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Unsteady Hydromagnetic Flow past a Moving Vertical Plate with Convective Surface Boundary Condition

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ABSTRACT

Investigation of unsteady MHD natural convection flow through a fluid-saturated porous medium of a viscous, incompressible, electrically-conducting and optically-thin radiating fluid past an impulsively moving semi-infinite vertical plate with convective surface boundary condition is carried out. With the aim to replicate practical situations, the heat transfer and thermal expansion coefficients are chosen to be constant and a new set of non-dimensional quantities and parameters are introduced to represent the governing equations along with initial and boundary conditions in dimensionless form. Solution of the initial boundary-value problem (IBVP) is obtained by an efficient implicit finite-difference scheme of the Crank-Nicolson type which is one of the most popular schemes to solve IBVPs. The numerical values of fluid velocity and fluid temperature are depicted graphically whereas those of the shear stress at the wall, wall temperature and the wall heat transfer are presented in tabular form for various values of the pertinent flow parameters. A comparison with previously published papers is made for validation of the numerical code and the results are found to be in good agreement.

Keywords: Unsteady MHD natural convection flow; Convective surface boundary condition; Porous medium; Optically thin fluid; Non-similar solution.

NOMENCLATURE

c_p	specific heat at constant pressure	T	fluid temperature
g	acceleration due to gravity,	T_f	hot fluid temperature
G_{rc}	convective Grashof number	T_∞	free stream temperature
h_f	heat transfer coefficient	u', v'	velocity components
k	thermal conductivity	u, v	dimensionless velocity components
K_{pc}	permeability parameter	x', y'	cartesian coordinates
K_p'	permeability of the porous medium	x, y	dimensionless coordinates
M_c^2	magnetic parameter	′	Thermal expansion coefficient
P_r	Prandtl number	³	kinematic coefficient of viscosity
q_r'	radiative flux vector	°	fluid density
R_c	radiation parameter	±	electrical conductivity

1. INTRODUCTION

Generally, in all the problems of fluid dynamics, unsteady flow is a natural phenomenon and steady state models are just simplifications of the real situation. Therefore, the investigation of unsteady Magnetohydrodynamic (MHD) flows is significant from practical point of view because fluid transients may be expected at the start-up time of many industrial processes and devices viz. electromagnetic stirring of molten metal, forging, casting and levitation processes, MHD energy generators, MHD pumps, MHD accelerators, MHD flow-meters, controlled thermonuclear reactors, etc. Keeping in mind the importance of such studies, considerable amount of investigations are carried out by a number of researchers on unsteady hydromagnetic natural convection flow past a flat plate through fluid saturated porous medium considering various aspects of the problem. A reference may be made to the research studies of Raptis (1986), Jha (1991), Chamkha (1997), Chamkha and Ahmed (2011), Eldabe *et al.* (2012), Samiulhaq *et al.* (2013), Das *et al.* (2014), Ghosh *et al*. (2015) and Seth *et al*. (2015a, b).

Nowadays hydromagnetic natural convection flows considering radiative heat transfer is a topic of much significance because thermal radiation plays an important role in the design of nuclear power plants, gas turbines, flight propulsion systems, automobile engines, high temperature heat exchangers and combustion chambers, which operate at elevated temperatures, in order to gain thermal efficiency (Howell *et al*., 2010). Besides, in several industrial processes viz. formation and tempering of glass, steel rolling, extraction of metals, semiconductor wafer processing and growth of crystals, the quality of the final product depends to a great extent on the heat controlling factors, and the knowledge of radiative heat transfer in the system may likely lead to a desired product with sought qualities. England and Emery (1969) were one of the premier investigators to study the effects of thermal radiation of an optically thin gray gas on the laminar free convection flow past a stationary vertical plate. Bestman and Adjepong (1988) investigated unsteady hydromagnetic free convection flow with radiative heat transfer of an optically thin fluid in a rotating system. Chamkha *et al*. (2001) considered laminar free convection flow of air past a semi-infinite vertical plate in the presence of chemical species concentration and thermal radiation using the optically thin limit for a gray-gas near equilibrium. Raptis *et al*. (2003) studied the effects of thermal radiation on hydromagnetic free convection flow of an optically thin fluid past an infinite vertical plate. Raptis (2011) investigated oscillatory natural convection heat and mass transfer flow past a porous plate in the presence of radiation for an optically thin fluid. Seth *et al*. (2014) investigated unsteady MHD natural convection flow with heat and mass transfer of an optically thin fluid past an impulsively moving vertical plate in the presence of radiation and chemical reaction.

