Effects of Particulate Diffusion on the Compressible Boundary-Layer Flow of a Two-Phase Suspension Over a Horizontal Surface

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The problem of steady, laminar, compressible flow and heat transfer of a particulate suspension past a semi-infinite horizontal flat surface is formulated and solved numerically using an implicit finite-difference scheme. The mathematical formulation of the governing equations is based on the Eulerian description familiar from fluid mechanics where both phases are treated as two separate interacting continua. These equations account for Brownian diffusion which is important when the particle phase consists of very tiny particles and allow for a general power-law fluid-phase viscosity-temperature and particle-phase diffusion-temperature relations. Obtained flow and heat transfer results are illustrated graphically to show interesting features of this type of flow.

Introduction

Boundary-layer flow and heat transfer have been the subject and of interest of many investigators for many years due to its direct application in many industries such as the aerospace, automotive, and petroleum industries. Other possible applications include fluidized beds and environmental pollutant motions, gas purification, dust collection, aerodynamic ablation, transport processes, and conveying of powdered materials. This type of problems is often complex due to the nature of the coupled nonlinear equations governing the behavior of the flow and heat transfer characteristics. There has been considerable work done on incompressible and compressible boundary-layer flow through and over many different geometries for a single-phase system (Stewartson, 1974, and Kuerti, 1951). The high possibility of the presence of solid particles in the fluid due to different processes, led to the consideration and analysis of two-phase systems. Modeling of two-phase particulate suspensions can be accomplished in two main ways. The first way is by assuming that both the fluid and the particles suspended in it behave as two interacting continua and can exchange momentum and heat transfer (Marble, 1970, and Ishii, 1975). This approach of representing the particle phase by continuous variables (similar to the fluid phase) is limited to problems of dilute dispersed systems. These consist of multiphase systems in the form of suspensions of particulates in predominating continuous fluid media (Soo, 1989). The second approach is by treating the fluid phase as a continuum while the particle phase is governed by the kinetic theory (Berlemont et al., 1990).

The presence of a second phase (like solid particles) adds more complexity to the problem and the equations governing it. The present work considers a fundamental problem in two-phase flow. This problem is that of steady, laminar, compressible, boundary-layer flow of a fluid-particle suspension over a semi-infinite horizontal flat surface or plate. The particle phase is assumed to consist of very tiny particles which allow for Brownian motion of particles. This leads to the consideration of the effects of particulate diffusion. Special cases of the present problem have been considered earlier by Singleton (1965) and Wang and Glass (1988). Both references obtained asymptotic solutions using the series expansion method. In addition, Wang and Glass (1988) reported numerical solutions based on the finite-difference methodology. Recently, Chamkha (1996) generalized the problem considered by Singleton (1965) and Wang and Glass (1988) for a dense suspension where particle-phase viscous effects are of importance. Review of the extensive work on the incompressible version of the present problem is available in the works of Osiptsov (1980), Prabha and Jain (1982), Datta and Mishra (1982), Chamkha and Peddieson (1989, 1992), and Chamkha (1994). A major conclusion of the work on the incompressible problem is that when the original dusty-gas model (a model meant for the description of particulate suspension having small particulate volume fraction and excludes particulate viscous and diffusive effects) discussed by Marble (1970) is used, a singularity (where the particle-phase density at the plate surface becomes infinite) exists. In contrast with this conclusion, the work of Chamkha (1996) has shown that for a compressible boundary-layer flow of a dense particulate suspension, a particle-free zone is formed somewhere downstream of the leading edge of the plate.

The presence of a particle-phase diffusivity in the dusty-gas model has shown to provide smoothing effects and is capable of removing the singularity predicted in the incompressible problem (Chamkha and Peddieson, 1989 and Chamkha, 1994). The present work investigates whether the inclusion of particle-phase diffusion effects in the dusty-gas model have the same influence on the compressible problem. Namely, the smoothing of the sharp peak in the particle-phase wall density predicted by Chamkha (1996). The fluid-phase dynamic viscosity, thermal conductivity, and particle-phase diffusivity are represented by general power-law functions of the fluid-phase temperature or the particle-phase temperature. The interaction between the phases is limited to drag and heat transfer. The particles are assumed very small and of spherical shape and their volume fraction is assumed small. Furthermore, the particle Reynolds number is assumed to be less than unity.

