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# Squeeze film behavior in porous transversely circular stepped plates with a couple stress fluid

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## Abstract

**Purpose** – The purpose of this paper is to carry out a study of the effect of surface roughness on squeeze film behavior between two transversely circular stepped plates with couple stress lubricant when the upper circular stepped plate has porous facing which approaches the lower plate with uniform velocity.

**Design/methodology/approach** – The modified Stochastic Reynolds equation is derived for Christensen Stochastic theory for the rough surfaces. Closed form solution of the Stochastic Reynolds equation is obtained in terms of Fourier-Bessel series.

**Findings** – It is found that the effect of couple stress fluid and surface roughness is more pronounced compared to classical case.

**Originality/value** – The problem is original that it consider a couple stress fluid in this type of applications.

**Keywords** Circular stepped plates, Couple stress fluid, Fourier-Bessel series, Squeeze film, Stochastic Reynolds equation, Christensen Stochastic theory

**Paper type** Research paper

## Nomenclature

$a$	outer radius of the plate	$p_1$	fluid film pressure in the region $0 \leq r \leq KR$
$E$	expectancy operator	$p_2$	fluid film pressure in the region $KR \leq r \leq R$
$f$	probability density function	$Q$	volume flow rate
$h_0$	initial film thickness	$R$	radius of the circular plate
$h_1$	maximum film thickness	$t$	time of approach
$h_2$	minimum film thickness	$t^*$	non-dimensional squeeze film time
$h_s^*$	stepheight ( $= h_s/h_0$ )	$u, w$	velocity components in r and z-directions, respectively
$H^*$	film thickness of the porous layer	$W$	load carrying capacity
$J_0$	Bessel function of first kind of 0th order	$W^*$	non-dimensional load carrying capacity
$l$	couple stress parameter		
$p$	pressure in the film region		
$p^*$	pressure in the porous region		



$u^*, w^*$	Darcy's velocity components in the porous region	$\tau$	non-dimensional couple stress parameter
$\bar{w}_0$	modified form of the Darcy's law	$\xi$	random variable
	<i>Greek symbols</i>	$\theta$	angular co-ordinate
$\alpha_n$	Nth eigenvalue	$\Phi$	permeability parameter
$\beta$	ratio of microstructure size of polar additives to the pore size	$\Psi$	non-dimensional permeability parameter
$\mu$	viscosity of the lubricant		
$\eta$	material constant		

## 1. Introduction

The squeeze film lubrication phenomenon is observed in several applications such as automotive engines, machine tools and rolling elements. The squeeze film phenomenon arises when the two lubricating surfaces move toward each other in the normal direction and generates a positive pressure and hence supports a load. This is due to the fact that a viscous lubricant present between the two surfaces cannot be instantaneously squeezed out when the two surfaces move toward each other and this action provides a cushioning effect in bearings. Self-lubricating porous bearings are widely used in industry due to their self-contained oil reservoir in addition to their low cost and other aspects concerned with lubrication mechanism. Porous bearings are extensively used in brakes, clutches, etc. due to their self-contained oil reservoir and favorable low friction characteristics. An analysis of the squeeze film between porous rectangular plates is studied by Wu (1961) and gives very useful predictions in respect of porous bearings. The squeeze film lubrication between two infinitely long parallel plates is studied by Cameron (1981).

It is well known in the tribology literature that the bearing surfaces develop roughness after having some run-in and wear. The chemical degradation of lubricants leading to the contamination of lubricants is also one of the plausible reasons for developing the roughness on bearing surfaces in some cases. Since the surface roughness distribution is random in nature, a stochastic approach to model the surface roughness mathematically has to be adopted. Many researchers modeled the roughness by a Fourier series type approximation and employed a saw-tooth curve to mathematically model the surface roughness. Mitchell (1950) described surface roughness by a high-frequency sine curve. Christensen (1969) developed a stochastic model for the study of rough surfaces in hydrodynamic lubrication of solid bearings. Prakash and Tiwari (1982) used the Christensen Stochastic theory to study the effect of surface roughness on the characteristics of porous bearings.

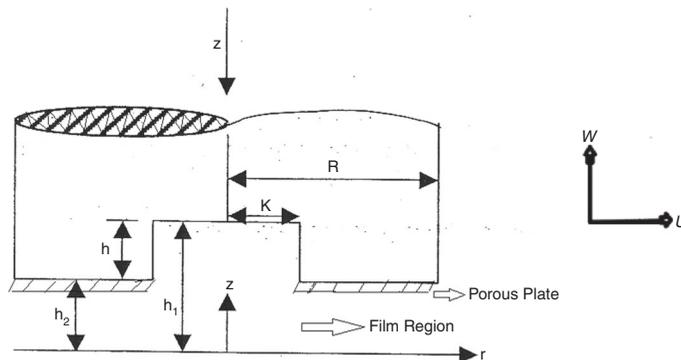
All the investigations mentioned above are confined to the study of surface roughness on porous bearings with Newtonian fluid as lubricant. The use of different liquids as lubricant under different circumstances has gained importance with the development of modern technology machines. It is observed fact that the fluids containing additives or contaminants enhance the lubrication process. These additives have a desirable effect of enhancement of viscosity and a consequent rise in load capacity. Usually, these additives are in the form of long chain organic compounds. In most of these lubricating oils the additives of high molecular weight polymers are present as a kind of viscosity index improvers. The base oils with high viscosity index exhibit improved response to additives of various chemical composition which upgrade their quality and results in reduced additives consumption in the production of additive blended oils. Hence nowadays, the trend has been toward increasing of the viscosity index to manifolds.