It is well known that the heat transfer characteristics of natural convection boundary layer flows are strongly dependent on the thermal boundary conditions. Most common heating processes specifying the wall-to-ambient temperature distributions are, usually, prescribed surface temperature distributions or prescribed surface heat flux distributions. Therefore, considerable amount of research works pertaining to these flows are available in the literature (Bejan, 1993; Gebhart *et al.* 1998) considering wide range of wall conditions and various fluid properties. However, there exists a class of thermal boundary conditions in which the surface heat flux depends on the local surface temperature. Usually, this situation arises in conjugate heat transfer problems when there is an interaction between the convective fluid and conduction through the bounding wall (Merkin and Pop, 1996), and, when there is Newtonian heating of the convecting fluid from the surface i.e. conjugate convective flow (Merkin, 1994). Another configuration, often arising in practical systems, is convective heat transfer for the Blasius flow with convective surface boundary condition, which is primarily investigated by Aziz (2009). It may be noted that conjugate/convective thermal boundary conditions are known to appear in numerous instances of the problems in science and engineering viz. optimization of turbine blade cooling systems (Nowak and Wróblewski, 2011), design of efficient heat exchangers (Zhang, 2013), combustion in gas turbines (Lefebvre, 1998), convective flows set up where the bounding surfaces absorb heat by solar radiation (seasonal thermal energy storage systems), etc. Therefore, a promising sense of applicability and the classic paper by Aziz (2009) prompted several researchers to investigate boundary layer flows with convective surface boundary condition considering various aspects of the problem. Mention may be made of the research studies of Ishak (2010), Khan and Gorla (2010), Makinde and Aziz (2010, 2011), Rahman (2011), Magyari (2011), Butt *et al*. (2012), Ferdows *et al*. (2013), Lok *et al*. (2013). However, in all the research works pertaining to convective surface boundary condition as mentioned above, the convective heat transfer coefficient associated with the hot surface is assumed to be a function of x (where x measures distance from the leading edge) so that the problems accept similarity solution. But the assumption of heat transfer coefficient to be a function on x implies that heat transfer coefficient varies along the plate surface which is unrealistic. In this regard, Merkin and Pop (2011) presented a non-similar solution of the Blasius flow with convective heat transfer by considering heat transfer coefficient to be a constant which is well-suited for real fluids. Merkin *et al*. (2013) investigated mixed convection boundary layer flow past a vertical surface in a porous medium with a constant convective boundary condition. Pantokratoras (2014) extended of the work of Merkin *et al*. (2013) to a Darcy–Brinkman porous medium.

Although there have been a lot of investigations pertaining to convective heat transfer problems, yet, so far no researcher has reported the study of unsteady hydromagnetic natural convection flow of a radiating fluid past an impulsively moving vertical plate with convective heating assuming constant heat transfer coefficient and constant thermal expansion coefficient: a problem which is of utmost importance from practical point of view. It may be noted that the governing equations for natural convection fluid flow problems are based on Boussinesq approximations. It is well known that the Boussinesq approximation is based on the assumption that fluid thermal expansion coefficient is constant and is equal to that of ambient fluid (Bejan, 1993; Schlichting and Gersten, 2000). Therefore, in view of the above, we propose to investigate unsteady hydromagnetic natural convection flow of a viscous, incompressible, electrically conducting and optically thin radiating fluid past an impulsively moving semi-infinite vertical plate with convective heating embedded in a fluid saturated porous medium. Non-similar solution to the initial boundary value problem (IBVP) is obtained using an implicit finite difference scheme of Crank-Nicolson type (Bapuji *et al*., 2008) which is a very popular scheme due to its stability and consistency.

2. FORMULATION OF THE PROBLEM

Consider unsteady hydromagnetic natural convection flow of an electrically conducting, viscous, incompressible and optically thin radiating fluid past a semi-infinite vertical plate embedded in a fluid saturated porous medium. Coordinate system is chosen in such a way that x'- axis is considered along the plate in upward direction and y'- axis normal to plane of the plate in the fluid. A uniform transverse magnetic field B_0 is applied in a direction which is parallel to y'- axis. Initially i.e. at time $t' \leq 0$, both the fluid and plate are at rest and are maintained at a uniform temperature T_∞'. At time $t' > 0$, plate starts moving in x'- direction with uniform velocity U_0 in its own plane and the right hand surface of the plate is heated by convection from a hot fluid with uniform temperature T_f' ($T_f' > T_\infty'$) which provides a constant heat transfer coefficient h_f. Physical model of the problem is presented in Fig. 1. No applied or polarized voltages exist so the effect of polarization of fluid is negligible. This corresponds to the case where no energy is added or extracted from the fluid by electrical means (Cramer and Pai, 1973). It is assumed that the induced magnetic field generated by fluid motion is negligible in comparison to the applied one. This assumption is justified because magnetic Reynolds number is very small for liquid metals and partially ionized fluids which are commonly used in industrial applications (Cramer and Pai, 1973).

