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HYDROMAGNETIC PLANE AND AXISYMMETRIC FLOW NEAR A STAGNATION POINT WITH HEAT GENERATION

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ABSTRACT

The steady, laminar, forced convection flow of an electrically-conducting and heat-generating/absorbing fluid at the stagnation point of an isothermal two-dimensional porous body and an axisymmetric body in the presence of a uniform applied magnetic field has been studied. The governing equations are transformed into ordinary differential equations using similarity variables. The obtained results are solved numerically by an implicit finite-difference method. A parametric study of all the physical parameters involved in the problem has been performed and graphical results for the velocity and temperature profiles as well as the skin-friction coefficient and the wall heat transfer are presented and discussed. © 1998 Elsevier Science Ltd

Introduction

This paper is concerned with steady, laminar flow and heat transfer of an electrically-conducting, heat generating/absorbing and viscous Newtonian fluid near a stagnation point in the presence of a transverse magnetic field. This flow and heat transfer process occurs in many industrial applications such as cooling of electronic devices by fans, heat exchangers design, cooling of nuclear reactors, and others.

Hiemenz [1] was the first to discover that the stagnation-point flow can be analyzed exactly by the Navier Stokes equations and he reported two-dimensional plane flow velocity distribution. Later, Goldstein [2] reported the corresponding temperature distribution. The axisymmetric flow velocity distribution was given by Homann [3] while the axisymmetric temperature distribution was reported by Sibulkin [4]. Since then, many investigators have considered various aspects of the problem (see, for instance, Wang [5], Ramachandran et al. [6], Takhar et al. [7], Kumari et al. [8], Mahmood and Merkin [9] and Merkin and Mahmood [10]). It has been observed that in a nuclear reactor, magnetic field affects considerably the flow

and heat transfer aspects. The steady hydromagnetic flow over a flat plate has been studied by Tan and Wang [11] and Na [12].

Problem Formulation

Consider a horizontal, semi-infinite, porous flat plate. Let the fluid flow be coming from top towards the plate, thus; forming a stagnation-point flow. A uniform magnetic field is applied normal to the flow direction. The flow situation and the coordinate system with the origin being the stagnation point and the y-axis is normal to the plane are shown in Fig. 1. The x-axis is the tangential direction and x is interpreted as the radial direction for axisymmetric flow situations. The fluid is assumed to be electrically-conducting, heat-generating/absorbing, viscous, Newtonian, and have constant properties. It is also assumed that the magnetic Reynolds number is small so that the induced magnetic field is neglected. In addition, the Hall effect is neglected and no applied voltage is assumed to exist which implies the absence of the electric field.

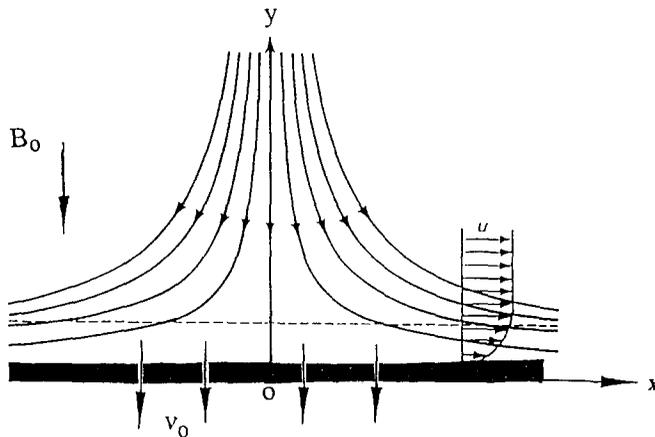


FIG. 1
Schematic of the Problem

The governing equations for this investigation are based on the balance laws of mass, linear momentum, and energy modified to account for the presence of the magnetic field, and heat generation/absorption effects.

The equations can be written as (see White [13])

$$\frac{\partial}{\partial x}(x^{n-2}u) + \frac{\partial}{\partial y}(x^{n-2}v) = 0 \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial}{\partial x} \left(\frac{u^{n-2}}{x^{n-2}} \right) + \frac{\partial^2 u}{\partial y^2} \right) - \frac{\sigma B_0^2}{\rho} u \quad (2)$$

