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Heat transfer enhancement in the boundary layer flow of hybrid nanofluids due to variable viscosity and natural convection

S. Manjunatha [a,*], B. Ammani Kuttan [a], S. Jayanthi [b], Ali Chamkha [c,d], B. J. Gireesha [e]

[a] *Department of Mathematics, Faculty of Engineering, CHRIST (Deemed to be University), Bengaluru, 560076, Karnataka, India*

[b] *Department of Mathematics, BMS College of Engineering, Bengaluru, 560019, Karnataka, India*

[c] *Mechanical Engineering Department, Prince Sultan Endowment for Energy and Environment, Prince Mohammad Bin Fahd University, Al-Khobar, 31952, Saudi Arabia*

[d] *RAK Research and Innovation Center, American University of Ras Al Khaimah, P.O. Box 10021, Ras Al Khaimah, United Arab Emirates*

[e] *Department of Studies and Research in Mathematics, Kuvempu University, Shankaraghatta, 577 451, Shimoga, Karnataka, India*

* Corresponding author.
E-mail address: manjubhushana@gmail.com (S. Manjunatha).

Abstract

The aim of the current work is to explore how heat transfer can be enhanced by variations in the basic properties of fluids in the presence of free convection with the aid of suspended hybrid nanofluids. Also, the influence of the Laurentz force on the flow is considered. The mathematical equations are converted into a pair of self-similarity equations by applying appropriate transformations. The reduced similarity equivalences are then solved numerically by Runge-Kutta-Fehlberg 45th-order method. To gain better perception of the problem, the flow and energy transfer characteristics are explored for distinct values of significant factors such as variable viscosity, convection, magnetic field, and volume fraction. The results acquired are in good agreement with previously published results. The

noteworthy finding is that the thermal conductivity is greater in hybrid nanofluid than that of a regular nanofluid in the presence of specified factors. The boundary layer thickness of both hybrid nanofluid and normal nanofluid diminishes due to decrease in variable viscosity. The fluid flow and temperature of the hybrid nanofluid and normal nanofluid increases as there is a rise in volume fraction.

Keywords: Applied mathematics, Computational mathematics, Mechanics

1. Introduction

Since its advent, nano technology has attracted many researchers who recently began using nanofluids in both experimental and theoretical works. Because of the heat transfer enhancement properties of nanoparticles, industries such as solar synthesis, gas sensing, biological sensing, chemical, nuclear reactors, the chemical industry etc. have adopted the concept of including nanoparticles in their relevant fields to improve the heat-transfer performance of normal fluids. The thermal conductivity gets doubled by adding nanoparticles as theoretically proved by Choi et al. [1]. Masuda et al. [2] and Minsta et al. [3] proved that adding a small amount of nano particles (< 5%) yielded a significant improvement (10−50 %) in the thermal conductivity of basic fluids. Kim et al. [4, 5] analyzed the crucial significance of nanofluids in the field of the nuclear science. They confirmed nanofluids can improve the performance of any water-cooled nuclear system applications. Pressurized water reactor, primary coolant, standby safety systems, accelerator targets, plasma divertors, and so forth, are such possible applications [6]. The use of nanofluids as a coolant could also be employed in emergency cooling systems where they could cool down overheat surfaces more quickly leading to an improvement in power plant safety. From Jackson′s [7] view, the critical heat flux could be improved by generating a structured surface from the accumulation of nanofluids, The use of nanofluid can increase the in-vessel retention capabilities of nuclear reactors by as much as 40% [8].

The fundamental physical properties of a fluid (viscosity) vary with temperature and play a vital role in nanofluids. The heat generated by internal friction increases the temperature which in turn has an effect on the stickiness of the fluid. Hence, fluid viscidness cannot be considered as constant. Thus, it is essential to assume that the viscosity will be temperature-dependent. Ammani Kuttan et al [9], Sedeek [10], Ali [11] and Manjunatha [12] supported this theory by carrying out research on variable fluid properties over a flat surface.

The two-dimensional flow of fluid in the immediate neighborhood of a surface under the effect of gravity is very important as it finds applications in different engineering

fields such as drawing, continuous stretching, glass blowing, paper production etc. Sakiadis [13] pioneered research on flow over a moving sheet in the immediate vicinity of a solid surface which led to the formulation of 2-D mathematical model. The theory proposed by him [13] was validated experimentally by Tsou et al. [14]. Mustafa et al. [15] estimated the impact of fluid flow of a nanofluid near the flow surface where the velocity of the fluid becomes zero. Since then, many researchers [16, 17, 18, 19] have focused on flow on a stretching sheet with different properties in the presence of additional effects.