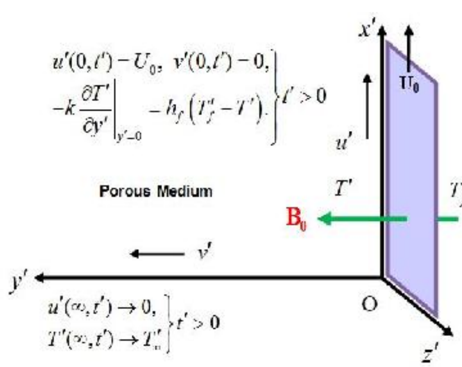

Fig. 1. Physical model of the problem.

Keeping in view the assumptions made above, the mathematical model for unsteady hydromagnetic natural convection flow of a viscous, incompressible, electrically conducting and optically thin radiating fluid past a semi-infinite vertical plate with convective heating in a fluid saturated porous medium, under Boussinesq approximation, is given by

$$\frac{\partial u'}{\partial x'} + \frac{\partial v'}{\partial y'} = 0, \tag{1}$$

$$\begin{aligned} &\frac{\partial u'}{\partial t'} + u'\frac{\partial u'}{\partial x'} + v'\frac{\partial u'}{\partial y'} = {}^{3}\,\frac{\partial^2 u'}{\partial y'^2} \\ &\quad - \frac{\pm B_0^2}{\circ}u' - \frac{{}^{3}\,u'}{K_p'} + g\quad'(T' - T_\infty') \end{aligned} \tag{2}$$

$$\frac{\partial T'}{\partial t'} + u'\frac{\partial T'}{\partial x'} + v'\frac{\partial T'}{\partial y'} = \frac{k}{\circ c_p}\frac{\partial^2 T'}{\partial y'^2} - \frac{1}{\circ c_p}\frac{\partial q_r'}{\partial y'} \tag{3}$$

subject to following initial and boundary conditions

$$t' \leq 0 \quad : u' = v' = 0,\ T' = T_\infty' \quad \text{for} \quad y' \geq 0, \tag{4a}$$

$$\begin{aligned} &t' > 0 : u' = U_0, \\ &v' = 0, -k\frac{\partial T'}{\partial y'} = h_f\left(T_f' - T'\right) \text{ at } y' = 0 \end{aligned} \tag{4b}$$

$$u' \to 0,\ T' \to T_\infty' \quad \text{as} \quad y' \to \infty \tag{4c}$$

where $u',\ v',\ {}^{3},\ \circ,\ \pm,\ g,\ \ ',\ T',\ c_p,\ k,\ q_r'$ and K_p' are, respectively, fluid velocity in x'-direction, fluid velocity in y'-direction, kinematic coefficient of viscosity, fluid density, electrical conductivity, acceleration due to gravity, thermal expansion coefficient, fluid temperature, specific heat at constant pressure, thermal conductivity of fluid, radiative flux vector and permeability of porous medium.

In the case of an optically thin fluid the local radiant absorption (Raptis, 2011) is expressed as:

$$\frac{\partial q_r'}{\partial y'} = -4a^*\pm^*\left(T_\infty'^4 - T'^4\right) \tag{5}$$

where a^* is absorption coefficient and $\pm^*$ is Stefan-Boltzmann constant.

It is assumed that the temperature difference within the fluid flow is sufficiently small such that fluid temperature T'^4 may be expressed as a linear function of the temperature. This is accomplished by expanding T'^4 in a Taylor series about free stream temperature T_∞'. Neglecting second and higher order terms in series, T'^4 is expressed as

$$T'^4 \cong 4T_\infty'^3 T' - 3T_\infty'^4. \tag{6}$$

Making use of Eqs. (5) and (6) in Eq. (3), we obtain

$$\frac{\partial T'}{\partial t'}+u'\frac{\partial T'}{\partial x'}+v'\frac{\partial T'}{\partial y'}=\frac{k}{\text{°}c_p}-\frac{16a^{*}\text{±}^{*}T_{\infty}'^{3}}{\text{°}c_p}\left(T'-T_{\infty}'\right) \quad (7)$$