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \nu \left(\frac{1}{x^{n-1}} \frac{\partial}{\partial x} (x^{n-1} v) + \frac{\partial^2 v}{\partial y^2} \right) \quad (3)$$

$$\rho c_p \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k \left(x^{2-n} \frac{\partial}{\partial x} (x^{n-2} T) + \frac{\partial^2 T}{\partial y^2} \right) + Q_0 (T - T_w) \quad (4)$$

where x and y are the tangential (or radial) distance and the normal distance, respectively. u , v , p , T are the fluid tangential (or radial) and normal velocity components, pressure, and temperature, respectively. ρ , ν , k , c_p and σ are the fluid density, kinematic viscosity, thermal conductivity, specific heat at constant pressure, and electrical conductivity, respectively. B_0 , Q_0 , T_w , and n are the respective magnetic induction, heat generation/absorption coefficient, wall temperature and the dimensionality index such that $n = 2$ corresponding to plane flow and $n = 3$ corresponding to axisymmetric flow.

The boundary conditions for this problem are

$$\begin{aligned} u(x,0) = 0, \quad v(x,0) = -v_0, \quad T(x,0) = T_w \\ u(x,\infty) = U_\infty, \quad T(x,\infty) = T_\infty \end{aligned} \quad (5)$$

where v_0 (a constant) is the suction or injection velocity, T_w (a constant) is the wall temperature, U_∞ and T_∞ are the freestream velocity and temperature, respectively.

It is convenient to introduce the following dimensionless variables

$$\eta = y \sqrt{\frac{B}{\nu}}, \quad \psi = \frac{x^{n-1}}{n-1} \sqrt{B\nu} f(\eta) = \frac{T - T_w}{T_\infty - T_w} \quad (6)$$

from which

$$u = \frac{Bx}{n-1} f'(\eta), \quad v = -\sqrt{B\nu} f(\eta) \quad (7)$$

where the constant B is a sort of a velocity gradient parallel to the wall and a prime denotes ordinary differentiation with respect to η .

Substituting Equations (6) and (7) into Equations (1) through (5) yields

$$f''' + ff'' + \frac{1}{n-1} (1-f'^2) - M^2 f' = 0 \quad (8)$$

$$\theta'' + Pr f \theta' + Pr \alpha \theta = 0 \quad (9)$$

$$f(0) = f_w, \quad f'(0) = 0, \quad \theta(0) = 0, \quad f'(\infty) = 1, \quad \theta(\infty) = 1 \quad (10)$$

where $M^2 = \sigma B_0^2 / (\rho B)$, $Pr = \mu c_p / k$ (μ is the dynamic viscosity of the fluid), and $\alpha = Q_0 / (\rho c_p B)$ are the square of the Hartmann number, the Prandtl number, the dimensionless heat generation / absorption coefficient, and the suction/injection parameter, respectively.

Important physical parameters for this flow and heat transfer situation are the skin friction coefficient and the wall heat transfer coefficient. They are given, respectively, by

$$C = \frac{\mu \frac{\partial u}{\partial y} \Big|_{y=0}}{\frac{1}{2} \rho U_\infty^2} = \frac{2\sqrt{n-1} f''(0)}{\sqrt{Re_x}} \quad (11)$$

$$H = - \frac{k \frac{\partial T}{\partial y} \Big|_{y=0}}{\rho U_\infty c_p (T_w - T_\infty)} = \frac{\sqrt{n-1} \theta'(0)}{Pr \sqrt{Re_x}} \quad (12)$$

where $Re_x = U_\infty x / \nu$ and $U_\infty = Bx / (n-1)$ are the Reynolds number and the freestream velocity, respectively

Results and Discussion

The plane and axisymmetric flow and heat transfer characteristics of a hydromagnetic stagnation point flow are governed by the similarity equations and boundary conditions (8) through (10). The equations are nonlinear and exhibit no closed-form solutions. Therefore, they must be solved numerically. The implicit, iterative, tri-diagonal finite-difference method discussed by Blottner [14] has proven to be successful for the solution of this type of equations. For this reason, this numerical method was employed in the present work.

The third-order ordinary differential equation in f (Equation (8)) is converted into a second-order equation in V by assuming $V = f'$. This equation along with Equation (9) are discretized using three-point central-difference quotients while the equation $f' = V$ is discretized using the trapezoidal rule.