New experimental work has been conducted recently by Suresh et al. [20] and Momin [21] on enhancing the thermal conductivity of a base fluid. It found that the mixture of two different types of nanoparticles (hybrid nanofluid) disseminated in a base fluid, improves the heat capacity of the principle fluid. For instance, Al_2O_3 exhibits more stability and chemical inertness, but shows lower thermal conductivity with respect to metallic nanoparticles. Furthermore, metallic nanoparticles like aluminum, zinc and copper possess high thermal conductivities. This hybrid nanofluid finds more applications in various engineering fields such as nuclear safety, micro fluidics, manufacturing, transportation, military, pharmaceutical, naval structures, acoustics, buildings or cooling of flush-mounted electronic heaters in modern electronic devices and supercomputers. Many researchers have shown interest due to the above mentioned benefits of hybrid nanofluids for developing nanotechnology. Mechanical and thermal properties can be significantly improved by adding nano composites to materials as demonstrated by Niihara [22]. The two-phase mixture model with forced convective heat transfer of nanofluid to enhance heat transfer was numerically analyzed by Nuim Labib et al. [23]. Mehdi Bahiraei et al. [24] explored that the merit of using the nanofluid in the liquid blocks is greater than pure water. Further, Mehdi Bahiraei et al. [25] found that employing nanofluid enhanced heat sink performance, and the flow experienced higher pumping power at higher Reynolds numbers and concentrations. The numerical investigation is conducted on hydrothermal attributes and energy efficiency of a new graphene−platinum nanofluid in tubes fitted with different twisted tapes by Mehidi et al. [26]. Also, many researchers have shown that the heat capacity of a fluid has been enhanced by considering a single and hybrid nano-additives in their work [27, 28, 29, 30].

The above literature survey demonstrates that researchers are interested in improving the thermal conductivity of a base fluid with the new technique of hybrid nanofluid without considering variable viscosity. Hence, we decided to analyze the effect of temperature dependent viscosity along with free convection and magnetic parameters on heat transfer enhancement in a boundary layer region with the aid of a hybrid nanofluid.

2. Model

We considered a steady two-dimensional MHD boundary layer flow of hybrid nano-fluid past a stretching sheet. The x-axis is chosen in the direction of the sheet motion, and the $y-$axis is normal to it. The wall is impermeable, as portrayed in Fig. 1. The impact of temperature $-$dependent viscosity along with natural convection has been integrated in the momentum equation. Using the foregoing assumptions, the prevailing equations of the current problem take the following form.

$$\frac{\partial a_1}{\partial x_1} + \frac{\partial b_1}{\partial x_2} = 0 \tag{1}$$

$$a_1 \frac{\partial a_1}{\partial x_1} + b_1 \frac{\partial b_1}{\partial x_2} = \frac{1}{\rho_{hnf}} \left[\mu_{hnf} \frac{\partial^2 a_1}{\partial x_2^2} + \frac{\partial a_1}{\partial x_2} \frac{\partial}{\partial x_2} \left(\mu_{hnf} \right) \right] - \frac{\sigma B_0^2 a_1}{\rho_{hnf}} + g \beta_{hnf} (T_2 - T_\infty) \tag{2}$$

$$a_1 \frac{\partial T_2}{\partial x_1} + b_1 \frac{\partial T_2}{\partial x_2} = \frac{K_{hnf}}{(\rho Cp)_{hnf}} \frac{\partial^2 T_2}{\partial x_2^2} \tag{3}$$

where (a_1 , b_1) - velocity components along (x_1 , x_2) axis respectively, ρ_{hnf}- density of hybrid nanofluid, μ_{hnf}- viscosity of hybrid nanofluid, B_0^2-induced magnetic field, g- acceleration due to gravity, β_{hnf}- co-efficient of thermal expansion of hybrid nanofluid, T_2- temperature of fluid, T_∞- ambient temperature, K_{hnf}- thermal conductivity of hybrid nanofluid, $(\rho Cp)_{hnf}$- heat capacity of hybrid nanofluid.

The viscosity of the fluid is assumed to be varying with temperature and is defined as

$$\mu_f = \mu_0 e^{-B\theta(\eta)} \tag{4}$$

where μ_0-reference viscosity of fluid, B-varible viscosity parameter. In general, $B > 0$ for liquids and $B < 0$ for gases.
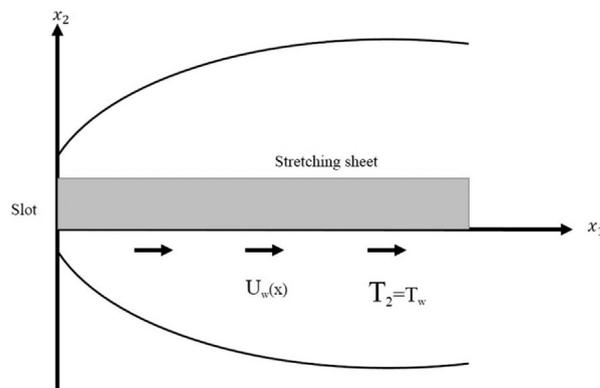

Fig. 1. Physical geometry of the problem.

The boundary conditions for the current problem are:

$$a_1 = U_w = cx_1 \, x_2 = 0, \, T_2 = T_w \text{ at } x_2 = 0$$
$$a_1 = 0, \, T_2 = T_\infty \text{ as } x_2 \to \infty \tag{5}$$

where U_w is velocity of stretching sheet, T_w is temperature of the wall.

Also, n $= 3$ is for spherical nanoparticles. The basic thermo physical properties of nanofluid at 25°C and particles are drawn from various standard literatures and are given in Tables 1 and 2 [29,30].