Governing Equations

Consider a steady, laminar, two-dimensional, boundary-layer flow of a two-phase particulate suspension bounded by

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a semi-infinite space in the x, y plane with a flat plate at zero angle of attack placed at $y = 0$. The flow is a uniform stream in the x-direction parallel to the plate. Far above the plate, both phases are in equilibrium. The governing equations for this investigation are based on the balance laws of mass, linear momentum, and energy for both phases. The boundary-layer form of these equations for the present problem can be written as

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0 \quad (1)$$

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) + \rho_p N(u_p - u) \quad (2)$$

$$\rho c\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \mu\left(\frac{\partial u}{\partial y}\right)^2 + \rho_p N(u_p - u)^2 + \rho_p c_p N_T(T_p - T) \quad (3)$$

$$P = \rho R T \quad (4)$$

$$\frac{\partial}{\partial x}(\rho_p u_p) + \frac{\partial}{\partial y}(\rho_p v_p) = \frac{\partial}{\partial y}\left(D_p\frac{\partial \rho_p}{\partial y}\right) \quad (5)$$

$$\rho_p\left(u_p\frac{\partial u_p}{\partial x} + v_p\frac{\partial u_p}{\partial y}\right) = \rho_p N(u - u_p) \quad (6)$$

$$\rho_p\left(u_p\frac{\partial v_p}{\partial x} + v_p\frac{\partial v_p}{\partial y}\right) = \rho_p N(v - v_p) \quad (7)$$

$$\rho_p c_p\left(u_p\frac{\partial T_p}{\partial x} + v_p\frac{\partial T_p}{\partial y}\right) = \rho_p c_p N_T(T - T_p) \quad (8)$$

where x and y are the distances along and normal to the plate, respectively. ρ, u, v, P, and T are the fluid-phase density, x-component of velocity, y-component of velocity, pressure, and temperature, respectively. μ, c, k, and R are the fluid-phase dynamic viscosity, specific heat, thermal conductivity, and gas constant, respectively. D_p is the particle-phase diffusivity. The subscript p refers to the same variable for the particle phase. $N = 18\mu/(\rho_s d^2)$ and $N_T = 12k/(\rho_s d^2 c_p)$ (where ρ_s and d are the density for the particle material and the particle diameter, respectively) are the momentum and temperature transfer coefficients, respectively.

Equation (4) assumes that the fluid phase is treated as an ideal gas. This equation is needed to render the problem determinant. The hydrodynamic and thermal coupling between the phases is accounted for by the interphase drag force and the interphase heat transfer. Other interphase mechanisms such as the virtual mass force (Zuber, 1964), the shear lift force (Saffman, 1965), and the spin-lift force (Rubinow and Keller, 1961) are neglected compared to the drag force. This is feasible when the particle Reynolds number is assumed to be small (Apazidis, 1985). Equation (5) for the particle-phase balance of mass is often used for flows with chemical reactions and mass generation processes. An alternative way to incorporate the particle-phase diffusivity is through the particle-phase momentum equation. However, since the present formulation has worked well for the incompressible problem (Chamkha and Peddieson, 1989), it is adopted in the present work.

Following Wang and Glass (1988), assume that

$$\Gamma = \frac{\mu}{\mu_\infty} = \left(\frac{T}{T_\infty}\right)^{\omega} \quad (0.5 \le \omega \le 1.0) \quad (9)$$

where ω is a fluid-phase power index coefficient and μ_∞ and T_∞ are the free-stream fluid-phase dynamic viscosity and temperature, respectively. Singleton (1965) employed $\omega = 0.5$ in his work on this problem.

In the absence of fundamental knowledge on how the particle-phase diffusivity varies with temperature (if so), it will be assumed without loss of generality that

$$D_p^* = \frac{D_p}{D_{p\infty}} = \left(\frac{T_p}{T_\infty}\right)^{\omega_p} \quad (0.5 \le \omega_p \le 1.0) \quad (10)$$

where ω_p is a particle-phase power index coefficient and $D_{p\infty}$ is the free-stream particle-phase diffusion, respectively. For convenience and simplicity, both the fluid and the particle phases will be assumed to have the same power index coefficient such that $\omega_p = \omega$. When ω and ω_p were varied separately, very similar results were predicted. For this reason and to cut down on the number of figures to be reported subsequently, they were assumed to be equal.