Linearization of the non-linear terms is achieved by evaluating them at the previous iteration. As a result, linear, tri-diagonal algebraic equations are obtained and solved by the Thomas algorithm (see, Blottner [14]). The computational domain was divided up into 195 nodes. Variable step sizes are used to accommodate sharp changes close to the wall. The initial step size and growth factor employed in the present work were $\Delta\eta_1 = 10^{-3}$ and $K^* = 1.03$ such that $\Delta\eta_i = K^* \Delta\eta_{i-1}$. A convergence criterion based on the relative difference between the current and the previous iterations was utilized. When this difference reached 10^{-5} , the solution was assumed converged and the iteration process was terminated. Many numerical results were obtained throughout the course of this work. Only a representative set of these results is presented in Figures 2 through 9.

Figures 2 through 4 present representative profiles for the tangential velocity V , normal velocity f , and temperature θ of both plane ($n = 2$) and axisymmetric ($n = 3$) flows for various values of the Hartmann number M , respectively. Application of a magnetic field has the tendency to warm up and slow down the movement of the fluid. This effect is depicted by the decreases in the values of V and f and increases in θ shown in Figures 2 through 4, respectively. It should be noted also from these figures that as M increases, the tangential velocity approach to the free-stream condition becomes slower and the temperature profile overshoots above the freestream value. These features are more pronounced for axisymmetric flows.