The following similarity transformation along with Eq. (4) have been employed in Eqs. (1), (2), and (3) to convert the non-linear partial differential equations into a pair of highly non-linear ordinary differential equations.

$$a_1 = cx_1 f'(\eta), \, b_1 = -\sqrt{cv_f} f(\eta), \, \eta = \sqrt{c/v_f} x_2 \, \theta(\eta) = \frac{(T_2 - T_\infty)}{(T_w - T_\infty)} \tag{6}$$

The converted highly coupled non-linear ordinary differential equations are of the form:

$$\left[(1 - \emptyset_2) \left\{ (1 - \emptyset_1) + \emptyset_1 \left(\frac{\rho_{s1}}{\rho_f} \right) \right\} + \emptyset_2 \left(\frac{\rho_{s2}}{\rho_f} \right) \right] * \left[(1 - \emptyset_1)^{2.5} (1 - \emptyset_2)^{2.5} \right]$$
$$* \left[f'(\eta)^2 - f''(\eta) f(\eta) - \left[(1 - \emptyset_2) * \left\{ (1 - \emptyset_1) + \emptyset_1 \left(\frac{\beta_{s1}}{\beta_f} \right) \right\} + \emptyset_2 \left(\frac{\beta_{s2}}{\beta_f} \right) \right] * \lambda \theta(\eta) \right]$$
$$- f'''(\eta) + B\theta'(\eta) f''(\eta) + Mf'(\eta)(1 - \emptyset_1)^{2.5} (1 - \emptyset_2)^{2.5} = 0. \tag{7}$$

$$\left[(1 - \emptyset_2) \left\{ (1 - \emptyset_1) + \emptyset_1 \left(\frac{(\rho Cp)_{s1}}{(\rho Cp)_f} \right) \right\} + \emptyset_2 \left(\frac{(\rho Cp)_{s2}}{(\rho Cp)_f} \right) \right] * k_f \text{Pr} f(\eta) \theta'(\eta)$$
$$+ K_{hnf} \theta''(\eta) = 0 \tag{8}$$

Also, the boundary condition takes the following form by employing (6):

Table 1. Thermo-physical properties of Al_2O_3, Cu (nanoparticles) and H_2O (base fluid) [29, 30].

Property	Al_2O_3	Cu	H_2O
Density $(kg \cdot m^{-3})$	3970	8933	997.1
Thermal conductivity $(W \cdot K^{-1} \cdot m^{-1})$	40	400	0.6071
Thermal expansion coefficient (K^{-1})	.000051	.000076	.000256
Heat capacitance (JK^{-1})	765	385	4179

Table 2. Thermo physical model.

Properties	Nanofluid Cu-H_2O	Hybrid nanofluid Cu-$Al_2O_3H_2O$
Density $(kg\ m^{-3})$	$\rho_{nf} = (1 - \phi_2)\rho_f + \phi_2\,\rho_s$	$\rho_{hnf} = [(1 - \text{Ø}_2)\{(1 - \text{Ø}_1)\rho_f + \text{Ø}_1\rho_{s1}\} + \text{Ø}_2\rho_{s2}]$
Heat capacity (JK^{-1})	$(\rho Cp)_{nf} = (1 - \phi_2)(\rho Cp)_f + \phi_2(\rho Cp)_s$	$(\rho Cp)_{hnf} = [(1 - \text{Ø}_2)\{(1 - \text{Ø}_1)(\rho Cp)_f + \text{Ø}_1(\rho Cp)_{s1}\} + \text{Ø}_2(\rho Cp)_{s2}]$
Viscosity $(Ns.m^{-2})$	$\mu_{nf} = \dfrac{\mu_f}{(1 - \phi_2)^{2.5}}$	$\mu_{hnf} = \dfrac{\mu_f}{(1 - \text{Ø}_1\ )^{2.5}(1 - \text{Ø}_2\ )^{2.5}}$
Thermal Conductivity $(W.K^{-1}.m^{-1})$	$\dfrac{K_{nf}}{K_f} = \dfrac{K_s + (n - 1)K_f - (n - 1)\phi_2(K_f - K_s)}{K_s + (n - 1)K_f + \phi_2(K_f - K_s)}$	$\dfrac{K_{hnf}}{K_{bf}} = \dfrac{K_{s2} + (n - 1)K_{bf} - (n - 1)\phi_2(K_{bf} - K_{s2})}{K_{s2} + (n - 1)K_{bf} + \phi_2(K_{bf} - K_{s2})}$ Where $\dfrac{K_{bf}}{K_f} = \dfrac{K_{s1} + (n - 1)K_f - (n - 1)\phi_1(K_f - K_{s1})}{K_{s1} + (n - 1)K_f + \phi_1(K_f - K_{s1})}$
Thermal Expansion Coefficient (K^{-1})	$\beta_{nf} = (1 - \phi_2)\beta_f + \phi_2\beta_s$	$\beta_{hnf} = [(1 - \text{Ø}_2)\{(1 - \text{Ø}_1)\beta_f + \text{Ø}_1\rho\beta_{s1}\} + \text{Ø}_2\beta_{s2}]$

where ρ_{nf}- density of nanofluid, ρ_f- density of fluid, ρ_s- density of solid, ρ_{s1}- density of alumina, ρ_{s2}- density of copper, K_f- thermal conductivity of fluid, K_{nf}- thermal conductivity of nanofluid, K_{s1}- thermal conductivity of alumina, K_{s2}- thermal conductivity of copper, β_f- thermal expansion coefficient of fluid, β_{nf}- thermal expansion coefficient of nanofluid, β_{s1}- thermal expansion coefficient of alumina, β_{s2}- thermal expansion coefficient of copper, Ø_1- volume fraction of alumina, Ø_2- volume fraction of copper.

