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Magnetohydrodynamics Boundary Layer Slip Casson Fluid Flow Over a Dissipated Stretched Cylinder

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Abstract: Magnetohydrodynamics (MHD) boundary layer slip Casson fluid flow over a dissipated moving cylinder is explored. Casson fluid model is employed as a non-Newtonian material that demonstrates the phenomenon of yield stress. Blood material is considered to be an example of Casson liquid. The non-linear partial differential quantities are transformed into expressions of ordinary derivatives through transformation of similarity variables. These equations are computed for numeric solutions by using Runge-Kutta method along with shooting scheme. The impact of pertinent constraints on the fluid velocity and temperature are examined through graphs. The coefficient of the skin friction and the rate of heat transfer are found numerically. Comparing of the present study with the earlier results is also presented. We observed that the coefficient of skin friction increases for higher values of Hartmann number.

Nomenclature

υ and u are the components of velocity in the r and x directions	$\alpha = \sqrt{\frac{\nu l}{u_0 R^2}}$ is the constraint of curvature
β is the Casson constraint	$M^2 = \frac{\sigma B_0^2 l}{\rho u_0}$ is the Hartmann number
σ is the constant electrical-conductivity	$\Pr = \frac{\mu c_p}{k}$ is the Prandtl number
B_0 is the applied magnetic strength	$Ec = \frac{U_w^2}{c_p T_\infty}$ is the Eckert number
ρ is the density of fluid	$\gamma = h_s\left(\sqrt{\frac{\nu l}{u_0}}\right)$ is the parameter of Newtonian heating
k is the temperature conductivity	$S_1 = L\sqrt{\frac{u_0}{\nu l}}$ is the constraint of velocity slip.
c_p is the specific heat	$\mathrm{Re}_x = \frac{u_0 x^2}{\nu l}$ signifies the local Reynolds number.
μ_B is the plastic-dynamic viscosity	
ν is the kinematic viscosity	
$U_w(x)$ is the stretching velocity	
h_s is the coefficient of heat transfer	
T is the fluid temperature	
T_∞ is the ambient temperature	
l is the characteristic length and	
L is the velocity-slip factor	

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Introduction

Dissipated Cylinder nothing but stretched Cylinder. Non- Newtonian fluids play key role in various industrial processes such as natural products, multiphase mixtures, biological fluids, food products, agricultural and daily food wastes. Due to its highly utility and wide range of applications in industries researchers have been interest to explore the characteristics of non-Newtonian fluids. The non-Newtonian fluids have nonlinear relationship between stress and rate of strain. Casson fluid is one of the non-Newtonian fluid. Human blood is taken as example of Casson fluid. The MHD flows with heat transportation induced by dissipated moving cylinder or plates have considerable importance of applications in industries and engineering like petroleum industry, MHD power-generators, plasma studies, extractions of geothermal energy etc. Such flow under condition of Newtonian heating, reactions of heterogeneous and homogeneous is characterized by Hayat *et al.* [1]. In another research, Hayat *et al.* [2] reported the MHD Casson liquid flow over moving cylinder. Rehman *et al.* [3] modeled the numerical results of magnetohydrodynamic dually convected Casson fluid flow past a stretched cylinder. Shalini *et al.* [4] examined the radiation effect in hydromagnetic Williamson fluid filled in porous medium with generation of heat. Theory of Cattaneo and Christov for heat diffusion in moving flow of Casson fluid is addressed by Makinde *et al.* [5]. Muhammad *et al.* [6] reported the Powell-Eyring radiated magneto-nanofluid under dual stratification and chemical reactions. Rehman *et al.* [7] investigated the thermal stratified non-Newtonian fluid by applying shooting method. Khan *et al.* [8] examined multiple slip aspects in MHD SA-Cu and SA-Al_2O_3 non-Newtonian nano-materials under radiative chemical reactions. Gireesha *et al.* [9] determined the radiation enhancement in Casson fluid flows with submersion of liquid-particle. Madaki *et al.* [10] described the aspects of transpiration in MHD Cattaneo-Christov flow over moving cylinder. Rehman *et al.* [11] discussed the aspects of dual stratifications in non-Newtonian material flow past a cylindrical sheet. Hayat *et al.* [12] reported the homogeneous and heterogeneous reaction in MHD micropolar fluid induced by curved sheet. Awais *et al.* [13] addressed the features of magneto Sisko fluid flow past a stretched cylinder. Ahmed *et al.* [14] discussed the combined phenomenon of heat transportation and entropy generation in nanofluid flows. Radiative MHD Casson nano-fluid flow induced by cylinder is reported by Reddy *et al.* [15]. Khan *et al.* [16] elaborated the characteristics of thermal slip, radiation and suction in flow of viscous fluid generated by non-linear movement of cylinder. Mukhopadhyay and Ishak [17] and Mukhopadhyay [18] reported the mixed convicted flows along under slip conditions induced by the stretching of cylinder. Hayat *et al.* [19] analyzed temperature-dependent stratification in Casson fluid flow. Khan *et al.* [20] developed the model of Casson fluid under homogeneous reaction and heterogeneous reaction. Ramadevi *et al.* [22] discussed cross-diffusion effects on MHD Cattaneo-Christov flow of Casson fluid past a convective linear/non-linear stretching sheet. Anantha Kumar *et al.* [23] developed impact of brownian motion and thermophoresis on bioconvective flow of nanoliquids past a variable thickness surface with slip effects. Influence of nonlinear radiation and cross diffusion on MHD flow of Casson and walters-B nanofluids past a variable thickness sheet was reported by Bhagya Lakshmi *et al.* [24].