In order to represent equations (1), (2) and (7) in dimensionless form, we introduce the following dimensionless quantities and parameters

$$x=\frac{\text{³}h_f^2}{U_0k^2}x',\ \ y=\frac{h_f}{k}y',\ t=\frac{\text{³}h_f^2}{k^2}t',\ u=\frac{u'}{U_0},$$

$$v=\frac{k}{\text{³}h_f}v',\ T=\frac{T'-T_{\infty}'}{T_f'-T_{\infty}'},\ \text{¦}=\frac{y}{\sqrt{x}},\ P_r=\frac{\text{°}\,\text{³}c_p}{k},$$

$$G_{rc}=\frac{g\ \ '\left(T_f'-T_{\infty}'\right)k^2}{U_0h_f^2\,\text{³}},\ M_c^2=\frac{\text{±}B_0^2k^2}{\text{°}\,\text{³}h_f^2},$$

$$K_{pc}=\frac{K_p'h_f^2}{k^2},\ R_c=\frac{16a^{*}\text{±}^{*}T_{\infty}'^{3}k^2}{\text{°}c_p\,\text{³}h_f^2} \quad (8)$$

where P_r and G_{rc} are, respectively, Prandtl number and convective Grashof number (Pantokratoras, 2014). M_c^2, K_{pc} and R_c are, respectively, magnetic parameter, permeability parameter, radiation parameter for convective surface boundary condition which are introduced in this paper for the first time.

Equations (1), (2) and (7) with the help of non-dimensional quantities and parameters defined in (8) assume the following form

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \quad (9)$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\partial^2u}{\partial y^2}-M_c^2u-\frac{u}{K_{pc}}+G_{rc}T, \quad (10)$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{1}{P_r}\frac{\partial^2T}{\partial y^2}-R_cT \quad (11)$$

Initial and boundary conditions (4a) to (4c), in non-dimensional form, are given by

$$t\le 0:\quad u=v=T=0\quad \text{for}\quad y\ge 0, \quad (12a)$$

$$t>0:\quad u=1,\ v=0,\ \frac{\partial T}{\partial y}=-\left(1-T\right)\ \text{at}\ y=0 \quad (12b)$$

$$u\to 0,\ T\to 0\quad \text{as}\quad y\to\infty \quad (12c)$$

3. NUMERICAL SOLUTION

The set of non-linear coupled equations (9) to (11) subject to the initial and boundary conditions (12a) to (12c) are solved numerically using an implicit finite difference technique of Crank-Nicolson type as described by Soundalgekar and Ganesan (1981), Muthukumarswamy and Ganesan (1998) and Bapuji *et al.* (2008). It is well known that the boundary layer thickness changes along x (Merkin and Pop, 2011). Therefore, the calculation domain must be wider than the momentum and thermal boundary layer thicknesses to ensure greater accuracy. For the present problem we have considered x_{max}=12, y_{max}=8 (corresponding to $y\to\infty$). The finite difference equations for each time step constitute a tridiagonal system of equations which are solved by Thomas algorithm (Carnahan *et al.*, 1969). Numerical solutions for the fluid temperature and fluid velocity are obtained corresponding to desired degree of accuracy for the required time by performing computations for a number of time steps. This is a well-established method for finding solution of any problem which is parabolic in nature and has been widely used to find accurate results by researchers and scientists in this field. Also, the stability and convergence of the scheme is analyzed in detail by Muthukumarswamy and Ganesan (1998) and Bapuji *et al.* (2008). Therefore, we skip these portions to avoid repetition.

Non-dimensional wall shear stress and non-dimensional wall heat transfer are expressed as:

$$u_{\text{¦}}\left(0,t\right)=\left[\frac{\partial u}{\partial \text{¦}}\right]_{\text{¦}=0}=\sqrt{x}\left[\frac{\partial u}{\partial y}\right]_{y=0} \quad (13)$$

$$T_{\text{¦}}\left(0,t\right)=\left[\frac{\partial T}{\partial \text{¦}}\right]_{\text{¦}=0}=\sqrt{x}\left[\frac{\partial T}{\partial y}\right]_{y=0} \quad (14)$$

3.1 Validation of Numerical Solution

In order to verify the correctness of our numerical results we have first applied our numerical scheme to the problem considered by Merkin and Pop (2011) and have compared our results with those of Merkin and Pop (2011) which are provided in tabular form by Pantokratoras (2014). For the sake of comparison we have considered steady flow past a stationary plate inside moving free stream in the absence of magnetic field, thermal buoyancy force, porous medium and thermal radiation. In our numerical scheme steady state solution is reached for large value of time t (i.e. t=7) when it was found that the absolute difference between the numerical values of fluid temperature and fluid velocity obtained for two consecutive time steps is less than 10^{-6}. A comparison is made between the values of wall temperature T(0) computed in this paper with the values of Merkin and Pop (2011) and Pantokratoras (2014). These values of wall temperature are presented in tabular form in Table 1.