$$f(0) = 0,\ \ f'(0) = 1,\ \ \theta(0) = 1,\ \ f'(\infty) = 0,\ \theta(\infty) = 0 \tag{9}$$

where $Pr = \dfrac{(C_p)_f\mu_f}{K_f}$ is Prandtl number, $M = \dfrac{\sigma B_0^2}{c\rho_f}$ is magnetic parameter, $\lambda = \dfrac{g\beta_f(T_w - T_\infty)}{c^2 x} -$ is convection parameter.

The physical quantities of interest, local skin friction coefficient C_f & local Nusselt number N_{ux} are given by

$$C_f = \frac{\mu_{hnf}}{\rho_f u_w^2}\left(\frac{\partial a_1}{\partial x_2}\right)_{x_2=0} \tag{10}$$

$$Nu_x = \frac{x_1 k_{hnf}}{k_f(T_w - T_\infty)}\left(-\frac{\partial T_2}{\partial x_2}\right)_{x_2=0} \tag{11}$$

with the aid of similarity variables, Eqs. (10) and (11) are reduced as

$$Re_x^{\frac{1}{2}}C_f = \frac{1}{(1 - \text{Ø}_1\ )^{2.5}(1 - \text{Ø}_2\ )^{2.5}}f''(0)$$

$$Re_x^{-\frac{1}{2}}Nu_x = -\frac{k_{hnf}}{k_f}\theta'(0)$$

where $Re_x = \frac{u_w x}{\nu_f} -$ Reynolds number.

3. Calculation

The Runge-Kutta-Fehlsberg 45th-order scheme is applied to solve the highly nonlinear Eqs. (7) and (8) with the prescribed boundary condition (9). The comparison results are shown in Table 3. The obtained outcomes are in excellent agreement with the previously published results of Khan and Pop [31], Wang [32], and Gorla and Sidwai [33].

4. Results and discussion

The flawless perception of the physical problem is discussed in this section. The prominent characteristics of the impact of a combination of nanofluids on the temperature augmentation are evaluated through graphical representation of numerical data and tables. Figs. 2, 3, 4, 5, 6, 7, 8, and 9 represent the fluid flow and temperature profiles of nanofluids and hybrid nanofluids for significant parameters such as variable viscosity, convection, magnetic field and solid volume fraction. Further, it is assumed that the concentration of both nano and hybrid nano fluid are same.

Fig. 2 demonstrates the influence of B on $f'(\eta)$. It shows that an increase in B decreases $f'(\eta)$, and hence, the associated thickness of the boundary layer decreases. It infers that the temperature difference between the surface and the ambient fluid is high due to the intensification in B. The effect of B on $\theta(\eta)$ is depicted in Fig. 3. It indicates that the thickness of the thermal boundary layer is greater for greater values of B as a result an increase in the profile $\theta(\eta)$. The impact of λ on $f'(\eta)$ is portrayed in Fig. 4. It conveys that $f'(\eta)$ and the thickness of fluid layer closest to the surface of a solid past in which the fluid flows is increased due to the rise in λ. Physically, augmentation in λ indicates the improvement of convection currents. The effect of an increase in λ decreases $\theta(\eta)$ is exposed in Fig. 5. Enhancement of λ assumes an improvement in the temperature of the fluid or a decline in the hotness of the surface. Physically, the free convection currents are carried away from the plate to the free stream.

The effect of M over $f'(\eta)$ is depicted through Fig. 6. The existence of a charismatic field produces the retarding force which is due to the presence of Lorentz force;

Table 3. Comparison result for temperature gradient $-\theta'(0)$ for the parameter Pr When $\phi_1 = \phi_2 = 0 = B = M = \lambda$.

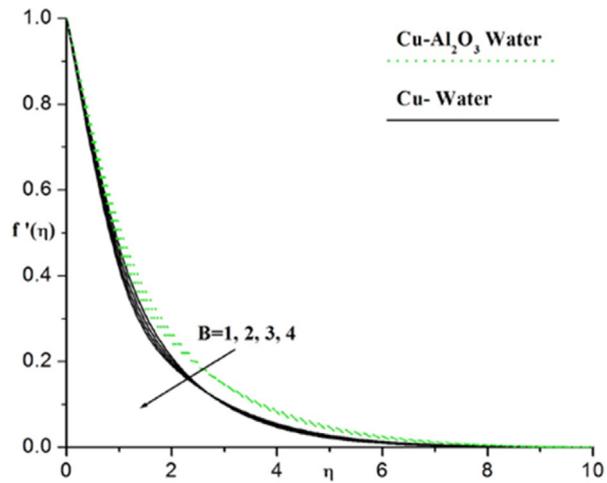| *Pr*	**Khan and Pop [31]**	**Wang [32]**	**Gorla and Sidwai [33]**	**Present study**
2.0	0.9113	0.9114	0.9114	0.91135
6.13	-	-	-	1.75968
7.0	1.8954	1.8954	1.8954	1.89540
20.0	1.3539	1.3539	1.3539	1.35390