Kragelsky and Alisin (1981) have given important advantages of the base oils of high viscosity index are highly reliable components of machine parts within a wide range of working temperature, a longer life and a good response to additives, etc. Hence, with the development of modern industry, the importance of non-Newtonian fluids as lubricant has been emphasized as Newtonian constitutive approximation is not a satisfactory engineering approach for most of the lubrication problems. Therefore, the effect of non-Newtonian property of lubricant must be taken into account in the realistic study of these bearings. The common lubricants exhibiting non-Newtonian behavior are polymer-thickened oils, greases and natural lubricating fluids, which appear in animal joints.

Stokes (1966) theory of couple stress fluids is the simplest generalization of classical theory of fluids which account for polar effects such as the presence of anti-symmetrical stresses, couple stresses and body couples. Many investigators have used this theory to analyze the lubrication characteristics of various bearing systems. An investigation on squeeze films between rough anisotropic porous rectangular plates is done by Bujurke and Naduvinamani (1998). Ramanaiah (1979) investigated squeeze films between finite plates lubricated by fluids with couple stresses. Naduvinamani and Siddangouda (2009) have investigated squeeze film lubrication between circular stepped plates of couple stress fluids. Bujurke *et al.* (2008) have investigated surface roughness effects on squeeze film behavior in porous circular disks with couple stress fluids. Squeeze films and thrust bearing lubricated by fluids with couple stress is investigated by Ramanaiah and Sarkar (1978). Sundarammal *et al.* (2014) have investigated magnetohydrodynamic squeeze film characteristics between finite porous parallel rectangular plates with surface roughness effect. Effect of bearing deformation on the characteristics of a slider bearing, (Ramanaiah and Sundarammal, 1982a) circular and rectangular plates is studied by Ramanaiah and Sundarammal (1982b). The purpose of this study is to investigate the performance of squeeze film behavior in porous circular stepped disks using couple stress fluid and surfaces with roughness.

## 2. Mathematical formulation of the problem

The schematic diagram of squeeze film geometry of the problem considered is shown in Figure 1. The squeeze film characteristics between the circular stepped plates are analyzed when one circular stepped plate has a porous facing which approaches the other rough plate with uniform velocity. The lubrication in the film region is assumed to be a Stokes couple stress fluid. It is also assumed that, the body forces and body couples are absent. The bearing surfaces are assumed to be transversely rough.



**Figure 1.**  
Configuration  
of the Squeeze film  
bearing system

Under the usual assumption of hydrodynamic lubrication, applicable to thin films, the equations of motion for couple stress fluid takes the forms:

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behavior

$$\frac{\partial p}{\partial r} = \mu \frac{\partial^2 u}{\partial z^2} - \eta \frac{\partial^4 u}{\partial z^4} \quad (1)$$

$$\frac{\partial p}{\partial z} = 0 \quad (2)$$

$$\frac{1}{r} \frac{\partial(ru)}{\partial r} + \frac{\partial w}{\partial z} = 0 \quad (3)$$

where  $u$  and  $w$  are the velocity components in  $r$  and  $z$  directions, respectively,  $p$  is the pressure,  $\mu$  is the Newtonian viscosity,  $\eta$  is the new material constant characterizing the couple stress and is of dimension of momentum. The ratio  $\eta/\mu$  has the dimension of length squared and hence characterizes the material length of the fluid. The film thickness of the lubricant film geometry is:

$$H = h(t) + h_s(r, \theta, \xi) \quad (4)$$

where  $h$  denotes the nominal smooth part of the film geometry, while  $h_s$  is the part due to the surface asperities measured from the nominal level and is regarded as a randomly varying quantity of zero mean,  $r$  and  $\theta$  are the radial and angular coordinates and  $\xi$  is the index parameter determining a definite roughness arrangement.

The relevant boundary conditions for the velocity components are:

$$u = 0, \quad w = 0, \quad \frac{\partial^2 u}{\partial z^2} = 0 \text{ at the lower surface } z = 0 \quad (5)$$

$$u = 0, \quad w = w^* + \frac{dH}{dt}, \quad \frac{\partial^2 u}{\partial z^2} = 0 \text{ at the upper surface } z = H \quad (6)$$

Here  $dH/dt$  is the constant velocity of the upper porous circular stepped plate approaching the lower impermeable circular stepped plate. Last conditions in Equations (5) and (6) are due to the vanishing of the couple stresses at  $z=0$  and  $H$ , respectively. Since,  $p$  is independent of  $z$ , the solution of Equation (1), subject to the boundary conditions (5) and (6) is:

$$u = \frac{1}{2\mu} \frac{dp}{dr} \left[ z(z-H) + 2l^2 \left[ 1 - \left( \frac{\cos h\left(\frac{2z-H}{2l}\right)}{\cos h\left(\frac{H}{2l}\right)} \right) \right] \right] \quad (7)$$

where  $l = 1/[\mu/\eta]^{1/2}$  is the couple stress parameter.

The volume flux of the lubricant is given by:

$$Q = 2\pi r \int_0^H u dz \quad (8)$$

where  $2\pi r$  is the circumference of the circle. On using Equations (7) and (8), we have:

$$Q = \frac{-\pi r}{6\mu} \frac{dp}{dr} f(H.l) \quad (9)$$

where:

$$f(H, l) = H^3 - 12l^2h + 24l^3 \tan h \left( \frac{H}{2l} \right)$$

Integration of the continuity Equation (3) over the film thickness and the use of boundary conditions  $w = w^* + dH/dt$ , and  $w = 0$  gives:

$$\frac{\partial Q}{\partial r} = 2\pi r \left( w^* + \frac{dH}{dt} \right) \quad (10)$$