The present study deals with MHD slip flow of Casson fluid over dissipated moving cylinder. The aspects of pertinent governing parameters on temperature and fluid velocity are depicted graphically. The skin-friction coefficients and the rate of heat transfer are found numerically.

Formulation of Problem

Consider steady-state MHD Casson fluid flow over a dissipated moving cylinder. The magnetic field is uniform of intensity B_0 acting in the radial way. The fluid flow in the x-direction and the radial direction is perpendicular to it is shown in Figure 1.The cylinder has linear stretching velocity.

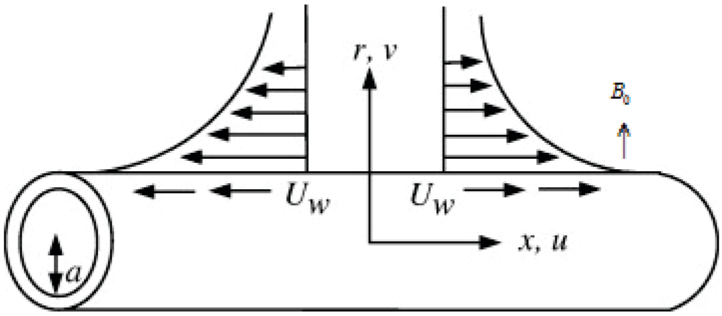

Figure 1 Physical model of the problem.

The boundary layer equations for continuity, momentum and heat transfer are

$$\frac{\partial(ru)}{\partial x}+\frac{\partial(r\upsilon)}{\partial r}=0 \quad (1)$$

$$u\frac{\partial u}{\partial x}+\upsilon\frac{\partial u}{\partial r}=\nu\left(1+\frac{1}{\beta}\right)\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}\right)-\frac{\sigma B_0^2}{\rho}u \quad (2)$$

$$u\frac{\partial T}{\partial x}+\upsilon\frac{\partial T}{\partial r}=\frac{k}{\rho c_p}\left(\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}\right)+\frac{\mu_B}{\rho c_p}\left(1+\frac{1}{\beta}\right)\left(\frac{\partial u}{\partial r}\right)^2+\frac{\sigma B_0^2 u^2}{\rho c_p}. \quad (3)$$

The imposed boundary conditions are

$$\left.\begin{aligned}&u(x,r)=U_w(x)=\frac{u_0 x}{l}+L\left(1+\frac{1}{\beta}\right)\frac{\partial u}{\partial r},\ \upsilon(x,r)=0,\frac{\partial T}{\partial r}=-h_s T\ \ at\ r=R,\\&u\to 0, T\to T_\infty\ as\ r\to\infty\end{aligned}\right\} \quad (4)$$

