium	β	coefficient of volumetric expansion
K_T	thermal conductivity	ν	kinematic viscosity
k	permeability parameter	ρ	density of the fluid
M	Hartmann number	σ	electrical conductivity
Pr	Prandtl number	ϕ	angle made by the plate with horizontal

gated the effects of non-Darcian and thermal dispersion of steady, laminar, double-diffusive natural convection boundary layer flow of a micropolar fluid over a vertical permeable semi-infinite plate embedded in a uniform porous medium. Chang and Lee (2008) analyzed the free convection on a vertical plate with uniform and constant heat flux in a thermally stratified micropolar fluid. Cheng (2009) considered the combined heat and mass transfer in natural convection flow from a vertical wavy surface in a power-law fluid saturated porous medium with thermal and mass stratification. Das (2011) analyzed the effect of first-order chemical reaction and thermal radiation on MHD free convection heat- and mass-transfer flow of a micropolar fluid via a porous medium bounded by a semi-infinite porous plate with constant heat source in a rotating frame of reference. Fan et al. (1998) considered the mixed convective heat and mass transfer over a horizontal moving plate with a chemical reaction effect. Gebhart and Pera (1971) analyzed the nature of vertical natural convection flows and mass diffusion on the free convection flow past a semi-infinite vertical plate. During a chemical reaction between two species, heat is also generated. In most cases of chemical reaction, the reaction rate depends on the concentration of the species itself. Gorla et al. (1996) have considered the effects of thermal dispersion and stratification on mixed convection about a vertical surface in a porous medium. Hayat et al. (2013) considered slip effects on the unsteady stagnation point flow with a variable free stream. Natural convection boundary layer flow along a heated vertical plate in a stratified environment was studied by Henkes et al. (1989). Ibrahim and Shankar (2013) examined MHD boundary layer flow and heat transfer of a nanofluid past a permeable stretching sheet with velocity, thermal, and solutal slip boundary conditions. Kandasamy et al. (2005) considered the effects of chemical reaction, heat, and mass transfer along a wedge with heat source and concentration in the presence of suction or injection. Kim (2000) studied unsteady MHD convective heat transfer past a semi-infinite vertical porous moving plate. Kumar et al. (2012) investigated thermal diffusion and radiation effects on unsteady MHD flow through a porous medium with variable temperature and mass diffusion in the presence of a heat source or sink. Mukhopadhyay (2011) discussed the effects of slip on unsteady mixed convective flow and heat transfer past a porous stretching surface. Muthucumarswamy and Kumar (2004) examined the thermal radiation effects on a moving infinite vertical plate in the presence of variable temperature and mass diffusion. Orhan and Ahmet (2008) presented radiation effects on MHD mixed convection flow about a permeable vertical plate. Pal and Talukdar (2010) studied perturbation analysis of unsteady magnetohydrodynamic convective heat and mass transfer in a boundary layer slip flow past a vertical permeable plate with thermal radiation and chemical reaction. Patil et al. (2009) studied double-diffusive mixed convection flow over a moving vertical plate in the presence of internal heat generation and chemical reaction. The radiation effects on an unsteady MHD convective heat and mass-transfer flow past a semi-infinite vertical permeable moving plate embedded in a porous medium was studied by Prasad and Reddy (2008). Rao et al. (2013) studied MHD transient free convection and chemically reactive flow past a porous vertical plate with a radiation and temperature-gradient–dependent heat source in a slip flow regime. Sajid et al. (2010) have obtained an analytical solution for the problem of fully developed mixed convection flow of viscoelastic fluid between two permeable parallel vertical walls. The effects of variable thermal conductivity and heat source/sink on MHD flow near a stagnation point on a linearly stretching sheet are studied by Sharma and Singh (2009). Sivaiah et al. (2012) studied the influence of thermal diffusion and radiation on unsteady magnetohydrodynamic free convection flow past an infinite heated vertical plate in a porous medium. The effect of chemical reaction and radiation absorption on free convection flow through a porous medium with variable suction in the presence of a uniform magnetic field was investigated by Babu and Satyanarayana (2009). The slip condition at the boundary gives interesting features related to engineering applications, for example, reverse osmosis filters. Double-diffusive convection (mass diffusion and thermal diffusion) is an important process in oceanography and plays a role ion mantle convection (magma chambers) and some technological applications. Keeping in sight the above studies and their importance in many areas, we made an attempt to analyze unsteady MHD free convection flow of a double-diffusive viscoelastic fluid past an inclined permeable plate in the presence of viscous dissipation and heat absorption. We have extended the work of Reddy et al. (2010), which gives an analytical solution of effects of unsteady free convective MHD non-Newtonian flow through a porous medium bounded by an infinite inclined porous plate. The novelty of the present study is the consideration of mass transfer in the presence of the Soret effect number.

2. MATHEMATICAL FORMULATION

We have considered an unsteady MHD free convective double-diffusive viscoelastic fluid flow past an inclined permeable plate. Let the $\bar{x}$ axis be taken in the direction of the flow along the infinite inclined plate and the $\bar{y}$ axis be taken perpendicular to the fluid flow. Let $\bar{u}$ be the velocity of the fluid along the $\bar{x}$ direction. A transverse magnetic field of uniform strength B_0 is applied perpendicular to the plate. The inclined plate makes an angle ϕ with the $\bar{x}$ axis. In the analysis of flow, the following assumptions are made:

(i) All the fluid properties are constant except the density in the buoyancy force term.