Integration of the continuity Equation (10) with respect to  $r$  and using the condition  $Q = 0$  at  $r = 0$  gives:

$$Q = \pi \left( w^* + \frac{dH}{dt} \right) r^2 \quad (11)$$

The modified Reynolds type equation for determining the pressure is obtained from Equations (9) and (11) in the form:

$$\frac{dp_i}{dr} = - \frac{6\mu r \left( w^* + \frac{dH}{dt} \right)}{S_i(H_i, l)} \quad (12)$$

where  $p_i = p_1, h_i = h_1$  for  $0 \leq r \leq KR$   
 $p_i = p_2, h_i = h_2$  for  $KR \leq r \leq R$

$$S_i(H_i, l) = H_i^3 - 12l^2H_i + 24l^3 \tan h \left( \frac{H_i}{2l} \right)$$

$p_1$  and  $p_2$  being the pressure in the region-I ( $0 \leq r \leq KR$ ) and in the region-II ( $KR \leq r \leq R$ ), respectively. The relevant boundary condition for the pressure are:

$$p_1 = p_2 \quad \text{at} \quad r = KR \quad (13)$$

$$p_2 = 0 \quad \text{at} \quad r = R \quad (14)$$

The flow of couple stress fluid in a porous medium is governed by the modified form of the Darcy's law, which accounts for the (Naduvanamani *et al*, 2002) polar effects:

$$\overline{w}_o = - \frac{\phi}{\mu(1-\beta)} \nabla p^* \quad (15)$$

where  $\overline{w}_o = (u^*, w^*)$ ;  $u^*, w^*$  are the Darcy's velocity components along  $r$  and  $z$  directions, respectively,  $p^*$  is the pressure in the porous region,  $\beta = \eta/\mu\phi$  and  $\phi$  is the permeability of the porous medium. The parameter  $\beta$  represents the ratio of microstructure size of polar additives to the pore size of the porous medium. If  $\sqrt{1/(\mu/\eta)} \approx \sqrt{\phi}$ , *i.e.*,  $\beta \approx 1$  then the microstructure additives present in the Newtonian fluid block the pores of the porous region and this reduces the Darcy flow through the porous matrix. When microstructure is very small compared to the porous size, *i.e.*,  $\beta \ll 1$ , the polar additives percolate in to the porous matrix.

The pressure  $p^*$  in the porous region satisfies the Laplace equation  $(\partial^2 p^*/\partial x^2) + (\partial^2 p^*/\partial y^2) + (\partial^2 p^*/\partial z^2) = 0$ . Since  $p^*$  is independent of  $\theta$  in cylindrical coordinates  $r$ ,  $\theta$  and  $z$ , it reduces to:

$$\frac{\partial^2 p^*}{\partial r^2} + \frac{1}{r} \frac{\partial p^*}{\partial r} + \frac{\partial^2 p^*}{\partial z^2} = 0 \quad (16)$$

Using Equation (15) in (12), we get:

$$\frac{dp_i}{dr} = -\frac{6\mu r}{S_i(H_i, l)} \left[ \frac{dH}{dt} - \frac{\varnothing}{\mu(1-\beta)} \frac{\partial p^*}{\partial z} \Big|_{z=H} \right] \quad (17)$$

Taking the expected values of both sides of Equation (17), we get:

$$E\left(\frac{dp_i}{dr}\right) = -\frac{6\mu r}{S_i(H_i, l)} \left[ \frac{dE(H)}{dt} - \frac{\varnothing}{\mu(1-\beta)} E\left(\frac{\partial p^*}{\partial z}\right) \Big|_{z=H} \right] \quad (18)$$

where expectancy operator  $E(\cdot)$  is defined by:

$$E(\cdot) = \int_{-\infty}^{\infty} (\cdot) f(h_s) dh_s \quad (19)$$

and  $f$  is the probability density function of the stochastic film thickness  $h_s$ . In many real engineering problems, sliding surfaces show a roughness in height distribution which is Gaussian in nature. Therefore, a polynomial form which approximates the Gaussian is chosen in the analysis. Such a probability density function is given by

$$f(h_s) = \begin{cases} \frac{35}{32c^4} (c^2 - h_s^2)^3, & -c < h_s < c \\ 0, & \text{otherwise} \end{cases} \quad (20)$$

where “ $c$ ” is the half total range of random film thickness variable and function terminates at  $c = \pm 3\sigma$  with  $\sigma$  being the standard deviation.

The relevant boundary conditions for pressure in the film region are:

$$\frac{dE(p)}{dr} = 0 \quad \text{at} \quad r = 0 \quad (21)$$

$$E(p) = 0 \quad \text{at} \quad r = a \quad (22)$$

$$p^*(r, z) = 0 \quad \text{at} \quad r = a \quad (23)$$

$$\frac{\partial p^*(r, z)}{\partial z} = 0 \quad \text{at} \quad z = H + H^* \quad (24)$$

$$p^*(r, z) = \varphi(r) \quad \text{at} \quad z = H + H^* \quad (25)$$

and the interface condition is:

$$E[p(r)] = E[p^*(r, H)] \quad (26)$$

where “ $a$ ” is the outer radius of the disk and  $H^*$  is the thickness of the porous layer. Conditions (22) and (23) show that both the film region and the porous facing are open to the ambient pressure. There is no flow through the impervious boundary at the top of the porous medium in condition (24). Pressure continuity at the film plate interface requires condition (25).