A set of boundary conditions suggested by the physics of the problem to be employed to solve the governing equations given previously are as follows:

$$u(x, 0) = 0, \quad v(x, 0) = 0, \quad T(x, 0) = T_w,$$

$$\frac{\partial \rho_p}{\partial y}(x, 0) = 0, \quad v_p(x, 0) = 0$$

$$u(x, \infty) = U_\infty, \quad u_p(x, \infty) = U_\infty,$$

$$v_p(x, \infty) = v(x, \infty), \quad T(x, \infty) = T_\infty$$

$$T_p(x, \infty) = T_\infty, \quad \rho(x, \infty) = \rho_\infty, \quad \rho_p(x, \infty) = \beta\rho_\infty \quad (11\ a\text{-}l)$$

where ρ_∞, U_∞, T_w, and β are the free-stream fluid-phase density, free-stream velocity, wall temperature, and the mass loading ratio of particles, respectively. Equations ($11a$–c) indicate that the fluid phase exhibits a no slip condition at the plate surface, no suction or injection condition, and is maintained at a uniform temperature T_w at the wall, respectively. Equations ($11d$, e) suggest that both the particle-phase normal variation of density and the normal velocity vanish at the plate surface, respectively. The rest of Eqs. (11) are matching conditions for both phases far above the plate and they indicate that both phases are in equilibrium with the free-stream conditions.

In the present work, a convenient set of modified Blasius transformations (similar to that employed previously by Chamkha and Peddieson (1994) converts the problem from semi-infinite in x ($0 \le x < \infty$) to finite in ξ ($0 \le \xi \le 1$) and eliminates the problems related to the singularities associated with the leading edge of the plate. In that it allows exact solution of the equations at the leading edge of the plate ($\xi = 0$) instead of assuming initial profiles of the dependent variables to start off the solution procedure as done previously by Wang and Glass (1988). This set of transformations is as follows

$$x = U_\infty\xi/(N(1-\xi)), \quad y = U_\infty/(N\,\mathrm{Re}_\infty^{1/2})(2\xi/(1-\xi))^{1/2}\eta$$

$$u = U_\infty F, \quad v = U_\infty((1-\xi)/(2\xi))^{1/2}(G + \eta F)/\mathrm{Re}_\infty^{1/2}$$

$$u_p = U_\infty F_p, \quad v_p = U_\infty((1-\xi)/(2\xi))^{1/2}(G_p + \eta F_p)/\mathrm{Re}_\infty^{1/2}$$

$$T = T_\infty H, \quad T_p = T_\infty H_p, \quad \rho = \rho_\infty Q, \quad \rho_p = \beta\rho_\infty Q_p$$

$$\mu = \mu_\infty\Gamma, \quad \delta = D_{p\infty}\rho_\infty/\mu_\infty, \quad \mathrm{Pr} = \mu c/k, \quad Ec = U_\infty^2/(cT_\infty)$$

$$\gamma = c/c_p, \quad \mathrm{Re}_\infty = \rho_\infty U_\infty^2/(N\mu_\infty) \quad (12)$$

Substituting Eqs. (12) (with $\beta = 1$ as done by Wang and Glass, 1988) along with Eqs. (9) and (10) into Eqs. (1) through (8) transforms the problem to

$$\frac{\partial(QG)}{\partial\eta} + QF + 2\xi(1-\xi)\frac{\partial(QF)}{\partial\xi} = 0 \quad (13)$$

$$\Gamma\frac{\partial^2 F}{\partial\eta^2} + \left(\frac{d\Gamma}{dH}\frac{\partial H}{\partial\eta} - QG\right)\frac{\partial F}{\partial\eta} - \frac{2\xi}{1-\xi}\left((1-\xi)^2 QF\frac{\partial F}{\partial\xi} - Q_p\Gamma(F_p - F)\right) = 0 \quad (14)$$

$$\Gamma\frac{\partial^2 H}{\partial\eta^2} + \left(\frac{d\Gamma}{dH}\frac{\partial H}{\partial\eta} - \mathrm{Pr}QG\right)\frac{\partial H}{\partial\eta} - 2\xi(1-\xi)\,\mathrm{Pr}QF\frac{\partial H}{\partial\xi} + \mathrm{Pr}Ec\Gamma\left(\frac{\partial F}{\partial\eta}\right)^2 + \left(\frac{2\xi}{1-\xi}\right) \times \left(\mathrm{Pr}Ec\Gamma Q_p(F_p - F)^2 + \frac{2\Gamma Q_p}{3}(H_p - H)\right) = 0 \quad (15)$$