(ii) The influence of the density variation in terms of momentum and energy equations and the variation of the expansion coefficient with temperature are negligible.

(iii) The Eckert number Ec and the magnetic Reynolds number R_m are assumed to be very small, so that the induced magnetic field can be neglected in comparison with the applied magnetic field.

(iv) The presence of viscous dissipation and thermal diffusion cannot be neglected.

Using the Boussinesq approximation with the above assumptions, the basic flow equations through the porous medium are:

Equation of continuity

$$\frac{\partial \bar{v}}{\partial \bar{y}} = 0 \Rightarrow \bar{v} = -v_0 \ (v_0 > 0). \tag{1}$$

Equation of motion

$$\frac{\partial \bar{u}}{\partial \bar{t}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} = g\beta \sin\phi \left(\bar{T} - \bar{T}_\infty\right) + g\bar{\beta}\sin\phi \left(\bar{C} - \bar{C}_\infty\right) + \upsilon\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + B_1\left(\frac{\partial^3 \bar{u}}{\partial \bar{t}\partial \bar{y}^2} + \bar{v}\frac{\partial^3 \bar{u}}{\partial \bar{y}^3}\right) - \left(\frac{\sigma B_0^2}{\rho} + \frac{\upsilon}{K}\right)\bar{u}. \tag{2}$$

Equation of energy

$$\frac{\partial \bar{T}}{\partial \bar{t}} + \bar{v}\frac{\partial \bar{T}}{\partial \bar{y}} = \frac{k_T}{\rho C_p}\frac{\partial^2 \bar{T}}{\partial \bar{y}^2} + \bar{Q}(\bar{T} - T_\infty) + \frac{\upsilon}{C_p}\left(\frac{\partial \bar{u}}{\partial \bar{y}}\right)^2. \tag{3}$$

Concentration equation

$$\bar{v}\frac{\partial \bar{C}}{\partial \bar{y}} = D\frac{\partial^2 \bar{C}}{\partial \bar{y}^2} + D_1\frac{\partial^2 \bar{T}}{\partial \bar{y}^2}. \tag{4}$$

The relevant boundary conditions are given by

$$\begin{aligned} &\bar{y} = 0; \quad \bar{u} = 0, \quad \bar{v} = -v_0, \\ &\bar{T} = T_W + \varepsilon(T_W - T_\infty)e^{i\omega t}, \\ &\bar{C} = C_W + \varepsilon\left(C_W - C_\infty\right)e^{i\omega t}, \\ &\bar{y} = \infty; \quad \bar{u} \to 0, \quad \bar{T} \to T_\infty, \quad \bar{C} \to C_\infty. \end{aligned} \tag{5}$$

On introducing the following nondimensional quantities,

$$\left.\begin{array}{l} y=\dfrac{\bar{y}v_0}{\upsilon}, \quad t=\dfrac{\bar{t}v_0^2}{4\upsilon}, \quad \omega=\dfrac{4v\bar{\omega}}{v_0^2}, \quad u=\dfrac{\bar{u}}{v_0}, \\ \upsilon=\dfrac{\mu}{\rho}, \quad \mathrm{Pr}=\dfrac{\upsilon}{\bar{k}}, \quad Q=\dfrac{4\bar{Q}\upsilon}{V_0^2}, \\ \bar{k}=\dfrac{k_T}{\rho C_p}, \quad T=\dfrac{\bar{T}-T_\infty}{T_W-T_\infty}, \quad k=\dfrac{Kv_0^2}{\upsilon^2}, \\ C=\dfrac{\bar{C}-C_\infty}{C_W-C_\infty}, \quad \mathrm{Gr}=\dfrac{\upsilon g\beta(T_W-T_\infty)}{v_0^3}, \\ M=\dfrac{\sigma B_0^2\upsilon}{\rho v_0^2}, \quad Gm=\dfrac{\upsilon g\bar{\beta}\left(C_W-C_\infty\right)}{v_0^3}, \\ \mathrm{Ec}=\dfrac{v_0^2}{C_p(T_W-T_\infty)}, \quad R_m=\dfrac{B_1v_0^2}{\upsilon^2}, \end{array}\right\} \quad (6)$$

and by substituting Eq. (6) into the set of Eqs. (2)–(5), we get

$$\frac{1}{4}\frac{\partial u}{\partial t}-\frac{\partial u}{\partial y}=\mathrm{Gr}\sin\phi T+Gm\sin\phi C+\frac{\partial^2 u}{\partial y^2} + R_m\left(\frac{1}{4}\frac{\partial^3 u}{\partial t\partial y^2}-\frac{\partial^3 u}{\partial y^3}\right)-\left(M+\frac{1}{K}\right)u, \quad (7)$$

$$\frac{\mathrm{Pr}}{4}\frac{\partial T}{\partial t}-\mathrm{Pr}\frac{\partial T}{\partial y}=\frac{\partial^2 T}{\partial y^2}+\frac{\mathrm{Pr}}{4}QT+\mathrm{Pr}\,\mathrm{Ec}\left(\frac{\partial u}{\partial y}\right)^2, \quad (8)$$

$$\frac{\partial^2 C}{\partial y^2}+\mathrm{Sc}\frac{\partial C}{\partial y}=S_0\frac{\partial^2 T}{\partial y^2}. \quad (9)$$