Using Equation (19) for the distribution function, we have:

$$E(H) = h \tag{27}$$

The solution of the Equation (16) by the method of separation of variables of the form  $p^* = RZ$ , where  $R$  is a function of  $r$  only and  $Z$  is a function of  $z$  only is:

$$\frac{1}{R} \left( \frac{d^2 R}{dr^2} + \frac{1}{r} \frac{dR}{dr} \right) = -\frac{1}{Z} \frac{d^2 Z}{dz^2} \tag{28}$$

Putting each side equal to  $-\alpha^2$ , we have:

$$p^* = \{AJ_0(\alpha_n r) + By_0(\alpha_n r)\} \{C \sin h \alpha_n z + D \cos h \alpha_n z\} \tag{29}$$

(or) the equivalent form:

$$p^* = \{AJ_0(\alpha_n r) + By_0(\alpha_n r)\} \{Ce^{-\alpha_n z} + De^{+\alpha_n z}\}$$

$J_0(\alpha_n r)$  are Bessel functions of first kind and zeroth order and  $\alpha_n$  is the  $n$ th eigen value which satisfies:

$$J_0(\alpha_n a) = 0 \tag{30}$$

Applying boundary condition (23) at  $r = a$ , the most general solution is:

$$p^* = \sum_a AJ_0\left(\frac{\alpha_n r}{a}\right) \sin h\left(\frac{\alpha_n z}{a}\right) \tag{31}$$

Also:

$$\frac{\partial p^*}{\partial z} = \sum_a A \frac{\alpha_n}{a} J_0\left(\frac{\alpha_n r}{a}\right) \cos h\left(\frac{\alpha_n z}{a}\right)$$

Applying condition (24) and (25):

$$\varphi(r) = \sum_a AJ_0\left(\frac{\alpha_n r}{a}\right) \sin h\left(\frac{\alpha_n(H+H^*)}{a}\right), \quad (0 < r < a)$$

Hence:

$$A \sin h\left[\frac{\alpha_n(H+H^*)}{a}\right] = \frac{2}{J_1^2(\alpha_n)} \int_0^1 r \varphi(r) J_0\left(\frac{\alpha_n r}{a}\right) dr, \quad \left(0 < \frac{r}{a} < 1\right)$$

Substituting A in Equation (31),

$$p^*(r, z) = \sum_a \frac{2J_0\left(\frac{\alpha_n r}{a}\right) \sin h\left(\frac{\alpha_n z}{a}\right)}{\sin h\left(\frac{\alpha_n(H+H^*)}{a}\right)} \int_0^1 r \varphi(r) J_0\left(\frac{\alpha_n r}{a}\right) dr \tag{32}$$

Substituting Equations (27) and (31) in (18), results in:

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$$\left(\frac{dE(p_i)}{dr}\right) = -\frac{6\mu r}{S_i(H_i, l)} \left(\frac{dh}{dt} - \frac{\varnothing}{(1-\beta)} \left\{ \sum_a A'_n \frac{\alpha_n}{a} J_0\left(\frac{\alpha_n r}{a}\right) \cos h\left(\frac{\alpha_n z}{a}\right) \right\}\right) \quad (33)$$

where  $A'_n = E(Ae^{\alpha_n H})$  are Fourier coefficients to be determined. Integrating Equation (33) with respect to  $r$  and making use of the boundary conditions (21):

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$$E(p_i) = \frac{-3\mu}{S_i(H_i, l)} \frac{dh}{dt} (r^2 - a^2) + \frac{3\varnothing}{(1-\beta)S_i(H_i, l)} \left\{ \sum_a A'_n \frac{\alpha_n}{a} J_0\left(\frac{\alpha_n r}{a}\right) \cos h\left(\frac{\alpha_n z}{a}\right) \right\} \quad (34)$$

Substituting Equations (32) and (34) in the interface condition (26) and using orthogonality of the eigen function  $J_0(\alpha_n r)$ , we get:

$$A'_n = \frac{E \left\{ \sum_a \frac{2J_0\left(\frac{\alpha_n r}{a}\right) \sin h\left(\frac{\alpha_n z}{a}\right)}{\sin h\left(\frac{\alpha_n(H+H^*)}{a}\right)} \int_0^1 r \varphi(r) J_0\left(\frac{\alpha_n r}{a}\right) dr \right\}}{\left\{ \frac{3\varnothing}{(1-\beta)S_i(H_i, l)} \left\{ \sum_a A'_n \frac{\alpha_n}{a} J_0\left(\frac{\alpha_n r}{a}\right) \cos h\left(\frac{\alpha_n z}{a}\right) \right\} \right\}} \quad (35)$$

### 3. Solution of the problem

The non-dimensional load carrying capacity is obtained by integrating pressure over the bearing surfaces and it is given by:

$$w = 2\pi \int_0^{KR} r p_1 dr + 2\pi \int_{KR}^R r p_2 dr$$

which in non-dimensional form is:

$$w^* = \frac{2wh_2^3}{3\pi\mu R^3 \left(w^* + \frac{dH}{dt}\right)} = \frac{K^4}{S_1(h^*, l^*)} + \frac{1-K^4}{S_2(1, l^*)} \quad (36)$$

where  $h^* = h_1/h_2$  and  $l^* = 2l/h_2$

$$S_1(h^*, l^*) = h^{*3} - 3l^{*2}h^* + 3l^{*3} \tan h\left(\frac{h^*}{l^*}\right)$$

$$S_2(1, l^*) = 1 - 3l^{*2} + 3l^{*3} \tan h\left(\frac{1}{l^*}\right)$$

The squeezing time for reducing the film thickness from an initial value  $h_0$  to  $h_2$  to a final value  $h_f$  is given by:

$$t = \frac{-3\mu\pi R^4}{2w} \int_{h_0}^{h_f} \left( \frac{K^4}{S_1(h^*, l^*)} + \frac{1-K^4}{S_2(1, l^*)} \right) dh_2$$

which in non-dimensional form is:

$$l^* = \int_{h_f}^1 \left\{ K^4 f_1(h_s^*, h_2^*, l^*) + f_2(h_2^*, l^*) \right\} dh_2^* \quad (37)$$

where:

$$f_1(h_s^*, h_2^*, l^*) =$$

$$\left[ h_2^{*3} + h_s^{*3} + 3h_2^{*2}h_s^* + 3h_2^*h_s^{*3} - 3l^{*2}h_2^* \left( 1 + \frac{h_s^*}{h_2^*} \right) + 3l^{*3} \tan h \left( \frac{h_2^* \left( 1 + \frac{h_s^*}{h_2^*} \right)}{l^*} \right) \right]^{-1}$$

$$f_2(h_2^*, l^*) = \frac{1 - K^4}{h_2^{*3} - 3l^{*2}h_2^* + 3l^{*3} \tan h \left( \frac{h_2^*}{l^*} \right)}$$

$$h_f^* = \frac{h_f}{h_0}, \quad h_2^* = \frac{h_2}{h_0}, \quad h_s^* = \frac{h_s}{h_0}, \quad l^* = \frac{2l}{h_0}$$