Fig. 2. The impact of B on $f'(\eta)$.
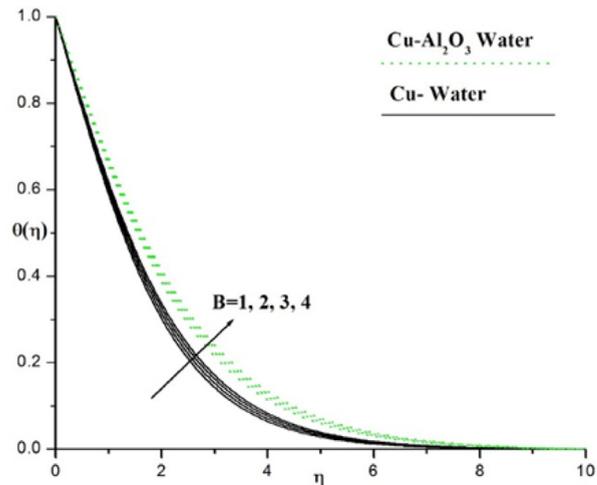
Fig. 3. The impact of B on $\theta(\eta)$.
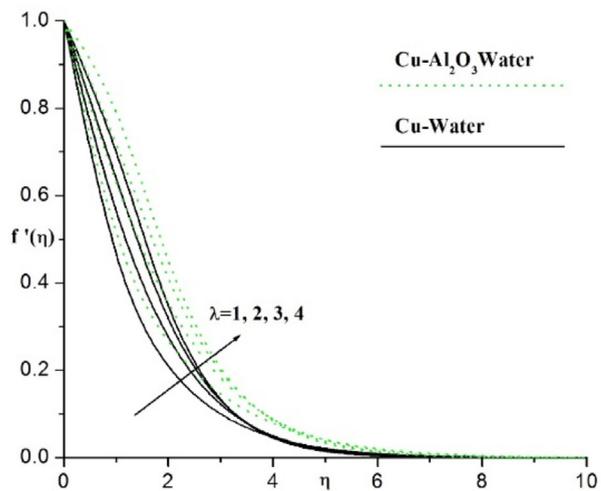
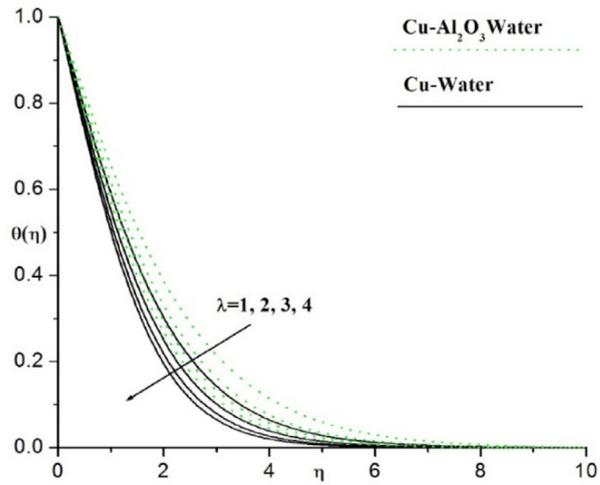**Fig. 4.** The impact of λ on $f'(\eta)$.

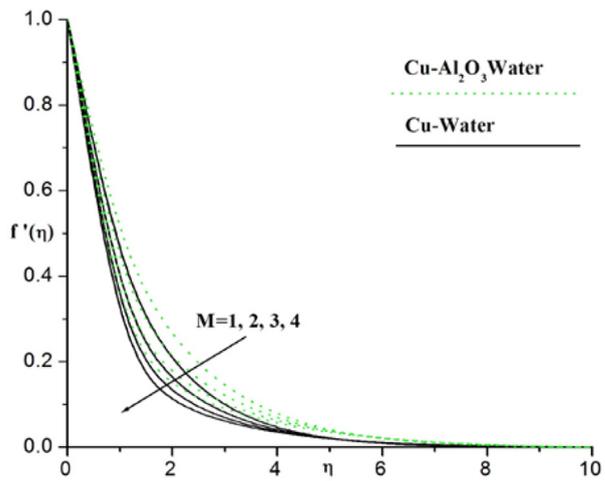**Fig. 5.** The impact of λ on $\theta(\eta)$.