Employing the following transformations

$$\eta=\sqrt{\frac{u_0}{\nu l}}\left(\frac{r^2-R^2}{2R}\right), u=\frac{u_0 x}{l}f'(\eta), \upsilon=-\frac{R}{r}\sqrt{\frac{u_0\nu}{l}}f(\eta), \theta=\frac{T-T_\infty}{T_\infty}. \quad (5)$$

Equation (1) is satisfied automatically while equations (2)-(4) are

$$\left(1+\frac{1}{\beta}\right)\left((1+2\alpha\eta)f'''+2\alpha f'\right)+ff''-f'^2-M^2 f'=0 \quad (6)$$

$$(1+2\alpha\eta)\theta''+\Pr EcM^2 f'^2+\Pr Ec(1+2\alpha\eta)\left(1+\frac{1}{\beta}\right)f''^2+2\alpha\theta'+\Pr f\theta'=0 \quad (7)$$

and the boundary conditions are

$$\left.\begin{aligned}&f(0)=0, f'(0)=1+S_1\left(1+\frac{1}{\beta}\right)f''(0), \theta'(0)=-\gamma\left(1+\theta(0)\right)\ \ at\ \ \eta=0,\\&f'\to 0, \theta\to 0\ \ when\ \ \eta\to\infty,\end{aligned}\right\} \quad (8)$$

The coefficient of skin friction and the rate of heat transfer are represents as follows

$$C_f = \frac{2\tau_w}{\rho u_w^2},\ Nu_x = \frac{xq_w}{k\left(T - T_\infty\right)} \text{ with } \tau_w = \mu\left(1+\frac{1}{\beta}\right)\left(\frac{\partial u}{\partial r}\right)_{r=R},\ q_w = -k\left(\frac{\partial T}{\partial r}\right)_{r=R}. \tag{9}$$

The coefficients of skin-factors and the rate of heat transportation in dimensionless form are

$$\frac{1}{2}C_f \,\mathrm{Re}_x^{1/2} = \left(1+\frac{1}{\beta}\right)f''(0),\quad Nu_x \,\mathrm{Re}_x^{-1/2} = \gamma\left(1+\frac{1}{\theta(0)}\right) \tag{10}$$

Results and Discussion

The present study deals with MHD slip boundary layer Casson fluid over a dissipated moving cylinder. The effects of pertinent parameters on fluid flow and temperature are examined both with slip condition and without slip condition. The coupled nonlinear expressions are tackled with help of Runge-Kutta scheme along with shooting procedure.

The effect of Casson parameter β on velocity $f'(\eta)$ is displayed in Figure 2.We revealed that the velocity and thickness of boundary-layer thickness decay for enhancing the non-Newtonian Casson constraint β. This reduction causes for higher values of Casson fluid constraint that corresponds reduction in factor of yield stress. The role of Hartmann number M on velocity $f'(\eta)$ and temperature $\theta(\eta)$ is displayed in Figures 3 and 4. It is found that $f'(\eta)$ decreases via higher values of Hartmann number. Since Lorentz force becomes stronger and higher resistance is observed for fluid flow and consequently velocity reduces and opposite behavior in temperature. The impact of velocity slip constraint S_1 on the fluid velocity $f'(\eta)$ is demonstrated in Figure 5. We found that $f'(\eta)$ and thickness of boundary-layer decreases with higher values of velocity slip constraint S_1. Here an increment in velocity slip constraint S_1 retarded the liquid velocity in boundary-layer. Figure 6 shows the influence of Prandtl number Pr on temperature distribution $\theta(\eta)$. It is observed that $\theta(\eta)$ and the temperature thickness of boundary-layer reduced for higher values of Prandtl number Pr.