#### 4. Results and discussions

The combined effects of surface roughness and couple stresses on the performance of porous squeeze film circular stepped plates are investigated. The squeeze film characteristics are analyzed with respect to various non-dimensional parameters, namely:

- couple stress parameter  $\tau (= l^* = (\eta/\mu)^{1/2}h_0)$ ;
- the permeability parameter  $\psi (= \phi h^*/h_0^3)$ ; and
- roughness parameter  $C = (c/h_0)$ .

The ratio  $(\eta/\mu)$  is of dimension length squared and this length may be regarded as the chain length of polar additives in the lubricant. This can also be characterized as rational length of the fluid.

Hence, the couple stress parameter  $l^*$  provides the mechanism of interaction of the lubricant with the bearing geometry. The numerical results computed are presented in graphical and tabular forms. This paper predicts the influence of couple stresses on the squeeze film characteristics of circular step bearings on the basis of Stokes couple stress fluid theory. The effect of couple stresses can be observed with the aid of non-dimensional couple stress parameter  $l^* = 2l/h_2$  where  $l = (\eta/\mu)^{1/2}$ . Hence the parameter  $l^*$  provides the mechanism of the interaction of the lubricant with the circular step bearing geometry.

Figure 2 shows the variation of non-dimensional squeeze film pressure  $p^*(r, z)$  as a function of dimensionless coordinates  $r$  and  $z$  for other fixed parameters. It is observed from this figure that, as coordinates of  $r$  and  $z$  increases, the pressure built-up in fluid film region also increases.

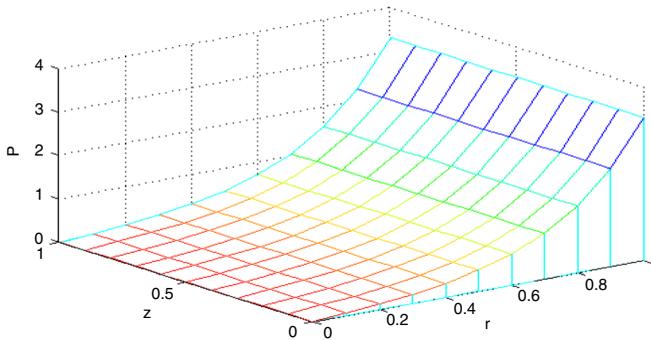
*Load carrying capacity*

The variation of non-dimensional load carrying capacity  $W^*$  as a function of  $H^*$  for different values of couple stress parameters  $l^* = 0.0, 0.2, 0.3, 0.4$  corresponds each with  $K = 0.6, 0.7, 0.8, 0.9, 1.0$  for circular stepped plates as shown in Figure 3. In the limiting case,  $C \rightarrow 0$  and  $l^* = 0.0$ , the present analysis reduce to classical case (i.e. Newtonian) studied by Murti (1974) as shown in Figure 3.

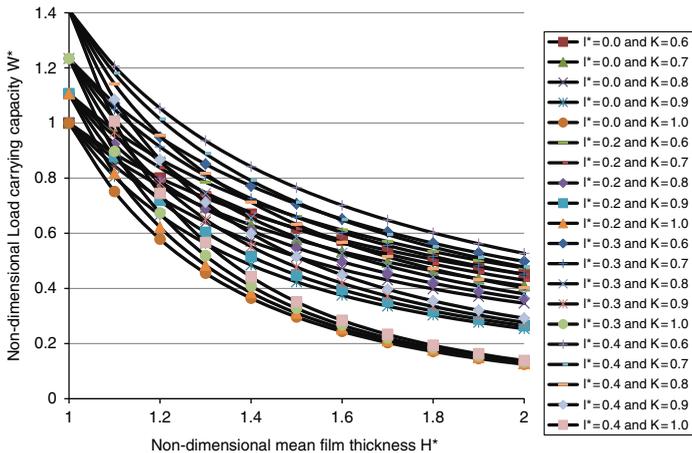
The variation of non-dimensional load carrying capacity  $W^*$  as a function of  $H^*$  for different values of couple stress parameters  $l^* = 0.0$  (studied by Murti, 1974),  $0.5, 0.8, 1.0$  corresponds each with  $K = 0.6, 0.7, 0.8, 0.9, 1.0$  for circular stepped plates as shown in Figure 4.

The variation of non-dimensional load carrying capacity  $W^*$  as a function of  $H^*$  for different values of couple stress parameters  $l^* = 0.0$  (studied by Murti, 1974),  $0.6, 0.7, 0.9$  corresponds each with  $K = 0.6, 0.7, 0.8, 0.9, 1.0$  for circular stepped plates as shown in Figure 5.

The couple stress parameter  $l^* = 0.0$  (studied by Murti, 1974) in the graph corresponds to the Newtonian case. Compared with the Newtonian lubricant case, the effect of couple stress in the present study increase the load carrying capacity and this increase in  $W^*$  is more accentuated for longer values of  $l^*$ .