Fig. 6. The impact of M on $f'(\eta)$.
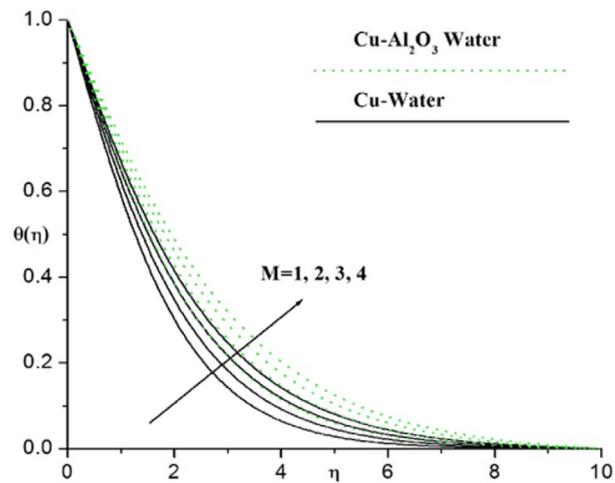
Fig. 7. The impact of M on $\theta(\eta)$.
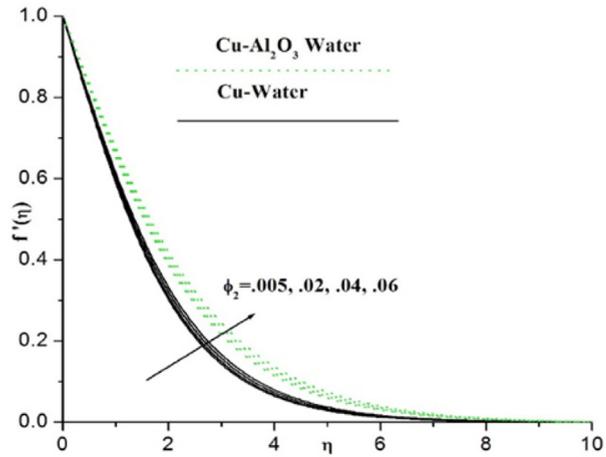
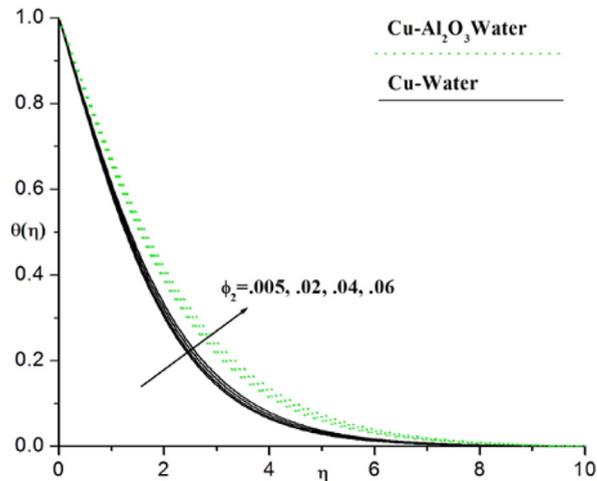**Fig. 8.** The impact of $\emptyset_2$ on $f'(\eta)$.

Fig. 9. The impact of $\emptyset_2$ on $\theta(\eta)$.

hence, there is a decline in moment of the fluid in the x_1 direction along with the momentum boundary layer thickness. Since the charismatic field has the inclination to subdue velocity, it is obvious that the hotness of the fluid increases as does the thermal boundary layer thickness is revealed through Fig. 7. It is observed that in the existence of a charismatic field, the change in the hotness of hybrid nanofluid $(Cu-Al_2O_3/H_2O)$ is greater than that of nanofluid (Cu/H_2O). Hence, we conclude that the anticipated hotness rate can be obtained by appropriate combination of nanoparticle proportions.

The effect of ϕ_2 is demonstrated through Figs. 8 and 9. Fig. 8 indicates that for greater values of ϕ_2, the flow in x_1 direction is quicker for both the nanofluid and the combination of nanofluids. This effect is caused by the collisions of suspended nanoparticles. The temperature distribution for (Cu/H_2O) and $(Cu-Al_2O_3/H_2O)$ is shown in Fig. 9. It is very clear from the figure that hybrid nanofluids has more

thermal conductivity that that of a nanofluids. Physically, this means that the nano-particles produces warmness thus, adding more and different nanoparticles adds energy which increases the temperature and stiffens the thermal boundary layer.

Numerical values of $f''(0)$ and $\theta'(0)$ for B, M, Pr, ϕ_2, and λ for a regular nanofluid and a hybrid nanofluid are presented in Table 4. It is observed that $f''(0)$ decreases for

Table 4. Numerical values of skin-friction coefficient and Nusselt number for variable viscosity and other parameters.

B	M	Pr	ϕ_2	λ	ϕ_1	$f''(0)$		$-\theta'(0)$	
						$Cu:H_2O$	$Cu-Al_2O_3:H_2O$	$Cu:H_2O$	$Cu-Al_2O_3:H_2O$
1	1	.72	0.005	1	0.1	-1.06696	-0.92348	0.47687	0.38516
2						-1.24485	-1.05834	0.4637	0.37673
3						-1.43412	-1.20196	0.45142	0.3688
4						-1.63225	-1.35298	0.44009	0.36139
1	1	.72	0.005	1	0.1	-1.06696	-0.92348	0.47687	0.38516
	2					-1.43143	-1.23538	0.43392	0.35304
	3					-1.73265	-1.49734	0.40092	0.32778
	4					-1.99326	-1.72559	0.3747	0.30751
1	1	.72	0.005	1	0.1	-1.06696	-0.92348	0.47687	0.38516
		2				-1.34491	-1.15886	0.84846	0.67937
		6				-1.76243	-1.52696	1.5865	1.26739
		10				-1.9842	-1.72937	2.11588	1.6921
1	1	.72	0.005	1	0.1	-1.06696	-0.92348	0.47687	0.38516
			.02			-1.04136	-0.90559	0.46713	0.37694
			.04			-1.01053	-0.88385	0.45423	0.3662
			.06			-0.983	-0.86421	0.44152	0.35574
1	1	.72	0.005	1	0.1	-1.06696	-0.92348	0.47687	0.38516
				2		-0.58266	-0.43557	0.53243	0.43418
				3		-0.12865	1.89E-02	0.57217	0.46874
				4		0.30387	0.45035	0.60389	0.49612