$$QH = 1 \quad (16)$$

$$\delta D_p^*\frac{\partial^2 Q_p}{\partial\eta^2} + \left(\delta\frac{dD_p^*}{dH_p}\frac{\partial H_p}{\partial\eta} - G_p\right)\frac{\partial Q_p}{\partial\eta} - \left(F_p + \frac{\partial G_p}{\partial\eta}\right)Q_p - 2\xi(1-\xi)\frac{\partial(Q_pF_p)}{\partial\xi} = 0 \quad (17)$$

$$G_p\frac{\partial F_p}{\partial\eta} + \left(\frac{2\xi}{1-\xi}\right) \times \left(\Gamma(F_p - F) + (1-\xi)^2 F_p\frac{\partial F_p}{\partial\xi}\right) = 0 \quad (18)$$

$$G_p\frac{\partial G_p}{\partial\eta} + \eta G_p\frac{\partial F_p}{\partial\eta} - \eta F_p^2 + \left(\frac{2\xi}{1-\xi}\right)\left((1-\xi)^2 F_p\frac{\partial}{\partial\xi}(G_p + \eta F_p) + \Gamma(G_p - G + \eta(F_p - F))\right) = 0 \quad (19)$$

$$G_p\frac{\partial H_p}{\partial\eta} + \left(\frac{2\xi}{1-\xi}\right) \times \left((1-\xi)^2 F_p\frac{\partial H_p}{\partial\xi} + \frac{2\Gamma\gamma}{3\,\mathrm{Pr}}(H_p - H)\right) = 0 \quad (20)$$

The dimensionless boundary and matching conditions become

$$F(\xi, 0) = 0, \quad G(\xi, 0) = 0, \quad H(\xi, 0) = t_0$$

$$\frac{\partial Q_p}{\partial\eta}(\xi, 0) = 0, \quad G_p(\xi, 0) = 0, \quad F(\xi, \infty) = 1$$

$$F_p(\xi, \infty) = 1, \quad G_p(\xi, \infty) = G(\xi, \infty), \quad H(\xi, \infty) = 1$$

$$H_p(\xi, \infty) = 1, \quad Q(\xi, \infty) = 1, \quad Q_p(\xi, \infty) = 1 \quad (21)$$

where $t_0 = T_w/T_\infty$ is a dimensionless fluid-phase surface temperature.

Of special practical significance for this type of flow are the fluid-phase displacement thickness Δ, the particle-phase displacement thickness Δ_p, the fluid-phase skin-friction coefficient C, and the wall heat transfer coefficient q_w. These physical parameters are defined in dimensionless form as

$$\Delta(\xi) = \int_0^\infty (1 - QF)d\eta, \quad \Delta_p(\xi) = \int_0^\infty (1 - Q_pF_p)d\eta$$

$$C(\xi) = \Gamma(\xi, 0)\frac{\partial F}{\partial\eta}(\xi, 0), \quad q_w(\xi) = \frac{\Gamma(\xi, 0)}{Ec\,\mathrm{Pr}}\frac{\partial H}{\partial\eta}(\xi, 0) \quad (22)$$

Results and Discussion

Equations (13) through (21) exhibit no self-similar solution and must be solved numerically. In this section, some representative numerical results are reported to elucidate the behavior of the flow and heat transfer aspects of the problem under consideration. These results are computed using an iterative, implicit, tri-diagonal second-order accurate, finite-difference method similar to that discussed by Blottner (1970) and Patankar (1980).

All first-order derivatives with respect to ξ are replaced by three-point backward difference formulas. Three-point central difference quotients are also employed to discretize all second-order differential equations with respect to η. First-order differential equations are discretized using the trapezoidal rule. A two-dimensional domain is divided into 1001 nodes in the ξ direction and 195 nodes in the η direction in which the governing equations are solved. Very small step sizes are used in the region close to the wall where significant changes in the dependent variables occur and these step sizes are gradually increased as the distance above the plate is increased. However, constant small step sizes are used in the ξ direction. After many numerical experimentations, it was decided to use an initial step size of 0.001 with a growth factor of 1.03 in the η direction, and a constant step size of 0.001 in the ξ direction. Smaller step sizes than these produced no significant changes in results. The governing equations are then converted into sets of linear algebraic equations and solved by iteration at each line of constant ξ using the Thomas' algorithm (Blottner, 1970). A convergence criterion based on the difference between the current and the previous iterations was employed. It required that the difference be 10^{-5} in the present work.