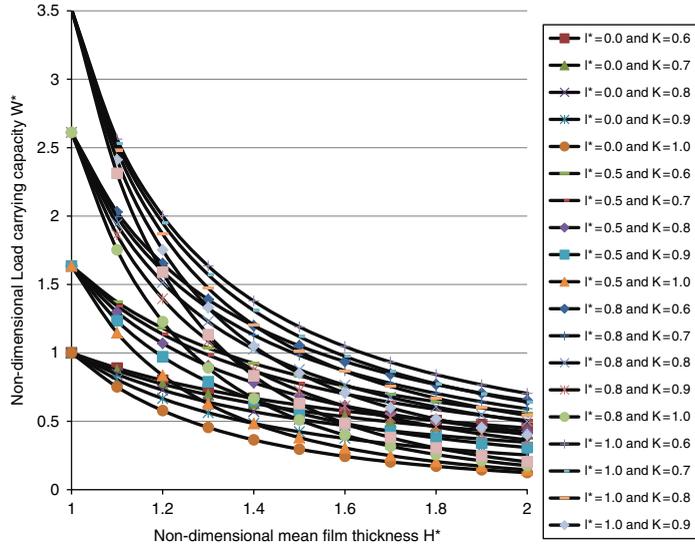


**Figure 2.** Variation of distribution of pressure  $p^*(r, z)$  for  $\alpha_n = 1.9, a = 0.5, H = 1.5, H^* = 0.5$  and using Bessel function of first kind

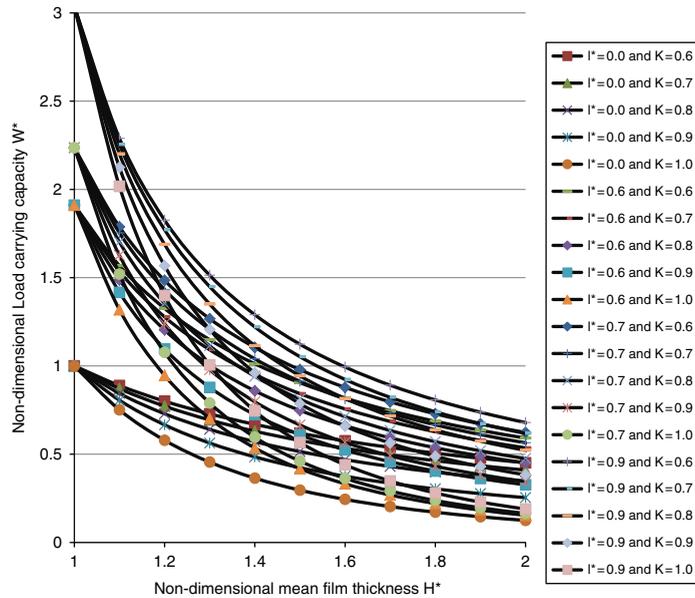


**Figure 3.** Couple stresses for larger values of  $l^* = 0.0$  (Newtonian),  $0.2, 0.3, 0.4$  increase the  $W^*$  for different values of  $K$

**Figure 4.** Couple stresses for larger values of  $\beta^* = 0.0$  (Newtonian), 0.5, 0.8, 1.0 increase the  $W^*$  for different values of  $K$



**Figure 5.** Couple stresses for larger values of  $\beta^* = 0.0$  (Newtonian), 0.5, 0.8, 1.0 increase the  $W^*$  for different values of  $K$

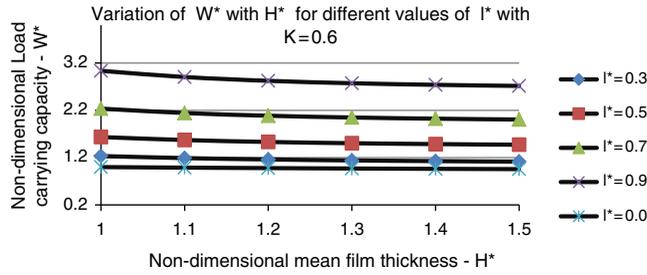


Figures 6-9 depicts the variation of non-dimensional load carrying capacity  $W^*$  as a function of  $H^*$  for both Newtonian lubricants ( $\beta^* = 0.0$ ) studied by Murti (1974) along with present study of different couple stress lubricants ( $\beta^* = 0.3, 0.5, 0.7$  and  $0.9$ ) with  $K = 0.6, 0.7, 0.8$  and  $1.0$ , respectively. It is observed that  $W^*$  increases for decreasing value of  $K$  and this increase in  $W^*$  is more pronounced for larger values of  $H^*$ . The relative percentage increase in  $W^* R_{w^*} = (W^*_{\text{Couplestress}} - W^*_{\text{Newtonian}}) / W^*_{\text{Newtonian}} \times 100$  for different values of  $K$  and  $H^*$  is given in Table I.

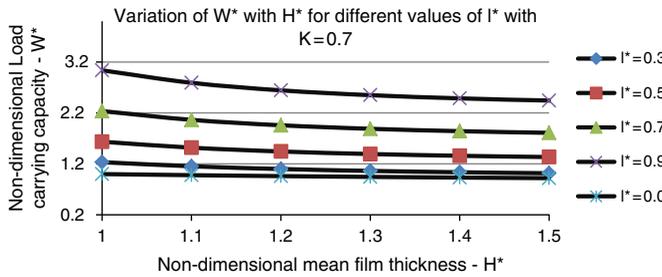
### Time height relationship

The most important characteristics of the squeeze film bearings is the time required for reducing the initial film thickness  $h_0$  to  $h_2$  to a final value  $h_f$ . The variations of the non-dimensional time of approach  $t^*$  as a function of  $h_f^*$  for different values of  $l^*$  for both Newtonian ( $l^* = 0.0$ ) studied by Murti (1974) and present study of couple stress lubricants ( $l^* = 0.3, 0.5, 0.7$  and  $0.9$ ) along with  $K = 0.5, 0.6, 0.7, 0.8$  and  $0.9$  are plotted in