superior values of B and M, and the reverse effect occurs for $-\theta'(0)$. This is due to the addition of nanoparticles in the normal fluid. $f''(0)$ and $-\theta'(0)$ decreases in the case of augmentation of Pr and ϕ_2. For increasing values of λ, an increasing trend is observed for $f''(0)$. On the other hand, a decreasing trend is noted for $-\theta'(0)$.

5. Conclusions

The impact of temperature dependent viscosity and free convection on MHD boundary layer flow over a stretching sheet is discussed through tables and graphs. Further, we conclude that the impact of combination of different nanofluids yields more heat capacity than normal fluid. The most noteworthy findings are as follows:

1. The fluid dynamic viscosity parameter B decreases the boundary layer thickness of both regular and hybrid nanofluids flows.

2. An increase in variable viscosity B strikingly increases the temperature of the hybrid nanofluid $Cu - Al_2O_3/H_2O$.

3. The momentum boundary layer thickness is increased by a rise in convection parameter λ.

4. An increase in natural convection parameter λ carries away the free convection currents from the plate.

5. An increase in volume fraction ϕ_2 results in the rise of both velocity and temperature of fluid flow.

6. The temperature profile $\theta(\eta)$ rises with a rise in magnetic parameter M.

7. Hybrid nanofluid flow plays a more substantial role in the process of heat transfer than a regular nanofluid flow.

Declarations

Author contribution statement

Manjunatha S: Conceived and designed the experiments.

Ammani Kuttan: Performed the experiments.

Jayanthi S: Wrote the paper.

Ali Chamkha: Contributed reagents, materials, analysis tools or data.

B. J. Gireesha: Analyzed and interpreted the data.

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The authors declare no conflict of interest.

Additional information

No additional information is available for this paper.

References

[1] S.U.S. Choi, Z.G. Zhang, W. Yu, F.E. Lockwood, E.A. Grulke, Anomalously thermal conductivity enhancement in nano-tube suspensions, Appl. Phys. Lett. 79 (2001) 2252−2254.

[2] H. Masuda, A. Ebata, K. Teramea, N. Hishinuma, Altering the thermal conductivity and viscosity of liquid by dispersing ultra-fine particles, Netsu Bussei 7 (4) (1993) 227−233.

[3] H.A. Minsta, G. Roy, C.T. Nguyen, D. Doucet, New temperature dependent thermal conductivity data for water-based nanofluids, Int. J. Therm. Sci. 48 (2009) 363−371.

[4] S.J. Kim, I.C. Bang, J. Buongiorno, L.W. Hu, Study of pool boiling and critical heat flux enhancement in nanofluids, Bull. Pol. Acad. Sci. Tech. Sci. 55 (2) (2007) 211−216. bulletin.pan.pl/(55-2)211.pdf.

[5] S.J. Kim, I.C. Bang, J. Buongiorno, L.W. Hu, Surface wettability change during pool boiling of nanofluids and its effect on critical heat flux, Int. J. Heat Mass Transf. 50 (19−20) (2007) 4105−4116.

[6] J. Boungiorno, L.-W. Hu, S.J. Kim, R. Hannink, B. Truong, E. Forrest, Nanofluids for enhanced economics and safety of nuclear reactors: an evaluation of the potential features issues, and research gaps, Nucl. Technol. 162 (1) (2017) 80−91, 2008.

[7] E. Jackson, Investigation into the Pool-Boiling Characteristics of Gold Nanofluids, M.S. thesis, University ofMissouri-Columbia, Columbia, Mo, USA, 2007.

[8] J. Buongiorno, L.W. Hu, G. Apostolakis, R. Hannink, T. Lucas, A. Chupin, A feasibility assessment of the use of nanofluids to enhance the in-vessel retention capability in light-water reactors, Nucl. Eng. Des. 239 (5) (2009) 941−948.

[9] B. Ammani Kuttan, S. Manjunatha, S. Jayanthy, B.J. Gireesha, M. Archana, Effect of variable viscosity on Marangoni convective boundary layer flow of

nanofluid in the presence of mixed convection, J. Nanofluids 8 (4) (2019) 845−851.

[10] M.A. Sedeek, Finite element method for the effects of chemical reaction, variable viscosity, thermophoresis and heat generation/absorption on a boundary layer hydro magnetic flow with heat and mass transfer over a heat surface, Acta Mech. 177 (1−4) (2005) 1−18.

[11] M.E. Ali, The effect of variable viscosity on mixed convection heat transfer along a vertical moving surface, Int. J. Therm. Sci. 45 (1) (2006) 60−69.