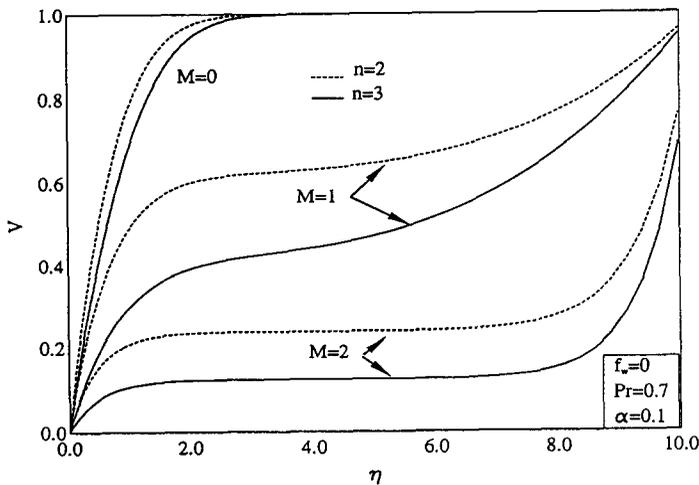


FIG. 2
Effects of M on Tangential Velocity Profiles

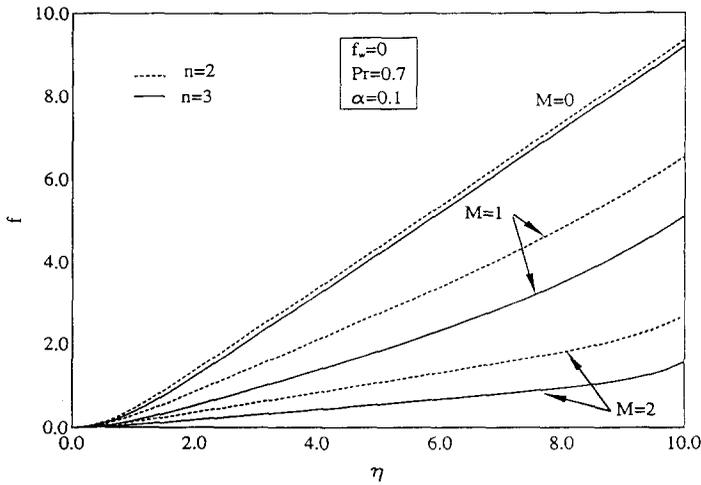


FIG. 3
Effects of M on Normal Velocity Profiles

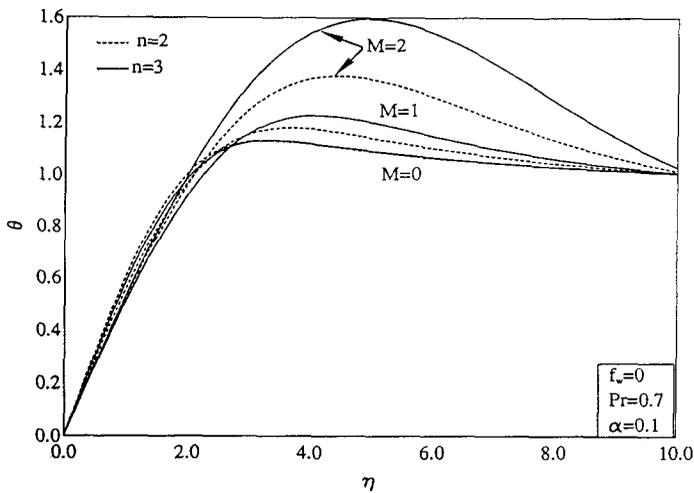


FIG. 4
Effects of M on Temperature Profiles

Figures 5 and 6 illustrate the influences of both the suction/injection parameter f_w and the Hartmann number M on the skin-friction coefficient $C_f = C Re_x^{1/2}$ and the wall heat transfer coefficient $C_h = H Re_x^{1/2}$, for plane and axisymmetric flows respectively. It is clearly seen that increases in the values of f_w produce increases in both C_f and C_h . As mentioned before increases in M cause reductions in the tangential velocity and its wall slope. This causes the skin-friction coefficient to decrease as M increases. In addition, it is evident from Figure 6 that for relatively small values of M , the wall heat transfer coefficient C_h tend to decrease. However, increasing M further causes C_h to increase. This is probably due to the fact that the effects of the convective terms in the energy equation become much smaller than those of the other effects.

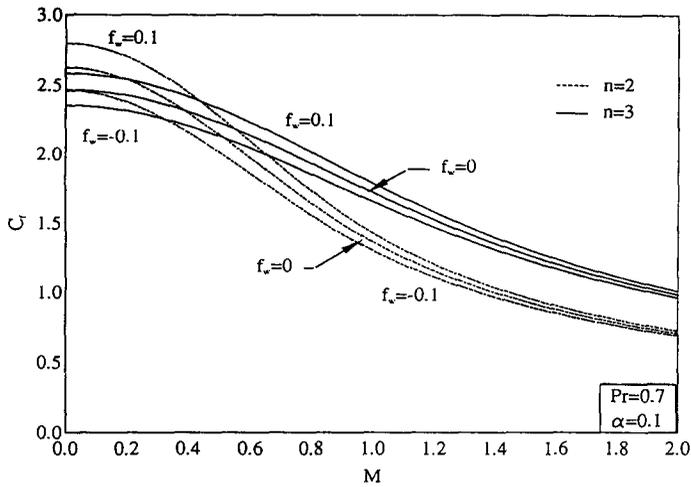


FIG. 5
Effects of M and f_w on Skin Friction Coefficient

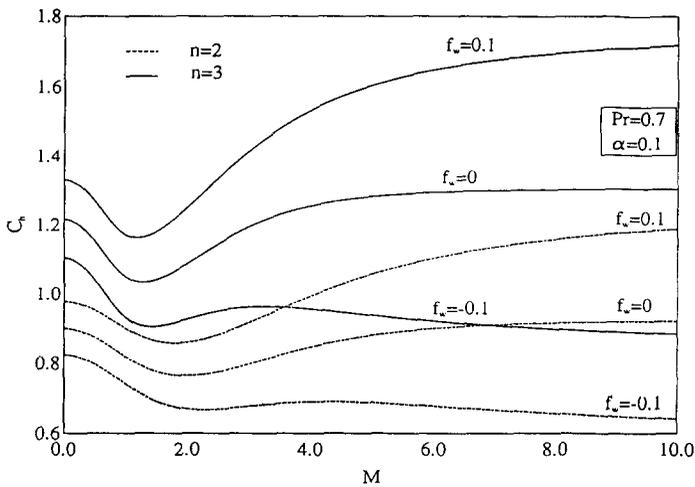


FIG. 6
Effects of M and f_w on Wall Heat Transfer

Figures 7 and 8 show the respective effects of the Prandtl number Pr and the heat generation/absorption coefficient α on the temperature profiles for both plane and axisymmetric stagnation-point flows. These figures illustrate the expected increases in θ as either Pr or α increases. It should be remarked that for the heat-generation case ($\alpha = 0.1$) in Figure 8 a sharp peak exists in the layer close the wall and this peak is more pronounced for axisymmetric stagnation-point flows than for its plane flow counterpart.

Finally, Figure 9 depicts the variations in C_h as a result of simultaneous increases in Pr and α . It is observed that C_h decreases as Pr increases and increases as α increases. Again, the axisymmetric flow values are higher than those for plane flows.