Many numerical experimentations were performed by altering the step sizes in both directions to ensure accuracy of the results and to assess grid independence. For example, when $\Delta\eta_1$ was set to 0.01 instead 0.001, an average error of about 8 percent was observed in the results with the maximum error being close to $\xi = 1$. Also, when $\Delta\eta_1$ was equated to 0.0001 no significant changes of results were observed. For this reason $\Delta\eta_1 = 0.001$ was chosen and employed in producing the numerical results. The flow and heat transfer parameter are not as sensitive to $\Delta\xi$ as they are sensitive to $\Delta\eta_1$. For this reason, a constant step size was used in the ξ direction. The sensitivity analysis of the results to changes in $\Delta\xi$ was also performed. For instance, when $\Delta\xi$ was set to 0.01, an average deviation of 5 percent from the results with $\Delta\xi = 0.001$. Smaller values of $\Delta\xi$ than 0.01 produced no changes in the results and, therefore, $\Delta\xi$ was set to 0.001 in all the produced results. Two types of convergence criterion were tried. One was based on the percentage error between the previous and the current iterations and the other was based on their difference. Since we are not dealing with very small numbers, the convergence criterion based on the difference between the previous and current iterations was employed in the present study. No convergence problems were encountered even with the small value of 10^{-5} used in this work. Equations (13) through (20) were solved for G, F, H, Q, Q_p, F_p, G_p, and H_p, respectively.

Many numerical computations were performed and a parametric study illustrating the influence of the various parameters on the solution was done throughout the course of this work. Only a small representative portion of the results obtained is presented in Figs. 1 through 10.

Figures 1 and 2 illustrate the development of the fluid-phase displacement thickness Δ and the particle-phase displacement

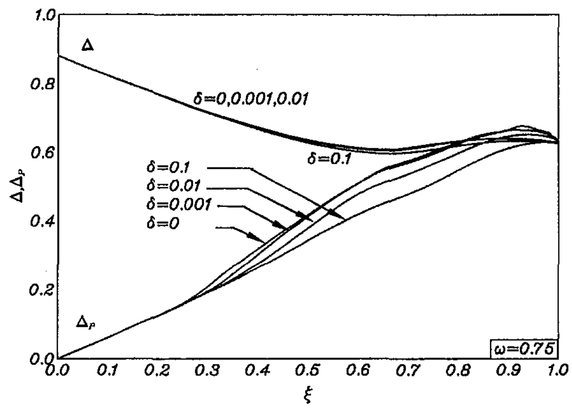

Fig. 1 Fluid and particle-phase displacement thicknesses profiles

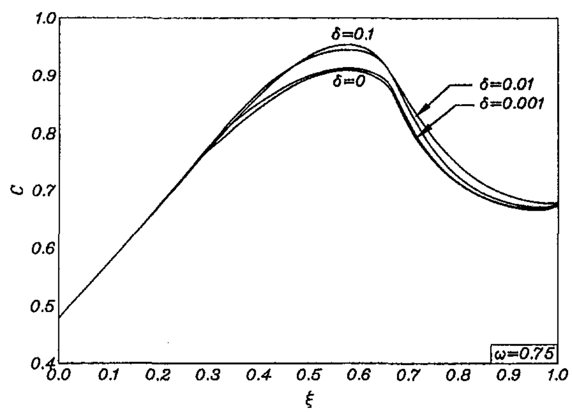

Fig. 2 Fluid-phase skin friction coefficient profiles

thickness Δ_p, and the fluid-phase skin-friction coefficient C along the plate's axial or tangential distance ξ for various inverse Schmidt's number δ, respectively. The four values of δ used to produce the numerical results are representative of non-diffusive suspensions ($\delta = 0$), small to moderate diffusive particulate phase ($\delta = 0.001$, 0.01) such as powdered gas flow and relatively high diffusive suspension such as smoke in air flow. Physically speaking, at the leading edge of the plate, a frozen flow condition exists where both phases move independently. As a result, the drag force between the phases is maximum. As the flow moves downstream of the plate's leading edge, the momentum exchange mechanism through the drag force increases causing Δ to decrease and Δ_p to increase until an equilibrium condition where both the fluid and the particle phases move together is reached at $\xi = 1$. However, the values of C tend to increase to a peak and then decrease to the equilibrium value. This behavior in Δ, Δ_p, and C are clearly depicted in Figs. 1 and 2. Furthermore, as the inverse Schmidt's number δ increases, a slight reduction in Δ, and a noticeable decrease in Δ_p and an increase in C are predicted as seen in Figs. 1 and 2.