Figure 7 signifies the variations of Eckert number Ec on temperature distribution $\theta(\eta)$. It is noted that $\theta(\eta)$ and its related thickness of temperature boundary-layer thickness increases for higher Eckert number. Eckert number defined as the ratio of advective heat transfer to heat dissipation potential. The work done against the viscous fluid stresses. Since increases the Eckert number Ec corresponds to more active liquid particles due to energy storage. The characteristics of the Newtonian heating γ on $\theta(\eta)$ are visualized in Figure 8. We revealed that curves of $\theta(\eta)$ are boosted with increasing γ. The conjugate heat transport coefficient h_s is stronger for larger γ. The impact of Hartmann number on coefficients of skin-friction factors and rate of heat transportation are displayed in Table 1 and Table 2. From Table1 we notice that the coefficient of skin-friction increases for larger M. The effects of $\alpha, M, \mathrm{Pr}, Ec$ and γ on $Nu\,\mathrm{Re}_x^{-1/2}$ are displayed in Table 2. We have seen that for higher values of γ, Pr and α, $Nu\,\mathrm{Re}_x^{-1/2}$ increases whereas Nusselt number reduces when M and Ec are increases. For special cases, the comparison of the existing results of Hayat *et al.* [2] and Fang *et al.* [21] shows good agreement between the results.

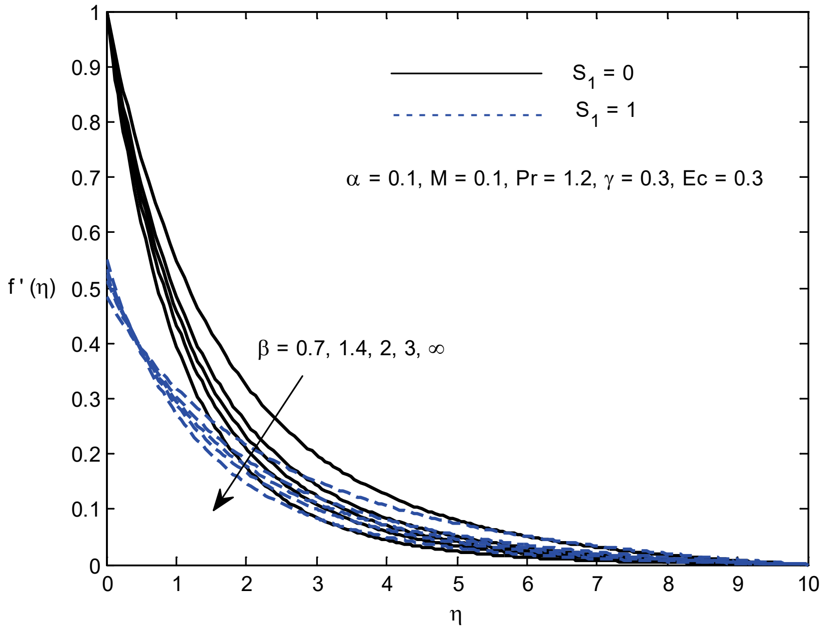

Figure 2 The velocity $f'(\eta)$ for distinct values of β.

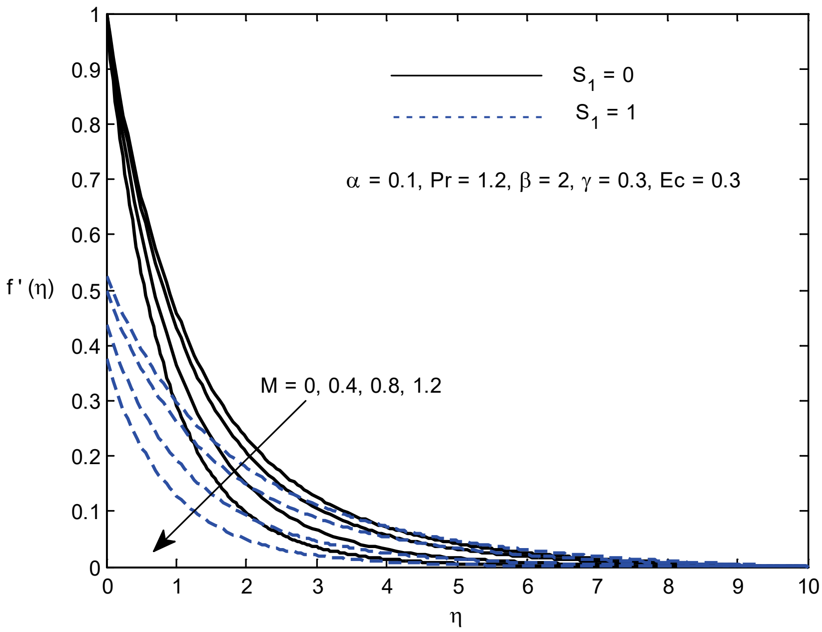

Figure 3 The velocity $f'(\eta)$ for distinct values of M.