[12] S. Manjunatha, B.J. Gireesha, Effects of variable viscosity and thermal conductivity on MHD flow and heat transfer of a dusty fluid, Ain Shams Eng. J. 7 (2016) 505−515.

[13] B.C. Sakiadis, Boundary layer behaviour on continuous solid surface; the boundary layer equations for two-dimensional and axisymmetric flow, AIChE J. 7 (1961) 26−28. Flow of a dusty fluid 611.

[14] F.K. Tsou, E.M. Sparrow, R.J. Glodstein, Flow and heat transfer in the boundary layer on a continuous moving surface, Int. J. Heat Mass Transf. 10 (1967) 219−235.

[15] M. Mustafa, T. Hayat, I. Pop, S. Asghar, S. Obaidat, Stagnation-point flow of a nanofluid towards a stretching sheet, Int. J. Heat Mass Transf. 54 (2011) 5588−5594.

[16] M.H. Hamza, N.A.C. Sidik, T.L. Ken, R. Mamat, G. Najafi, Factors affecting the performance of hybrid nanofluids: a comprehensive review, Int. J. Heat Mass Transf. 115 (2017) 630−646.

[17] N.A.C. Siddik, M.J. Muhammad, M.A.A.J. Wan, M.A. Isa, A review on preparation methods, stability and applications of hybrid nanofluids, Renew. Sustain. Energy Rev. 80 (2017) 1112−1122.

[18] O.D. Makinde, A. Aziz, Boundary layer flow of a nanofluid past a stretching sheet with a convective boundary condition, Int. J. Therm. Sci. 50 (2011) 1326−1332.

[19] S. Jana, A.S. Khojin, W.H. Zhong, Enhancement of fluid thermal conductivity by the addition of single and hybrid nano-additives, Thermochim. Acta 462 (2007) 45−55.

[20] S. Suresh, K.P. Venkitaraj, P. Selvakumar, andM. Chandrasekar, Synthesis of Al_2O_3-Cu/water hybrid nanofluids using two step method and its thermo physical properties, Colloid. Surf. A Physicochem. Eng. Asp. 388 (1−3) (2011) 41−48.

[21] G.G. Momin, Experimental investigation of mixed convection with water-Al_2O_3 & hybrid nanofluid in inclined tube for laminar flow, IJSTR 2 (2013) 195−202.

[22] K. Niihara, New design concept of structural ceramics-ceramic nano composites, J. Ceram. Soc. Jpn. 99 (1991) 974−982.

[23] M. Nuim Labib, Md. J. Nine, Handry Afrianto, Hanshik Chung, Hyomin Jeong, Numerical investigation on effect of base fluids and hybrid nanofluid in forced convective heat transfer, Int. J. Therm. Sci. 71 (2013) 163−171.

[24] Mehdi Bahiraei, Saeed Heshmatian, Efficacy of a novel liquid block working with a nanofluid containing graphene nanoplatelets decorated with silver nanoparticles compared with conventional CPU coolers, Appl. Therm. Eng. 127 (2017) 1233−1245.

[25] Mehdi Bahiraei, Mohammad Jamshidmofid, Marjan Goodarz, Efficacy of a hybrid nanofluid in a new microchannel heat sink equipped with both secondary channels and ribs, J. Mol. Liq. 273 (2019) 88−98.

[26] Mehdi Bahiraei, Nima Mazaheri, Seyed Mohammadhossein Hassanzamani, Efficacy of a new graphene−platinum nanofluid in tubes fitted with single and twin twisted tapes regarding counter and co-swirling flows for efficient use of energy, Int. J. Mech. Sci. 150 (2019) 290−303.

[27] Mehdi Bahiraei, Saeed Heshmatian, Thermal performance and second law characteristics of two new microchannel heat sinks operated with hybrid nanofluid containing graphene−silver nanoparticles, Energy Convers. Manag. 168 (2018) 357−370.

[28] Mehdi Bahiraei, Nima Mazaheri, Application of a novel hybrid nanofluid containing graphene−platinum nanoparticles in a chaotic twisted geometry for utilization in miniature devices: thermal and energy efficiency considerations, Int. J. Mech. Sci. 138−139 (2018) 337−349.

[29] S. Suriya Uma Devi, S.P. Anjali Devi, Numerical investigation of three-dimensional hybrid Cu−Al2O3/water nanofluid flow over a stretching sheet with effecting Lorentz force subject to Newtonian heating, Can. J. Phys. 94 (5) (2016) 490−496.

[30] Tanzila Hayat, S. Nadeem, Heat transfer enhancement with Ag−CuO/water hybrid nanofluid, Results Phys. 7 (2017) 2317−2324.

[31] W.A. Khan, I. Pop, Boundary-layer flow of a nanofluid past a stretching sheet, Int. J. Heat Mass Transf. 53 (2010) 2477−2483.

[32] C.Y. Wang, Free convection on a vertical stretching surface, J. Appl. Math. Mech. 69 (1989) 418−420.

[33] R.S.R. Gorla, I. Sidawi, Free convection on a vertical stretching surface with suction and blowing, Appl. Sci. Res. 52 (1994) 247−257.