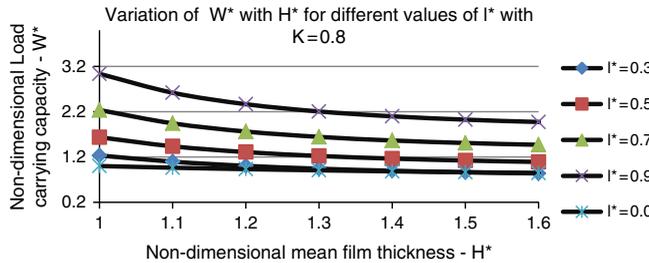
### Squeeze film behavior



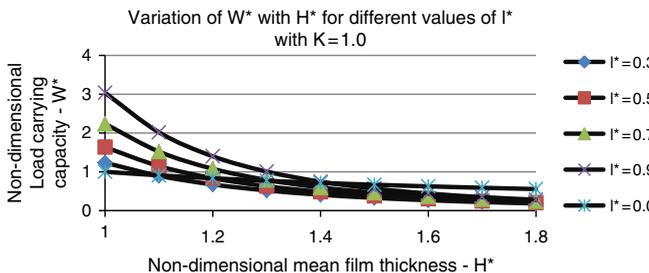
**Figure 6.** Depicts  $W^*$  as a function of  $H^*$  for both Newtonian lubricants  $l^* = 0.0$  along with different couple stress lubricants with  $K = 0.6$



**Figure 7.** Depicts  $W^*$  as a function of  $H^*$  for both Newtonian lubricants  $l^* = 0.0$  along with different couple stress lubricants with  $K = 0.7$



**Figure 8.** Depicts  $W^*$  as a function of  $H^*$  for both Newtonian lubricants  $l^* = 0.0$  along with different couple stress lubricants with  $K = 0.8$



**Figure 9.** Depicts  $W^*$  as a function of  $H^*$  for both Newtonian lubricants  $l^* = 0.0$  along with different couple stress lubricants with  $K = 1.0$

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33,2

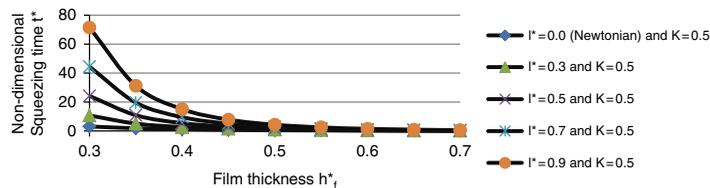
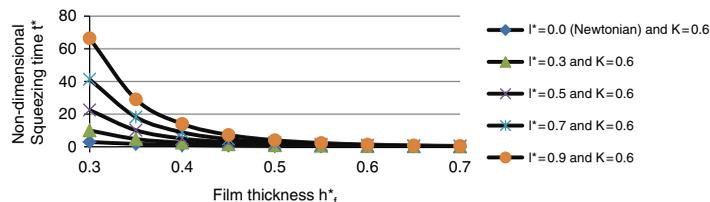
340

**Table I.**  
Variation of  $R_{W^*}$  for  
different values of  
 $t^*$  and  $K$ 

	$K$	$H^* = 1.1$	$H^* = 1.3$	$R_w^*$ $H^* = 1.5$	$H^* = 1.7$	$H^* = 1.9$
$t^* = 0.3$	0.6	92.0220458	45.11117843	16.63811068	16.42215466	16.52121564
	0.7	17.83052657	12.41134794	10.43063379	9.880627426	9.96525679
	0.8	13.79575267	2.373906994	2.19576907	6.874868183	23.06849387
	0.9	7.686872615	13.67466143	7.410321771	5.341671076	1.018730434
	0.6	99.55120781	55.11130473	53.92635629	53.77229343	54.02263814
$t^* = 0.5$	0.7	55.19093163	47.51270619	45.02615039	44.54032759	44.87664985
	0.8	49.07657158	35.02517457	29.89809646	28.41513109	28.53861616
	0.9	39.82698845	14.81038902	3.993039421	14.53254961	12.49463468
$t^* = 0.7$	0.6	116.8788099	116.3297486	109.6494273	109.589404	110.0672187
	0.7	111.0185935	99.95912649	96.71825751	96.33121461	102.6879344
	0.8	101.7917269	81.52598197	74.73860482	73.17353495	73.78625102
$t^* = 0.9$	0.9	87.8336564	51.68645283	37.10097082	31.59167578	30.28838447
	0.6	142.2804929	185.8657288	183.8850777	183.9507828	184.732121
	0.7	185.3929074	169.8264134	165.5807584	165.3261191	166.5467111
	0.8	172.0166144	143.4674138	134.4681376	132.7949804	134.0610103
	0.9	151.7814415	100.79753	81.19134353	74.38216695	73.31055746

Figures 10-14, respectively. It is found that  $t^*$  increases for decreasing values of  $K$ . The variations of  $t^*$  with  $h_f^*$  for different values of step height  $h_s^*$  with different values of  $K$  shows that  $t^*$  decreases for increasing values of non-dimensional step height  $h_s^*$ .

It is observed that the presence of couple stresses provides an increase in the response time compared to the Newtonian lubricant case. The relative in  $t^*$ ,  $R_{t^*} = (t_{\text{Couple stress}}^* - t_{\text{Newtonian}}^* / t_{\text{Newtonian}}^*) \times 100$  for different values of  $K$  and  $H^*$  is given in Table II.