Table 1 Validation of numerical code by comparing values of wall temperature taking $P_r=1$

x	T(0) Merkin and Pop (2011)	T(0) Pantokratoras (2014)	T(0) Present code
0.001	0.062	0.062	0.062
0.124	0.457	0.446	0.455
1.867	0.784	0.780	0.783
8.335	0.895	0.892	0.892
110.335	1.0	0.970	0.985

It is perceived from Table 1 that the numerical values of wall temperature T(0) obtained through our numerical scheme are in a good agreement with the values of Merkin and Pop (2011) and Pantokratoras (2014). This favorable comparison lends confidence and justifies the correctness of the results to be presented subsequently**.**

4. RESULTS AND DISCUSSION

In order to analyze the effects of the magnetic field, thermal buoyancy force, permeability of the medium, radiation, thermal diffusivity and time on the flow field, the numerical solution of the fluid velocity $u(\eta,t)$ is depicted graphically versus the boundary layer coordinate η in Figs. 2 to 7 for various values of the magnetic parameter M_c^2, convective Grashof number G_{rc}, permeability parameter K_{pc}, radiation parameter R_c, Prandtl number P_r and time *t*. It is revealed from Figs. 2 to 7 that, the fluid velocity is maximum at the surface of the plate and it decreases uniformly upon increasing the boundary layer coordinate η to approach the free stream value.

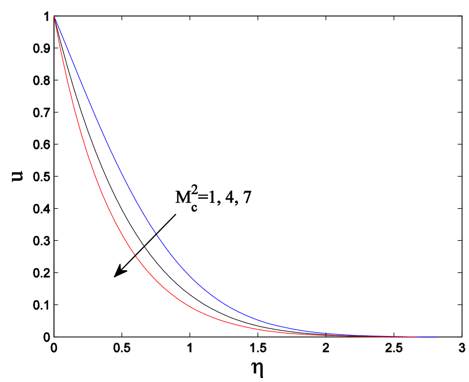

Fig. 2. Velocity profiles when G_{rc}=10, K_{pc}=0.4, R_c=2, P_r=0.71, *x*=1 and *t*=0.3.

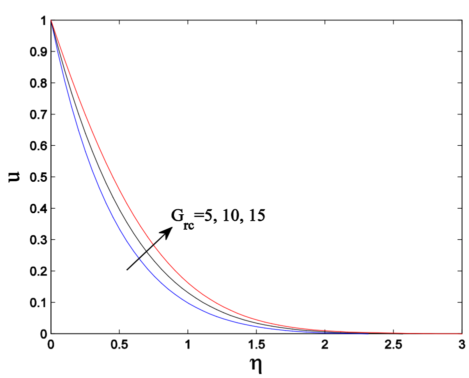

Fig. 3. Velocity profiles when $M_c^2=4$, K_{pc}=0.4, R_c=2, P_r=0.71, *x*=1 and *t*=0.3.

It is revealed from Fig. 2 that *u* decreases by increasing M_c^2 throughout the boundary layer region. M_c^2 signifies the relative strength of the magnetic force to the viscous force, M_c^2 increases upon increasing the strength of the magnetic force. This implies that the magnetic field tends to retard the fluid flow throughout the boundary layer region. This phenomenon is attributed to the Lorentz force, induced due to the movement of an electrically-conducting fluid in the presence of a magnetic field, which has a tendency to resist the fluid motion.

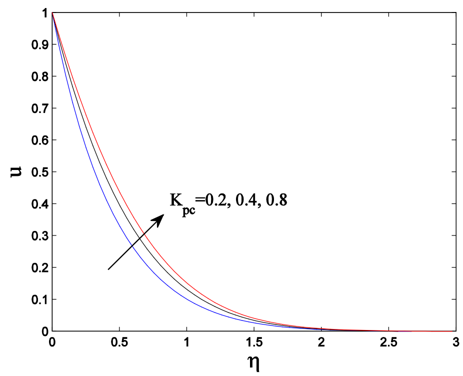

Fig. 4. Velocity profiles when $M_c^2=4$, G_{rc}=10, R_c=2, P_r=0.71, *x*=1 and *t*=0.3.

It is evident from Fig. 3 that *u* increases upon increasing G_{rc} throughout the boundary layer region. G_{rc} represents the relative strength of the thermal buoyancy force to the viscous force, G_{rc} increases upon increasing the strength of the thermal buoyancy force. This implies that the thermal buoyancy force tends to accelerate the fluid flow throughout the boundary layer region. It is perceived from Fig. 4 that *u* increases as K_{pc} increases. It may be noted that an increase in K_{pc} implies that there is a decrease in the resistance of the porous medium. Due to this reason, the permeability of the medium tends to accelerate the fluid flow throughout the boundary layer region. It is observed from Fig. 5 that *u* decreases upon increasing R_c throughout the boundary layer region. This implies that the thermal radiation tends to retard the fluid flow for an optically-thin fluid. It is noticed from Fig. 6 that *u* decreases upon increasing P_r throughout the boundary layer region. P_r is a measure of the relative strength of the viscosity to thermal diffusivity of the fluid and therefore, P_r decreases upon increasing the thermal diffusivity. It is widely known that natural convection flow is induced in a fluid with low Prandtl number. If the Prandtl number decreases, then the strength of the thermal buoyancy force increases due to the thermal diffusion which tends to accelerate the fluid flow throughout the boundary layer region. Fig. 7 reveals that *u* increases upon increasing *t* throughout the boundary layer region. This implies that the fluid velocity gets accelerated with the progress of time.