Figures 3 and 4 present representative profiles for the particle-phase tangential velocity and density at the wall for various inverse Schmidt's numbers δ, respectively. At $\xi = 0$, the particle phase experiences a perfect slip condition at the wall with a uniform density distribution. As ξ increases and the interaction between the phases takes place, the drag force begins to decrease. As a result, the particle-phase wall tangential velocity $F_p(\xi, 0)$ starts to decrease and the particle-phase wall density $Q_p(\xi, 0)$ starts to increase slightly until equilibrium between the phases at the plate surface occurs. At this wall equilibrium position ($\xi = 0.67$), $F_p(\xi, 0)$ vanishes and $Q_p(\xi, 0)$ becomes maximum. This type of behavior was predicted in analysis of the incompressible version of the present problem where $Q_p(\xi, 0)$ became infinite (suggesting the presence of a singularity) when $F_p(\xi, 0)$ vanished (Osiptsov, 1980; Datta and Mishra, 1982; and Chamkha and Peddieson, 1989). However, in this problem continuous nonsingular solution exists in the whole computational domain. This is because as the particle phase continues to adjust with the fluid phase, its wall density begins to decrease until complete equilibrium between the phases is attained at $\xi = 1$. These behaviors in $F_p(\xi, 0)$ and $Q_p(\xi, 0)$ are clearly illustrated in Figs. 3 and 4, respectively. The effect of particulate diffusion is limited to spreading the particles away from the wall causing a significant reduction in the peak values of $Q_p(\xi, 0)$ without affecting the wall velocities of the particles as shown in Figs. 3 and 4. This problem serves as a special example of how a small change in the mathematical model (by addition of particulate diffusion in this case) produces a significant change in results.

In Fig. 5, the wall heat transfer coefficient q_w is presented along the plate for various values of δ. It is seen from this figure that q_w increases to a peak existing in the vicinity of $\xi = 0.67$ where $Q_p(\xi, 0)$ is maximum and then decreases to a limiting value at the equilibrium condition existing between the phases at $\xi = 1$. It is also seen that for $\delta = 0$ (nondiffusive particle phase) a sharp peak in q_w (corresponding to a sharp peak in $Q_p(\xi, 0)$) exists and this peak flattens out as δ increases as predicted for the profiles of $Q_p(\xi, 0)$. This suggests that the energy transfer between the phases increases as the density of the particles increases which, in turn, augments the wall heat transfer as depicted in Fig. 5. While ω was set to 0.75 in Figs. 1 through 5, similar trends are observed for other values of ω.

Figures 6 through 10 show the influence of the power index coefficient ω on the flow and heat transfer properties reported