The corresponding boundary conditions in nondimensional form are

$$\begin{array}{l} y=0:\quad u=0,\ T=1+\varepsilon e^{i\omega t},\ C=1+\varepsilon e^{i\omega t}, \\ y\to\infty:\quad u\to 0,\ \ T\to 0,\ \ C\to 0. \end{array} \quad (10)$$

3. METHOD OF SOLUTION

To solve the Eqs. (7)–(9) subject the boundary conditions (10), velocity and temperature are assumed in the neighborhood of the plate as in the following form:

$$\begin{array}{l} u\,(y,t)=u_0\,(y)+\varepsilon e^{i\omega t}u_1\,(y)+o(\varepsilon^2), \\ T\,(y,t)=T_0\,(y)+\varepsilon e^{i\omega t}T_1\,(y)+o(\varepsilon^2), \\ C\,(y,t)=C_0\,(y)+\varepsilon e^{i\omega t}C_1\,(y)+o(\varepsilon^2). \end{array} \quad (11)$$

On substituting relations (11) in Eqs. (7)–(9) and equating the coefficients of the zeroth- and first-order terms and also neglecting second and higher powers of ε, we get the following set of equations:

Zeroth order equations

$$R_m u_0^{111}-u_0^{11}-u_0^1+\left(M+\frac{1}{K}\right)u_0=\mathrm{Gr}\sin\phi\,T_0 + Gm\sin\phi\,C_0, \quad (12)$$

$$T_0^{11}+\mathrm{Pr}\,T_0^1+\frac{\mathrm{Pr}}{4}QT_0=-\mathrm{Pr}\,\mathrm{Ec}\left(u_0^1\right)^2, \quad (13)$$

$$C_0^{11}+\mathrm{Sc}\,C_0^1=S_0T_0^{11}. \quad (14)$$

First-order equations

$$R_m u_1^{111}-\left(1+\frac{iR_m\omega}{4}\right)u_1^{11}-u_1^1+\left(M+\frac{1}{K}+\frac{i\omega}{4}\right)u_1 = \mathrm{Gr}\sin\phi T_1+Gm\sin\phi C_1, \quad (15)$$

$$T_1^{11}+\mathrm{Pr}\,T_1^1+\frac{\mathrm{Pr}}{4}\left(Q-i\omega\right)T_1=-2\mathrm{Pr}\,\mathrm{Ec}\,u_0^1u_1^1, \quad (16)$$

$$C_1^{11}+\mathrm{Sc}C_1^1=S_0\,T_1^{11}. \quad (17)$$

The corresponding boundary conditions are

$$\left.\begin{array}{l} y=0;\ u_0=u_1=0,\ T_0=T_1=1,\ C_0=C_1=1, \\ y\to\infty;\ u_0\to 0,\ u_1\to 0,\ T_0\to 0,\ T_1\to 0, \\ C_0\to 0,\ C_1\to 0. \end{array}\right\} \quad (18)$$

To solve Eqs. (12) and (15), we need three boundary conditions but we have two. So we assume that the solutions of these equations to be of the form

$$\begin{array}{l} u_0=u_{00}+R_mu_{01}+o\left(R_m^2\right), \\ u_1=u_{10}+R_mu_{11}+o\left(R_m^2\right); \\ T_0=T_{00}+R_mT_{01}+o\left(R_m^2\right), \\ T_1=T_{10}+R_mT_{11}+o\left(R_m^2\right); \\ C_0=C_{00}+R_mC_{01}+o\left(R_m^2\right), \\ C_1=C_{10}+R_mC_{11}+o\left(R_m^2\right). \end{array} \quad (19)$$

Substituting Eq. (19) into the equations from (12) to (17), we get the following equations after neglecting second and higher powers of R_m:

Zero order of R_m

$$u_{00}^{11}+u_{00}^1-\left(M+\frac{1}{K}\right)u_{00}=-\mathrm{Gr}\,\sin\phi\,T_{00} - Gm\,\sin\phi\,C_{00}, \quad (20)$$

$$u_{10}^{11}+u_{10}^1-\left(M+\frac{1}{K}+\frac{i\omega}{4}\right)u_{10}=-\mathrm{Gr}\sin\phi T_{10} - Gm\sin\phi C_{10}, \quad (21)$$

$$T_{00}^{11} + \text{Pr}T_{00}^{1} + \frac{\text{Pr}}{4}QT_{00} = -\text{Pr}\,\text{Ec}\left(u_{00}^{1}\right)^{2}, \quad (22)$$

$$T_{10}^{11} + \text{Pr}T_{10}^{1} + \frac{\text{Pr}}{4}\left(Q - i\omega\right)T_{10} = -2\text{Pr}\,\text{Ec}\,u_{00}^{1}u_{10}^{1}, \quad (23)$$

$$C_{00}^{11} + \text{Sc}\,C_{00}^{1} = S_0\,T_{00}^{11}, \quad (24)$$

$$C_{10}^{11} + \text{Sc}\,C_{10}^{1} = S_0\,T_{10}^{11}. \quad (25)$$

First order of R_m

$$u_{01}^{11} + u_{01}^{1} - \left(M + \frac{1}{K}\right)u_{01} = u_{00}^{111} - \text{Gr}\sin\phi\,T_{01} - Gm\sin\phi\,C_{01}, \quad (26)$$