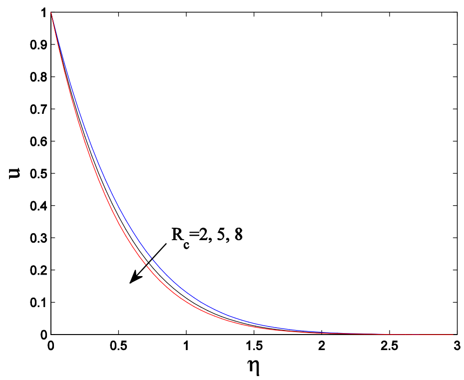

Fig. 5. Velocity profiles when $M_c^2 = 4$, G_{rc}=10, K_{pc}=0.4, P_r=0.71, x=1 and t=0.3.

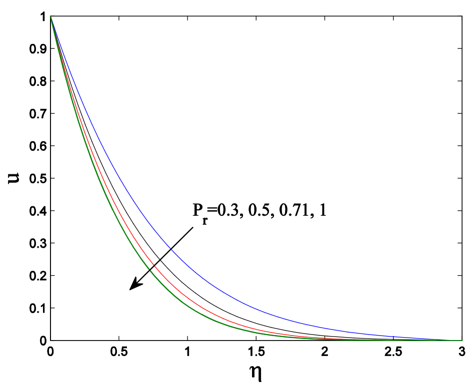

Fig. 6. Velocity profiles when $M_c^2 = 4$, G_{rc}=10, K_{pc}=0.4, R_c=2, x=1 and t=0.3.

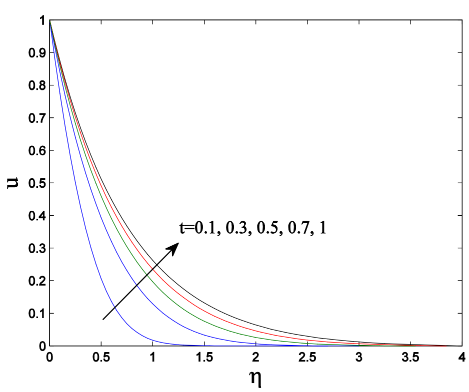

Fig. 7. Velocity profiles when $M_c^2 = 4$, G_{rc}=10, K_{pc}=0.4, R_c=2, x=1 and P_r=0.71

In order to investigate the effects of radiation, thermal diffusion and time on the temperature field, numerical solutions for the fluid temperature $T(\eta, t)$ are depicted graphically versus the boundary layer coordinate η in Figs. 8 to 10 for various values of R_c, P_r and t taking $M_c^2 = 4$, $G_{rc} = 10$ and $K_{pc} = 0.4$. It is revealed from Figs. 8 to 10 that the fluid temperature is maximum at the surface of the plate and it decreases uniformly upon increasing the boundary layer coordinate η to approach the free stream value. Figs. 8 to 10 show that the fluid temperature T decreases upon increasing either R_c or P_r whereas it increases upon increasing t.

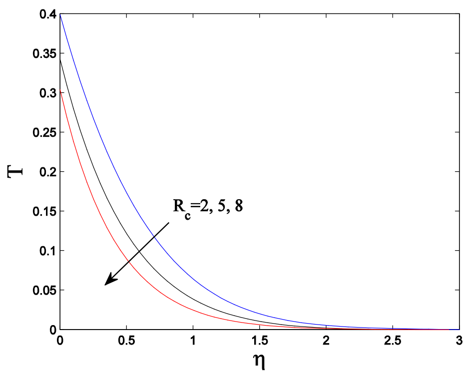

Fig. 8. Temperature profiles when P_r=0.71, x=1 and t=0.3.

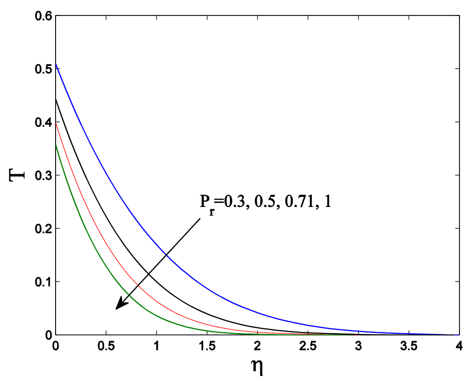

Fig. 9. Temperature profiles when R_c=2, x=1 and t=0.3.