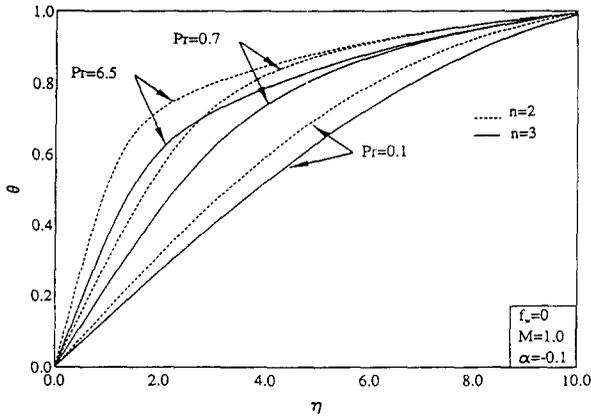


FIG. 7
Effects of Pr on Temperature Profiles

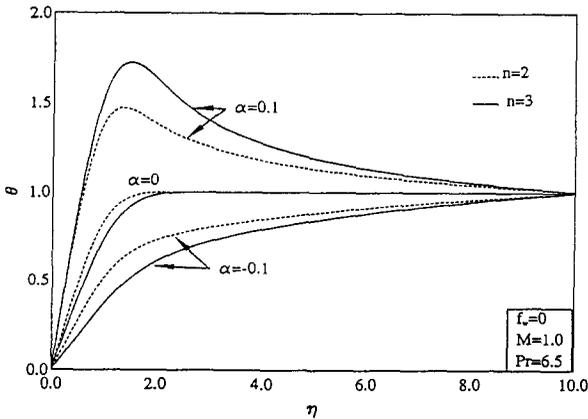


FIG. 8
Effects of α on Temperature Profiles

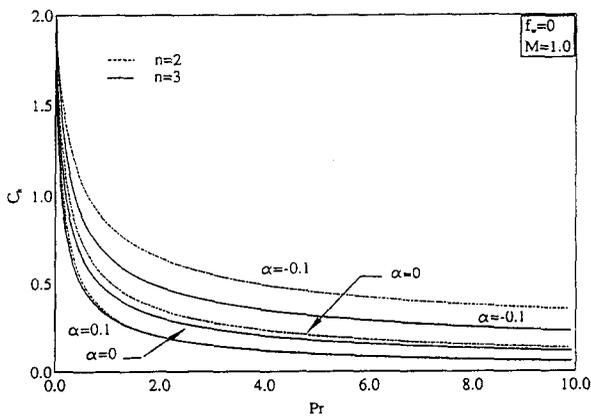


FIG. 9
Effects of Pr and α on Wall Heat Transfer

It should be noted that all the results associated with $M = 0$ and $\alpha = 0$ are consistent with those reported by White [13] as evident from Table 1. It can be seen from Table 1. that the comparison with the approximate correlations of White [13] which are applicable for gases ($Pr \approx 1$) is excellent. However, it is not as good for values of $Pr > 1$. This is because the correlations given by White [13] do not apply for this range of Pr values. The values of C_f also compare well since the obtained values for $n = 2$ and $n = 3$ are 2.4695 and 2.6240 while the values based on White's correlations $C_f \approx 2 Pr^{2/3} C_h$ are 2.4782 and 2.6275, respectively.

TABLE 1.

Comparison for C_h values
 $M = 0, \alpha = 0$

n=2			n=3		
Pr	White [13]*	Present Results	Pr	White [13]**	Present Results
0.7	0.7060	0.7080	0.7	0.9438	0.9507
1	0.5700	0.5705	1	0.7620	0.7624
10	0.1432	0.1339	10	0.1914	0.1752
100	0.0360	0.0299	100	0.0481	0.0387

* $C_h = 0.570 Pr^{-0.6}$

** $C_h = 0.762 Pr^{-0.6}$

Conclusion

The problem of steady, laminar plane and axisymmetric flow and heat transfer of an electrically-conducting and heat-generating/absorbing fluid near a stagnation point was solved numerically. Similarity transformations were employed to convert the governing partial differential equations into ordinary differential equations. The resulting equations were solved numerically by an implicit iterative finite-difference method. The obtained results were presented graphically to elucidate interesting features of the solutions. It was found that the imposition of wall suction produced higher skin-friction and wall heat transfer coefficients. Also, the application of a small to moderate magnetic field normal to the flow direction caused reductions in both the skin-friction and wall heat transfer coefficients. However, for large magnetic field strengths, the wall heat transfer was augmented. Furthermore, it was concluded that fluid heat generation produced higher wall heat transfer rates while fluid heat absorption caused reductions in the wall heat transfer values. The effects of all the parameters mentioned above were more pronounced for axisymmetric flows than for plane flows. It is hoped that the results obtained in this paper be of use for understanding of more complicated problems involving stagnation-point flows.

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