$$u_{11}^{11} + u_{11}^{1} - \left(M + \frac{1}{K} + \frac{i\omega}{4}\right)u_{11} = u_{10}^{111} - \text{Gr}\sin\phi\,T_{11} - \frac{i\omega}{4}u_{10}^{11} - Gm\sin\phi\,C_{11}, \quad (27)$$

$$T_{01}^{11} + \text{Pr}T_{01}^{1} + \frac{\text{Pr}}{4}Q\,T_{01} = -2\text{Pr}\,\text{Ec}\,u_{00}^{1}\,u_{01}^{1}, \quad (28)$$

$$T_{11}^{11} + \text{Pr}T_{11}^{1} + \frac{\text{Pr}}{4}\left(Q - i\omega\right)T_{11} = -2\text{Pr}\,\text{Ec} \times \left(u_{00}^{1}u_{11}^{1} + u_{01}^{1}\,u_{10}^{1}\right), \quad (29)$$

$$C_{01}^{11} + \text{Sc}\,C_{01}^{1} = S_0\,T_{01}^{11}, \quad (30)$$

$$C_{11}^{11} + \text{Sc}\,C_{11}^{1} = S_0\,T_{11}^{11}. \quad (31)$$

The corresponding boundary conditions are

$$\begin{aligned} &y = 0;\ \ u_{00} = u_{01} = u_{10} = u_{11} = T_{01} = T_{11} \\ &= C_{01} = C_{11} = 0,\ \ T_{00} = T_{10} = C_{00} = C_{10} = 1, \\ &y \to \infty;\ \ u_{00} \to u_{01} \to u_{10} \to u_{11} \to T_{00} \to T_{01} \\ &\to T_{10} \to T_{11} \to C_{00} \to C_{01} \to C_{10} \to C_{11} \to 0. \end{aligned} \quad (32)$$

In order to solve Eqs. (20)–(31) the multiperturbation scheme for Ec $\ll 1$ (for all incompressible fluid) has been introduced as follows:

$$\left.\begin{aligned} u_{00} &= u_{000} + \text{Ec}\,u_{001} + o(\text{Ec}^2), \\ u_{01} &= u_{010} + \text{Ec}\,u_{011} + o(\text{Ec}^2); \\ u_{10} &= u_{100} + \text{Ec}\,u_{101} + o(\text{Ec}^2), \\ u_{11} &= u_{110} + \text{Ec}\,u_{111} + o(\text{Ec}^2); \\ T_{00} &= T_{000} + \text{Ec}T_{001} + o(\text{Ec}^2), \\ T_{01} &= T_{010} + \text{Ec}T_{011} + o(\text{Ec}^2); \\ T_{10} &= T_{100} + \text{Ec}T_{101} + o(\text{Ec}^2), \\ T_{11} &= T_{110} + \text{Ec}T_{111} + o(\text{Ec}^2); \\ C_{00} &= C_{000} + \text{Ec}\,C_{001} + o(\text{Ec}^2), \\ C_{01} &= C_{010} + \text{Ec}\,C_{011} + o(\text{Ec}^2); \\ C_{10} &= C_{100} + \text{Ec}\,C_{101} + o(\text{Ec}^2), \\ C_{11} &= C_{110} + \text{Ec}\,C_{111} + o(\text{Ec}^2). \end{aligned}\right\} \quad (33)$$

On substituting relations (33) in Eqs. (20)–(31) and equating the coefficients of zeroth and first-order terms and also neglecting second and higher powers of Ec, the following equations are obtained:

Zero order of Ec

$$u_{000}^{11} + u_{000}^{1} - \left(M + \frac{1}{K}\right)u_{000} = -\text{Gr}\sin\phi\,T_{000} - Gm\sin\phi\,C_{000}, \quad (34)$$

$$u_{100}^{11} + u_{100}^{1} - \left(M + \frac{1}{K} + \frac{i\omega}{4}\right)u_{100} = -\text{Gr}\sin\phi\,T_{100} - Gm\sin\phi\,C_{100}, \quad (35)$$

$$T_{000}^{11} + \text{Pr}\,T_{000}^{1} + \frac{\text{Pr}}{4}Q\,T_{000} = 0, \quad (36)$$

$$T_{100}^{11} + \text{Pr}\,T_{100}^{1} + \frac{\text{Pr}}{4}\left(Q - i\omega\right)T_{100} = 0, \quad (37)$$

$$u_{010}^{11} + u_{010}^{1} - \left(M + \frac{1}{K}\right)u_{010} = u_{000}^{111} - \text{Gr}\sin\phi\,T_{010} - Gm\sin\phi\,C_{010}, \quad (38)$$

$$u_{110}^{11} + u_{110}^{1} - \left(M + \frac{1}{K} + \frac{i\omega}{4}\right)u_{110} = u_{100}^{111} - \text{Gr}\sin\phi\,T_{110} - Gm\sin\phi\,C_{110} - \frac{i\omega}{4}u_{100}^{11}, \quad (39)$$

$$T_{010}^{11} + \text{Pr}\,T_{010}^{1} + \frac{\text{Pr}}{4}Q\,T_{010} = 0, \quad (40)$$