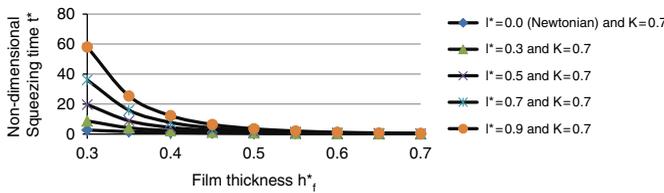
**Figure 10.**  
Depicts  $t^*$  as a  
function of  $h_f^*$  for  
both Newtonian  
 $t^* = 0.0$  along with  
different couple  
stress lubricants  
with  $K = 0.5$ **Note:** Variation of non-dimensional time of approach  $t^*$  with  $h_f^*$  for  
different values of couple stress lubricant  $t^*$  with  $K=0.5$  and  $h_s^*=0.15$ **Figure 11.**  
Depicts  $t^*$  as a  
function of  $h_f^*$  for  
both Newtonian  
 $t^* = 0.0$  along with  
different couple  
stress lubricants  
with  $K = 0.6$ **Note:** Variation of non-dimensional time of approach  $t^*$  with  $h_f^*$  for  
different values of couple stress lubricant  $t^*$  with  $K=0.6$  and  $h_s^*=0.15$

5. Conclusion

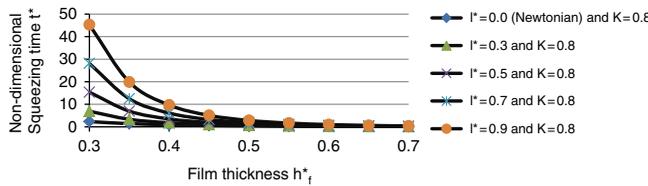
The squeeze film lubrication between porous transversely circular stepped plates with couple stress fluid is studied on the basis of Stokes microcontinuum theory for couple stress fluids. On the basis of the theoretical results presented, the following conclusions are drawn:

- The effect of couple stresses enhances the load carrying capacity significantly. This increase in load is three-fourth (nearly 75 percent) higher in comparison with the corresponding Newtonian case.
- The relative increase in load carrying capacity  $R_{w*}$  is found to be a function of  $K$  and  $l^*$ .
- The relative squeeze film time  $R_{t*}$  is found to be a function of  $l^*$  and  $K$  and  $R_{t*}$  increases for increasing values of  $l^*$  and decreases for increasing values of  $K$ .

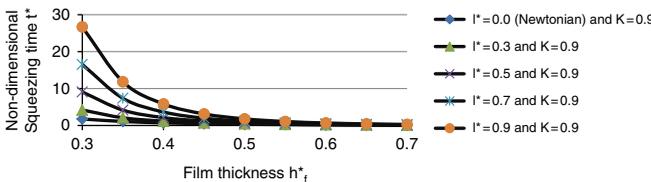
Therefore, the theoretical results presented here suggest that the squeeze film characteristics between porous transversely circular stepped plates can be improved by the use of lubricants with microstructure additives.



**Note:** Variation of non-dimensional time of approach  $t^*$  with  $h_f^*$  for different values of couple stress lubricant  $l^*$  with  $K=0.7$  and  $h_s^*=0.15$



**Note:** Variation of non-dimensional time of approach  $t^*$  with  $h_f^*$  for different values of couple stress lubricant  $l^*$  with  $K=0.8$  and  $h_s^*=0.15$



**Note:** Variation of non-dimensional time of approach  $t^*$  with  $h_f^*$  for different values of couple stress lubricant  $l^*$  with  $K=0.9$  and  $h_s^*=0.15$

**Figure 12.** Depicts  $t^*$  as a function of  $h_f^*$  for both Newtonian  $l^* = 0.0$  along with different couple stress lubricants with  $K = 0.7$

**Figure 13.** Depicts  $t^*$  as a function of  $h_f^*$  for both Newtonian  $l^* = 0.0$  along with different couple stress lubricants with  $K = 0.8$

**Figure 14.** Depicts  $t^*$  as a function of  $h_f^*$  for both Newtonian  $l^* = 0.0$  along with different couple stress lubricants with  $K = 0.9$

**Table II.**  
Variation of  $R_{t^*}$  for  
different values of  
 $l^*$  and K

		K	$h_f^* = 0.3$	$h_f^* = 0.5$	$R_t^*$	$h_f^* = 0.7$	$h_f^* = 0.9$
$l^* = 0.3$	0.6	238.096695	83.75710094	42.84505176	26.31075202		
	0.7	224.7211938	76.58243086	38.97743636	24.10964023		
	0.8	198.5241211	63.41589838	32.14875856	20.31890077		
	0.9	139.7069522	37.51322378	19.65229448	13.68619913		
$l^* = 0.5$	0.6	663.3007932	232.8757154	118.0517292	71.97963297		
	0.7	630.1709388	214.9582039	107.9402752	66.13882677		
	0.8	565.2828176	182.0767735	90.08778013	56.08734099		
	0.9	419.598165	117.3899652	57.41350997	38.49872436		
$l^* = 0.7$	0.6	1,301.92705	458.083872	232.0894966	141.2211301		
	0.7	1,240.04292	424.7178058	213.9172462	130.8084691		
	0.8	1,118.837416	366.0947913	181.8355439	172.7145923		
	0.9	846.7093786	248.9564542	123.1177159	81.50658788		
$l^* = 0.9$	0.6	2,154.962707	762.1564803	388.7905291	237.4045104		
	0.7	2,056.321234	714.0812651	364.3422785	224.545205		
	0.8	1,863.122853	625.8553269	321.1817381	202.4127782		
	0.9	1,429.357396	452.2902743	242.184136	163.6706345		

The variations of non-dimensional time  $t^*$  for various values of roughness parameters  $K$  and  $l^*$  are plotted in Figures 10-14. The squeeze film time is lengthened for the couple stress fluids compared to Newtonian fluid. The squeezing time for couple stress fluid in the presence of surface roughness is found to be longer for all values of couple stress parameter  $l^*$  roughness compared to Newtonian fluid. The couple stress fluid provides more resistance to the moving plate and hence takes longer time to reduce the required height compared to Newtonian case. This delayed squeezing time reduces the coefficient of friction and also results in negligible rate of wear of the bearing surfaces.

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