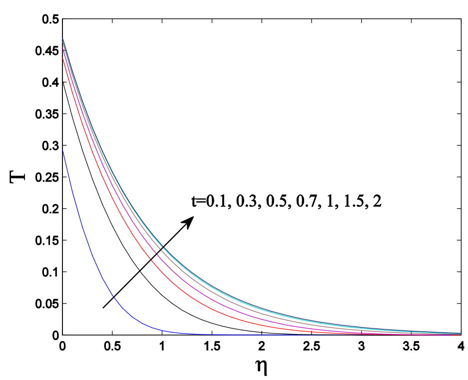

Fig. 10. Temperature profiles when R_c=2, x=1 and P_r=0.71.

This implies that the thermal diffusion tends to enhance the fluid temperature whereas thermal radiation has a reverse effect on it. The fluid temperature enhances with the progress of time and it gradually attains the steady-state value for large time i.e. t>1.5.

Figures 11 and 12 demonstrate the variation of the fluid velocity and the fluid temperature for different values of the distance x (where x is the distance along the surface measured from the leading edge). It is seen from Figs. 11 and 12 that as x increases, the velocity and temperature profiles become steeper near the surface of the plate. This implies that the momentum and thermal boundary layer thicknesses decrease along the distance from the leading edge which is in agreement with the results obtained by Pantokratoras (2014).

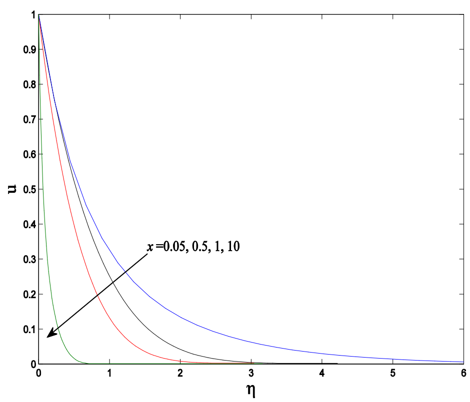

Fig. 11. Velocity profiles when $M_c^2=4$, G_{rc}=10, K_{pc}=0.4, R_c=2, P_r=0.71 and t=0.3

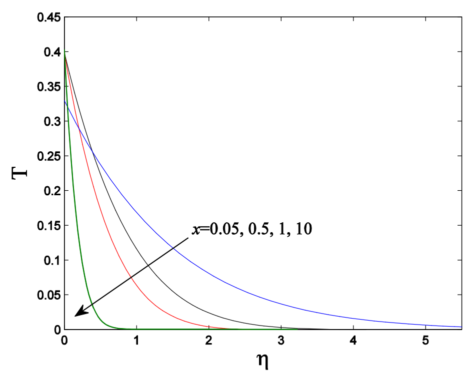

Fig. 12. Temperature profiles when $M_c^2=4$, G_{rc}=10, K_{pc}=0.4, R_c=2, P_r=0.71 and t=0.3.

However, it is observed from Fig. 11 that the fluid velocity decreases uniformly starting from its maximum value at the plate surface which is not seen in the velocity profiles presented by Pantokratoras (2014) (where the velocity profiles attain maximum value near the plate but not at the plate). This nature of velocity profile in the present problem is due to the flow of an electrically-conducting fluid in the presence of a magnetic field whose tendency is to retard the fluid flow by virtue of the Lorentz force which tends to stabilize the fluid motion.

The non-dimensional wall temperature $T(0,t)$ and the non-dimensional wall heat transfer $T'(0,t)$ are the most important entities in the problems concerning convective surface boundary conditions because the wall temperature although not known a priori, but it plays a key role in inducing natural convection due to the difference between wall temperature and the ambient fluid temperature. Therefore, we present the numerical values of $T(0,t)$ and $T'(0,t)$ in tabular form in Table 2 for various values of R_c, P_r, x, t, M_c^2, G_{rc} and K_{pc} in order to study the effects of various agencies on them. It is evident from Table 2 that $T(0,t)$ decreases upon increasing R_c, P_r, G_{rc} and K_{pc} whereas it increases upon increasing t and M_c^2. $T'(0,t)$ increases upon increasing R_c, P_r, G_{rc} and K_{pc} whereas it decreases upon increasing t and M_c^2. Both $T(0,t)$ and $T'(0,t)$ increase upon increasing x. This implies that the thermal radiation, thermal buoyancy force and the permeability of the medium tend to reduce the wall temperature whereas these parameters have the reverse effect on the wall heat transfer. The magnetic field and the thermal diffusion tend to enhance the wall temperature whereas they have the reverse effect on the wall heat transfer. As we move along the leading edge, the wall temperature and the wall heat transfer become enhanced. The wall temperature gets enhanced and the wall heat transfer gets reduced with the progress of time.