$$T_{110}^{11} + \text{Pr}\,T_{110}^{1} + \frac{\text{Pr}}{4}\left(Q - i\omega\right)T_{110} = 0, \quad (41)$$

$$C_{000}^{11} + \text{Sc}\,C_{000}^{1} = S_0\,T_{000}^{11}, \quad (42)$$

$$C_{100}^{11} + \text{Sc}\,C_{100}^{1} = S_0 T_{100}^{11}, \quad (43)$$

$$C_{010}^{11} + \text{Sc}\,C_{010}^{1} = S_0\,T_{010}^{11}, \quad (44)$$

$$C_{110}^{11} + \text{Sc}\,C_{110}^{1} = S_0\,T_{110}^{11}. \quad (45)$$

First order of Ec

$$u_{001}^{11} + u_{001}^{1} - \left(M + \frac{1}{K}\right)u_{001} = -\text{Gr}\sin\phi\,T_{001} - Gm\sin\phi\,C_{001}, \quad (46)$$

$$u_{101}^{11} + u_{101}^{1} - \left(M + \frac{1}{K} + \frac{i\omega}{4}\right)u_{101} = -\text{Gr}\sin\phi\,T_{101} - Gm\sin\phi\,C_{101}, \quad (47)$$

$$T_{001}^{11} + \text{Pr}T_{001}^{1} + \frac{\text{Pr}}{4}Q\,T_{001} = -\text{Pr}\,(u_{000}^{1})^2, \quad (48)$$

$$T_{101}^{11} + \text{Pr}T_{101}^{1} + \frac{\text{Pr}}{4}\,(Q - i\omega)\,T_{101} = -2\text{Pr}u_{000}^{1}\,u_{100}^{1}, \quad (49)$$

$$u_{011}^{11} + u_{011}^{1} - \left(M + \frac{1}{K}\right)u_{011} = u_{001}^{111} - \text{Gr}\sin\phi\,T_{011} - Gm\sin\phi\,C_{011}, \quad (50)$$

$$u_{111}^{11} + u_{111}^{1} - \left(M + \frac{1}{K} + \frac{i\omega}{4}\right)u_{111} = u_{101}^{111} - \text{Gr}\sin\phi\,T_{111} - Gm\sin\phi\,C_{111} - \frac{i\omega}{4}u_{101}^{11}, \quad (51)$$

$$T_{011}^{11} + \text{Pr}\,T_{011}^{1} + \frac{\text{Pr}}{4}Q\,T_{011} = -2\text{Pr}\,u_{000}^{1}\,u_{010}^{1}, \quad (52)$$

$$T_{111}^{11} + \text{Pr}T_{111}^{1} + \frac{\text{Pr}}{4}\,(Q - i\omega)\,T_{111} = -2\text{Pr}\left(u_{000}^{1}u_{110}^{1} + u_{010}^{1}u_{100}^{1}\right), \quad (53)$$

$$C_{001}^{11} + \text{Sc}\,C_{001}^{1} = S_0\,T_{001}^{11}, \quad (54)$$

$$C_{101}^{11} + \text{Sc}\,C_{101}^{1} = S_0\,T_{101}^{11}, \quad (55)$$

$$C_{011}^{11} + \text{Sc}\,C_{011}^{1} = S_0\,T_{011}^{11}, \quad (56)$$

$$C_{111}^{11} + \text{Sc}\,C_{111}^{1} = S_0\,T_{111}^{11}. \quad (57)$$

The corresponding boundary conditions are

$$\left.\begin{array}{l} y = 0: \;\; u_{000} = u_{001} = u_{010} = u_{011} = 0, \\ T_{000} = 1,\; T_{010} = T_{001} = T_{011} = 0, \\ u_{100} = u_{110} = u_{101} = u_{111} = 0,\; T_{100} = 1, \\ T_{110} = T_{101} = T_{111} = 0, \\ C_{000} = 1,\; C_{001} = C_{010} = C_{011} = 0, \\ C_{100} = 1,\; C_{110} = C_{101} = C_{111} = 0, \\ y \to \infty: \;\; u_{000} \to u_{001} \to u_{010} \to u_{011} \to 0, \\ T_{000} \to T_{010} \to T_{001} \to T_{011} \to 0, \\ u_{100} \to u_{110} \to u_{101} \to u_{111} \to 0, \\ T_{100} \to T_{110} \to T_{101} \to T_{111} \to 0, \\ C_{000} \to C_{001} \to C_{010} \to C_{011} \to 0, \\ C_{100} \to C_{110} \to C_{101} \to C_{111} \to 0. \end{array}\right\} \quad (58)$$