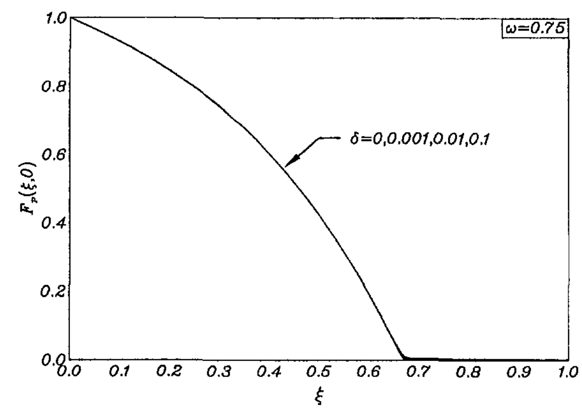

Fig. 3 Wall particle-phase tangential velocity profiles

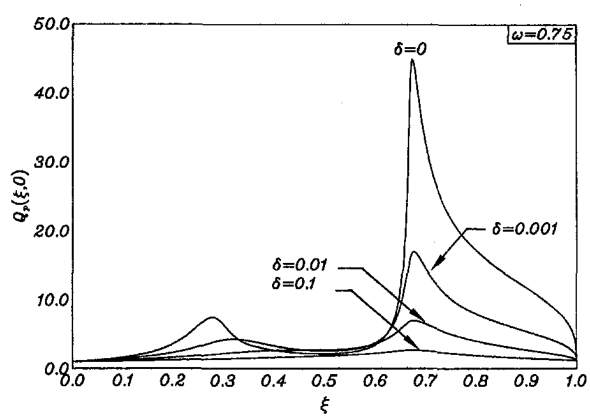

Fig. 4 Wall particle-phase density profiles

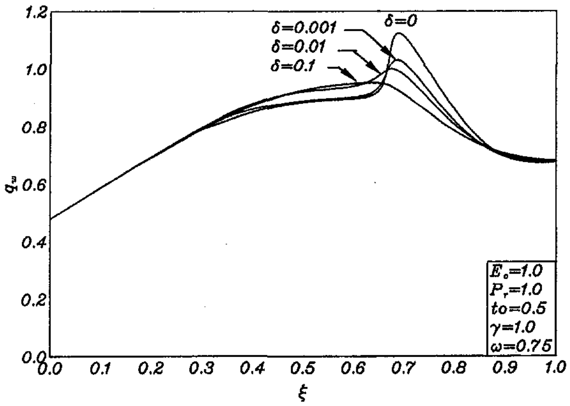

Fig. 5 Wall heat transfer coefficient profiles

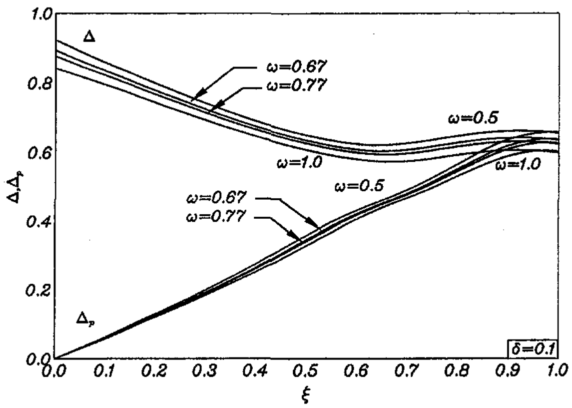

Fig. 6 Fluid and particle-phase displacement thicknesses profiles

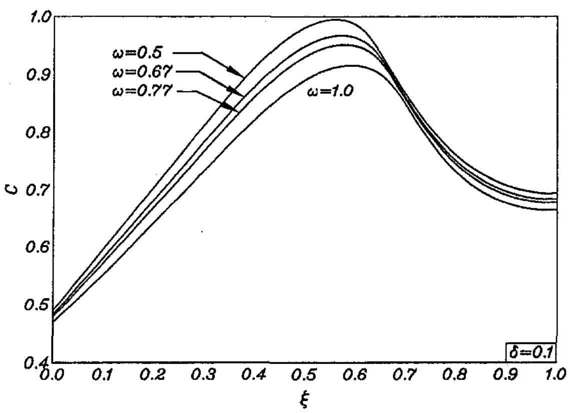

Fig. 7 Fluid-phase skin friction coefficient profiles

in Figs. 1 through 5, respectively. In general, increasing ω has a tendency to decrease the domain of viscous effects along the plate causing both Δ and Δ_p to decrease as clearly depicted in Fig. 6. The fluid-phase skin-friction coefficient C is related to the fluid wall viscosity which is dependent on the fluid-phase wall temperature (a constant value of 0.5 in this case) raised to the power index ω. Thus, increasing ω causes a decrease in C as shown in Fig. 7. In addition, as ω is increased, higher particle-phase wall tangential velocities are predicted which delay the approach to a no slip condition far downstream. This, in turn, causes the peak in $Q_p(\xi, 0)$ discussed earlier to move downstream with a reduced value. These phenomena are due to the fact that the effect of the particle-phase diffusion increases as ω increases and they are clearly depicted in Figs. 8 and 9. Finally, similar to the skin-friction coefficient C, the wall heat transfer coefficient q_W is directly proportional to the wall fluid viscosity. Therefore, increasing ω results in decreasing q_W for the reasons discussed earlier. These reductions in q_W as ω increases are apparent in Fig. 10.