The numerical values of the wall shear stress $u'(0,t)$ are presented in tabular form in Table 3 for various values of M_c^2, G_{rc}, K_{pc}, R_c, P_r, x and t. It is noticed from Table 3 that $u'(0,t)$ increases upon increasing x, M_c^2, R_c and P_r whereas it decreases upon increasing G_{rc}, K_{pc} and t. This implies that the magnetic field and the thermal radiation tend to enhance the wall shear stress whereas the thermal diffusion, thermal buoyancy force and the permeability of the medium have the reverse effect on it. The wall shear stress increases along the distance from the leading edge and it reduces with the progress of time.

5. CONCLUSION

A non-similar solution to the fundamental problem concerning unsteady hydromagnetic natural convection flow through a fluid-saturated porous medium of a viscous, incompressible, electrically-conducting and optically-thin radiating fluid past an impulsively-moving semi-infinite vertical plate with a convective surface boundary condition is obtained using an efficient implicit finite-difference scheme of the Crank-Nicolson type. Significant findings are predicted and they are summarized as follows:

- The momentum and thermal boundary layer thicknesses decrease along the distance from the leading edge.

Table 2 Wall Temperature and Wall heat transfer

R_c	P_r	x	t	M_c^2	G_{rc}	K_{pc}	$T(0,t)$	$-T'(0,t)$
2	0.71	0.5	0.3	4	10	0.4	0.39842	0.43518
5	0.71	0.5	0.3	4	10	0.4	0.34222	0.48589
8	0.71	0.5	0.3	4	10	0.4	0.30362	0.52166
2	0.3	0.5	0.3	4	10	0.4	0.50865	0.35285
2	0.5	0.5	0.3	4	10	0.4	0.44254	0.40188
2	1	0.5	0.3	4	10	0.4	0.35710	0.46691
2	0.71	0.05	0.3	4	10	0.4	0.32938	0.15239
2	0.71	1	0.3	4	10	0.4	0.39861	0.61523
2	0.71	10	0.3	4	10	0.4	0.40273	1.97847
2	0.71	0.5	0.5	4	10	0.4	0.43022	0.41339
2	0.71	0.5	0.7	4	10	0.4	0.44312	0.40443
2	0.71	0.5	0.7	1	10	0.4	0.43873	0.40718
2	0.71	0.5	0.7	7	10	0.4	0.44523	0.40313
2	0.71	0.5	0.7	4	5	0.4	0.44478	0.40341
2	0.71	0.5	0.7	4	15	0.4	0.44116	0.40564
2	0.71	0.5	0.7	4	10	0.2	0.44497	0.40329
2	0.71	0.5	0.7	4	10	0.8	0.44166	0.40533

Table 3 Wall shear stress

M_c^2	G_{rc}	K_{pc}	R_c	P_r	x	t	$-u'(0,t)$
1	10	0.4	2	0.71	1	0.3	1.02719
4	10	0.4	2	0.71	1	0.3	1.72580
7	10	0.4	2	0.71	1	0.3	2.30646
4	5	0.4	2	0.71	1	0.3	2.14879
4	15	0.4	2	0.71	1	0.3	1.30282
4	10	0.2	2	0.71	1	0.3	2.21610
4	10	0.8	2	0.71	1	0.3	1.45187
4	10	0.4	5	0.71	1	0.3	1.88720
4	10	0.4	8	0.71	1	0.3	1.99922
4	10	0.4	2	0.3	1	0.3	1.30325
4	10	0.4	2	0.5	1	0.3	1.56344
4	10	0.4	2	1	1	0.3	1.86901
4	10	0.4	2	0.71	0.05	0.3	1.03608
4	10	0.4	2	0.71	0.5	0.3	1.22089
4	10	0.4	2	0.71	10	0.3	5.50286
4	10	0.4	2	0.71	1	0.5	1.51218
4	10	0.4	2	0.71	1	0.7	1.41621

- The thermal buoyancy force, permeability of the porous medium and the thermal diffusion tend to accelerate the fluid flow throughout the boundary layer region whereas the magnetic field and the thermal radiation have the reverse effect on it.
- The fluid flow gets accelerated and the fluid temperature becomes enhanced with the progress of time.
- Thermal diffusion tends to enhance the fluid temperature whereas thermal radiation has the reverse effect on it.
- The thermal radiation, thermal buoyancy force and the permeability of the porous medium tend to reduce the wall temperature whereas these parameters have the reverse effect on the wall heat transfer. The magnetic field and the thermal diffusion tend to enhance the wall temperature whereas these parameters have the reverse effect on the wall heat transfer. The wall temperature becomes enhanced and the wall heat transfer tends to reduce with the progress of time.

The magnetic field and the thermal radiation tend to enhance the wall shear stress whereas the thermal diffusion, thermal buoyancy force and the permeability of the porous medium have the reverse effect on it. The wall shear stress reduces with the progress of time.

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