The solution of the problem is

$$u\,(y,t) = R_1e^{-r_3y} + R_2e^{-q_1y} + R_3e^{-Scy} + R_4e^{-2r_3y} + R_5e^{-2q_1y} + R_6e^{-2\text{Sc}y} + R_7e^{-(r_3+q_1)y} + R_8e^{-(q_1+\text{Sc})y} + R_9e^{-(r_3+Sc)y} + R_{10}e^{-r_5y} + R_{11}e^{-r_1y} + R_{12}e^{-(r_3+r_5)y} + R_{13}e^{-(r_3+r_1)y} + R_{14}e^{-(r_5+q_1)y} + R_{15}e^{-(r_1+q_1)y} + +R_{16}e^{-(r_5+Sc)y} + R_{17}e^{-(r_1+Sc)y}, \quad (59)$$

$$T\,(y,t) = R_{19}e^{-q_1y} + R_{20}e^{-2r_3y} + R_{21}e^{-2q_1y} + R_{22}e^{-2Scy} + R_{23}e^{-(r_3+q_1)y} + R_{24}e^{-(q_1+Sc)y} + R_{25}e^{-(r_3+Sc)y} + R_{26}e^{-r_1y} + R_{27}e^{-(r_3+r_5)y} + R_{28}e^{-(r_3+r_1)y} + R_{29}e^{-(r_5+q_1)y} + R_{30}e^{-(r_1+q_1)y} + R_{31}e^{-(r_5+Sc)y} + R_{32}e^{-(r_1+Sc)y}, \quad (60)$$

$$C\,(y,t) = R_{34}e^{-\text{Sc}y} + R_{35}e^{-q_1y} + R_{36}e^{-2r_3y} + R_{37}e^{-(r_3+q_1)y} + R_{38}e^{-(r_3+\text{Sc})y} + R_{39}e^{-(q_1+\text{Sc})y} + R_{40}e^{-2q_1y} + R_{41}e^{-2\text{Sc}y} + R_{42}e^{-r_1y} + R_{43}e^{-(r_3+r_5)y} + R_{44}e^{-(r_3+r_1)y} + R_{45}e^{-(r_5+q_1)y} + R_{46}e^{-(r_1+q_1)y} + R_{47}e^{-(r_5+Sc)y} + R_{48}e^{-(r_1+Sc)y}. \quad (61)$$

Skin friction: The skin friction is obtained from Eq. (59), which is defined as

$$\tau = \left(\frac{\partial u}{\partial y}\right)_{y=0} = -R_1r_3 - R_2q_1 - R_3\text{Sc} - 2R_4r_3 - 2R_5q_1 - 2R_6\text{Sc} - R_7\,(r_3 + q_1) - R_8\,(q_1 + \text{Sc}) - R_9\,(r_3 + \text{Sc}) - R_{10}r_5 - R_{11}r_1 - R_{12}\,(r_3 + r_5) - R_{13}\,(r_1 + r_3) - R_{14}\,(r_5 + q_1) - R_{15}\,(r_1 + q_1) - R_{16}\,(r_5 + \text{Sc}) - R_{17}\,(r_1 + \text{Sc}). \quad (62)$$

Rate of heat transfer: The rate of mass transfer in the form of Nusselt number is given by

$$\text{Nu} = \left(\frac{\partial T}{\partial y}\right)_{y=0} = -R_{19}q_1 - 2R_{20}r_3 - 2R_{21}q_1 - 2R_{22}\text{Sc} - R_{23}\,(r_3 + q_1) - R_{24}\,(q_1 + \text{Sc}) - R_{25}\,(r_3 + \text{Sc}) - R_{26}r_1 - R_{27}\,(r_3 + r_5) - R_{28}\,(r_3 + r_1) - R_{29}\,(r_5 + q_1) - R_{30}\,(r_1 + q_1) - R_{31}\,(r_5 + \text{Sc}) - R_{32}\,(r_1 + \text{Sc}). \quad (63)$$

Sherwood: The Sherwood is obtained from Eq. (61), which is defined as

$$\text{Sh} = \left(\frac{\partial C}{\partial y}\right)_{y=0} = -R_{34}\text{Sc} - R_{35}q_1 - 2R_{36}r_3 - R_{37}\,(r_3 + q_1) - R_{38}\,(r_3 + \text{Sc}) - R_{39}\,(q_1 + \text{Sc}) - 2R_{40}q_1 - 2R_{41}\text{Sc} - R_{42}r_1 - R_{43}\,(r_3 + r_5) - R_{44}\,(r_3 + r_1) - R_{45}\,(r_5 + q_1) - R_{46}\,(r_1 + q_1) - R_{47}\,(r_5 + \text{Sc}) - R_{48}\,(r_1 + \text{Sc}). \quad (64)$$

4. RESULTS AND DISCUSSION

In order to get physical insight into the problem, the velocity, temperature, concentration distributions along with skin friction, Nusselt number, and Sherwood number have been studied by assigning numerical values for M, K, ϕ, Sc, Pr, Q, Gr, Gm while keeping $R_m = 0.01$, Ec $= 0.01$, $w = \pi/6$, $t = 1$, and $\varepsilon = 0.01$ constant. The results obtained are illustrated through Figs. 1–8 and Table 1. Figure 1 shows the velocity profiles u against y for different values of M. From this figure, we observe that as M increases, the velocity u decreases, which indicates that magnetic field restrains the free convection. Figure 2 depicts the velocity profile u against y for different values of K. From this figure, it is observed that as K increases, the velocity u increases. In Fig. 3, as ϕ increases, the velocity u increases. From Fig. 4, as Schmidt number increases, the velocity (u) decreases. From Fig. 5, it is noticed that velocity decreases with the increase in Prandtl