The results associated with $\delta = 0$ (diffusionless theory) were put in terms of their primitive untransformed variables and compared with those reported by Wang and Glass (1988) and were found to be in good agreement. Furthermore, additional favorable comparisons were performed with the incompressible results reported by Chamkha and Peddieson (1989) and Chamkha (1994). In the absence of reported experimental results on the problem considered in this paper, and inspite of the favorable

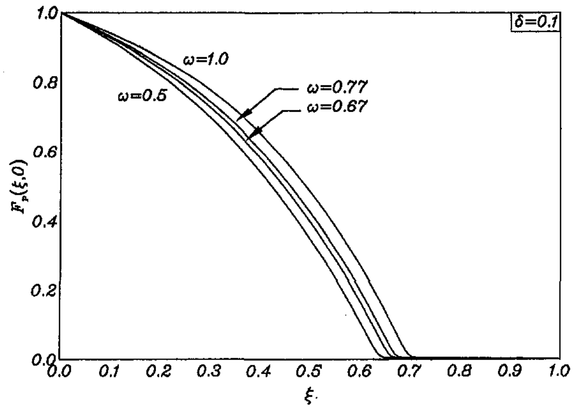

Fig. 8 Wall particle-phase tangential velocity profiles

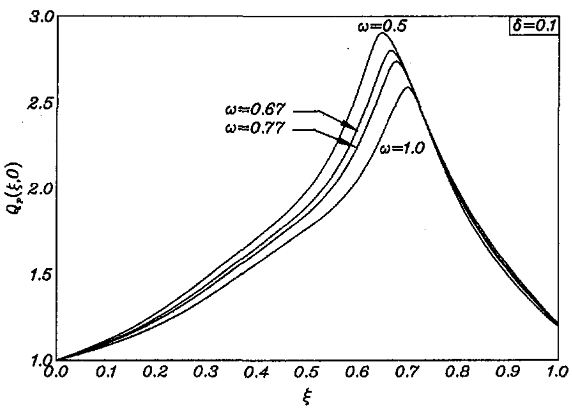

Fig. 9 Wall particle-phase density profiles

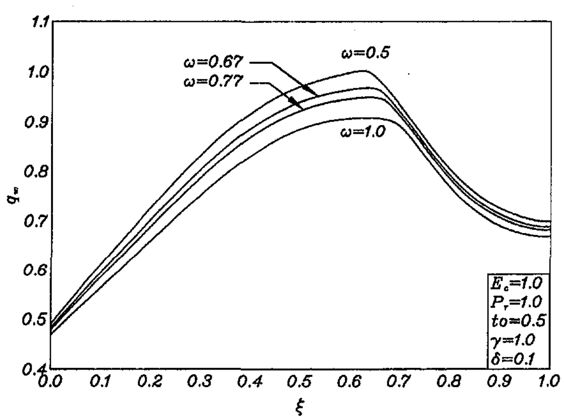

Fig. 10 Wall heat transfer coefficient profiles

comparisons made (which lend confidence in the numerical procedure), it is difficult to be certain that the interesting phenomena predicted in this work and the work reported previously (Chamkha, 1996) are physically possible. This is difficult to evaluate because of the contrast that these results offer when a small change in the mathematical model occurs. Therefore, it is highly recommended that experimental investigation of this problem be undertaken. The present and the previously reported results (Chamkha, 1996) can serve as a stimulus for this investigation by identifying a special phenomenon to look for.

Conclusion

In the present work, a two-phase (particle-fluid) theory based on the continuum approach modified to include particle-phase diffusion effects is formulated and applied to the problem of steady, laminar, compressible boundary-layer flow over a semi-infinite flat plate. General power-law viscosity- and diffusion-temperature relations are utilized. An implicit numerical scheme based on the tri-diagonal finite-difference methodology is employed in the solution of the governing equations. The predicted results show that the presence of the particulate diffusion in the model is capable of reducing significant wall heat transfer and particle concentration occurring at the plate surface somewhere downstream of the leading edge. Also, in contrast with the incompressible version of the problem (where a singularity in the particle-phase wall density is predicted for a diffusionless theory), nonsingular continuous solutions were obtained throughout the computational region. Favorable comparisons with previously published numerical results are performed which serve as a check on the correctness of the reported results. It is hoped that experimental investigations on this problem be undertaken to verify the physical validity of the predictions.

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