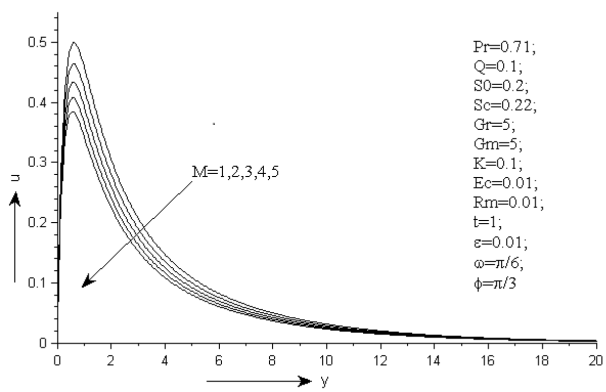

FIG. 1: Effect of M on velocity

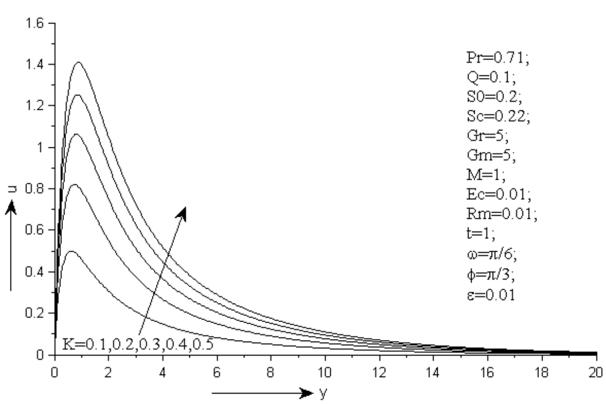

FIG. 2: Effect of K on velocity

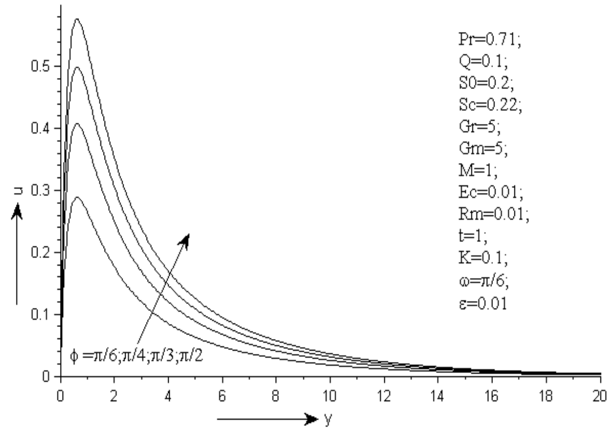

FIG. 3: Effect of ϕ on velocity

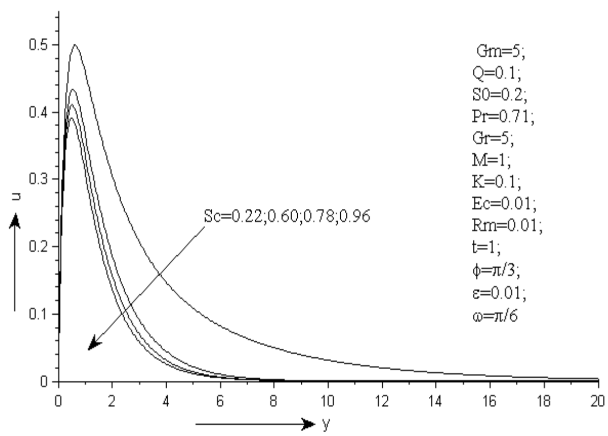

FIG. 4: Effect of Sc on velocity

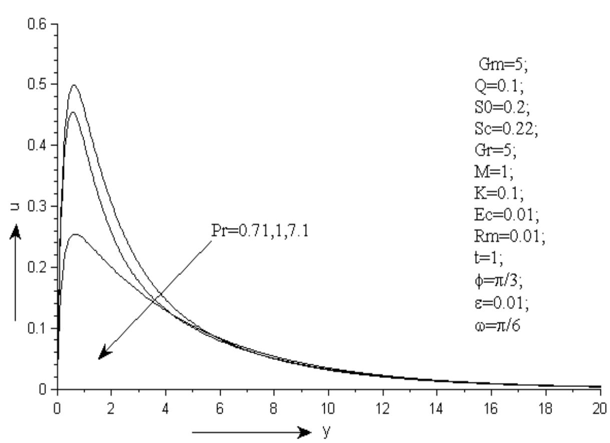

FIG. 5: Effect of Pr on velocity

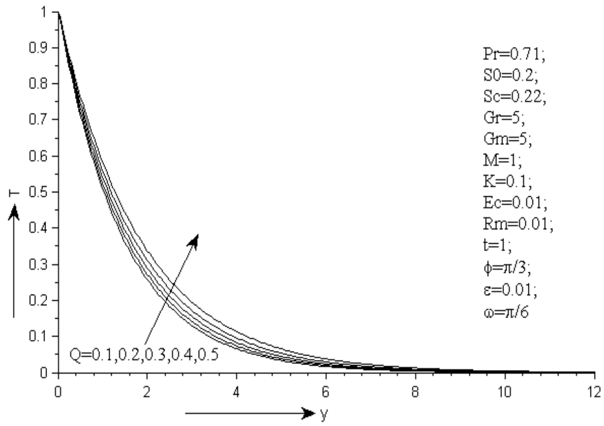

FIG. 6: Effect of Q on temperature

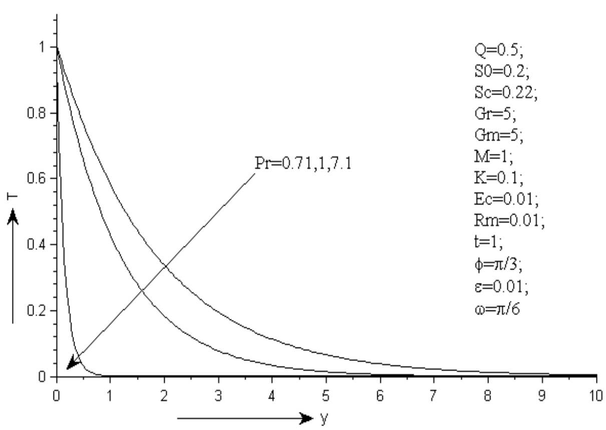

FIG. 7: Effect of Pr on temperature

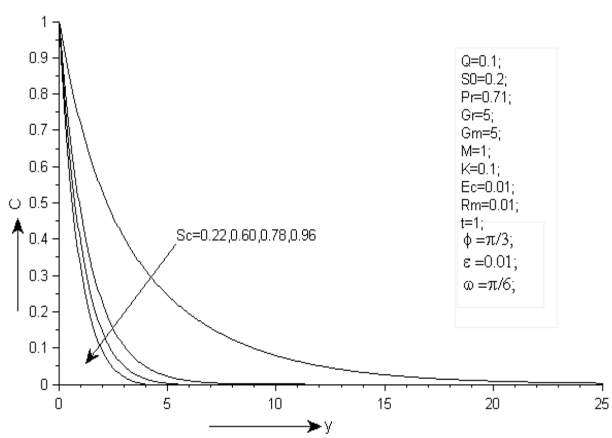

FIG. 8: Effect of Sc on concentration

number (Pr). From this figure, we observe that velocity (u) is greater for mercury (Pr $=$ 0.025) than that of the electrolytic solution (Pr $=$ 1), i.e., velocity (u) for various viscous fluid is more than the viscoelastic fluid.

From Fig. 6, it is noticed that the temperature (T) increases as the heat source parameter (Q) increases. From Fig. 7, it is observed that temperature (T) decreases as the Prandtl number (Pr) increases. Figure 8 shows that the concentration (C) decreases as the Schmidt number (Sc) increases. From Table 1, we observe that the skin friction, Nusselt number, and Sherwood number decrease with an increase in Schmidt number (Sc), Prandtl number (Pr), and magnetic field (M), but they show the reverse effect in the case of Gr, Gm, and Q.

5. CONCLUSION

In this paper, an unsteady MHD free convective double-diffusive viscoelastic fluid flow past an inclined permeable plate has been studied numerically. T;e equations governing the velocity and temperature of the fluid are solved by using a multiparameter perturbation technique in terms of dimensionless parameters. In the analysis of the flow the following conclusions are made:

(i) Velocity increases with an increase in K and ϕ but it shows the reverse effects in the case of M, Pr, and Sc.

(ii) Temperature increases with an increase in Q, but it shows the reverse effects in the case of Pr.

(iii) Concentration decreases with an decrease in Sc.

(iv) Skin friction increases with an increase in Gr, Gm, and Q but it shows the reverse effects in the case of M, Sc, and Pr.

(v) Nusselt number increases with an increase in Gr, Gm, and Q but it shows the reverse effects in the case of M, Sc, and Pr.

(vi) Sherwood number increases with an increase in Gr, Gm, and Q but it shows the reverse effects in the case of M, Sc, and Pr.

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TABLE 1: Variations in skin friction, Nusselt number and Sherwood number in the presence of different physical parameters

Sc	***Q***	**Gr**	***Gm***	**Pr**	***M***	τ	**Nu**	**Sh**
0.22	0.1	5	5	0.71	1	2.6015	−0.6844	−0.3588
0.60	0.1	5	5	0.71	1	2.4515	−0.6850	−0.7422
0.78	0.1	5	5	0.71	1	2.3914	−0.6853	−0.9244
0.96	0.1	5	5	0.71	1	2.3374	−0.6855	−1.1055
0.22	0.2	5	5	0.71	1	2.6132	−0.6558	−0.3530
0.22	0.3	5	5	0.71	1	2.6264	−0.6244	−0.3467
0.22	0.4	5	5	0.71	1	2.6416	−0.5888	−0.3396
0.22	0.5	5	5	0.71	1	2.6599	−0.5468	−0.3312
0.22	0.1	10	5	0.71	1	3.8378	−0.6786	−0.3576
0.22	0.1	15	5	0.71	1	5.0754	−0.6706	−0.3560
0.22	0.1	20	5	0.71	1	6.3149	−0.6605	−0.3540
0.22	0.1	5	10	0.71	1	3.9685	−0.6772	−0.3573
0.22	0.1	5	15	0.71	1	5.3364	−0.6669	−0.3551
0.22	0.1	5	20	0.71	1	6.7052	−0.6536	−0.3523
0.22	0.1	5	5	1	1	2.4911	−0.9761	−0.4171
0.22	0.1	5	5	7.1	1	1.606	−7.1188	−1.6455
0.22	0.1	5	5	0.71	2	2.4926	−0.6849	−0.3589
0.22	0.1	5	5	0.71	3	2.3964	−0.6854	−0.3590
0.22	0.1	5	5	0.71	4	2.3105	−0.6858	−0.3591

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