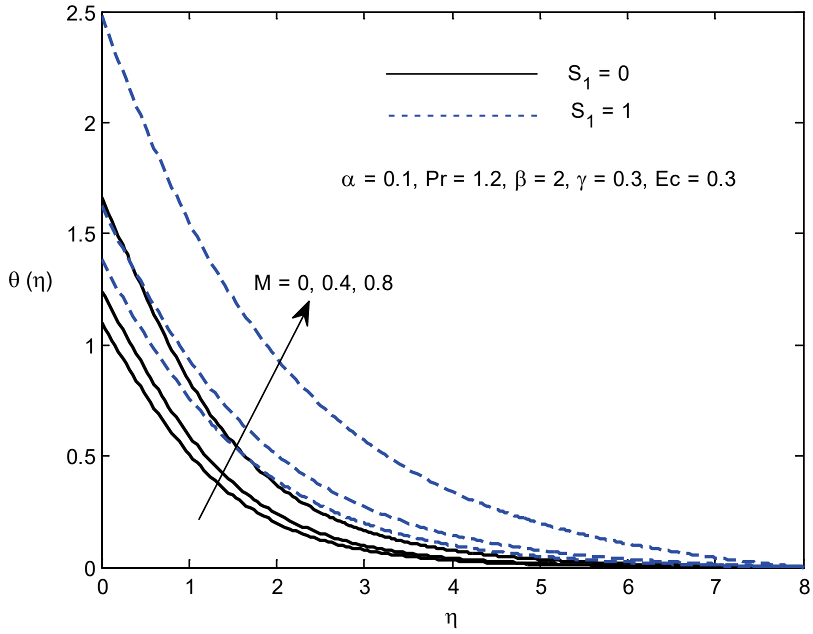

Figure 4 The temperature $\theta(\eta)$ for distinct values of M.

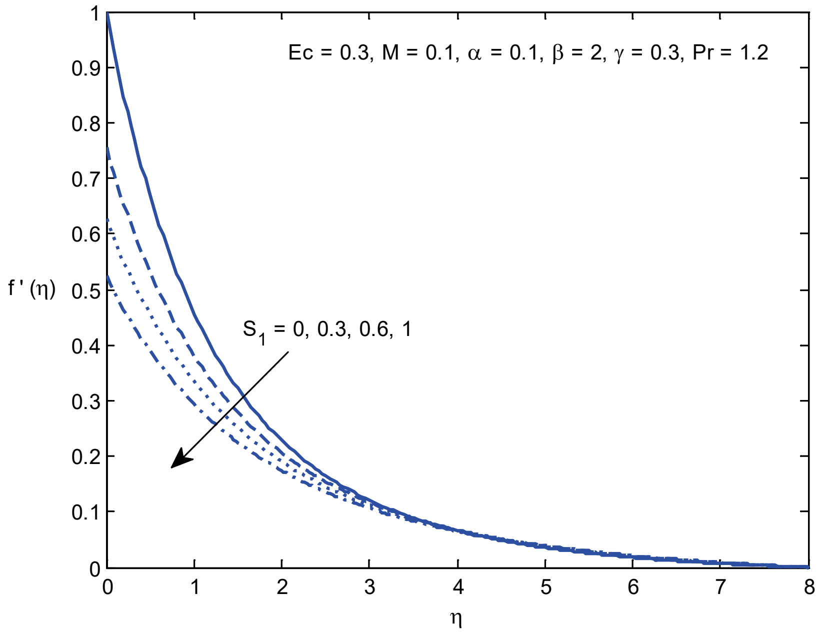

Figure 5 The velocity $f'(\eta)$ for distinct values of S_1.

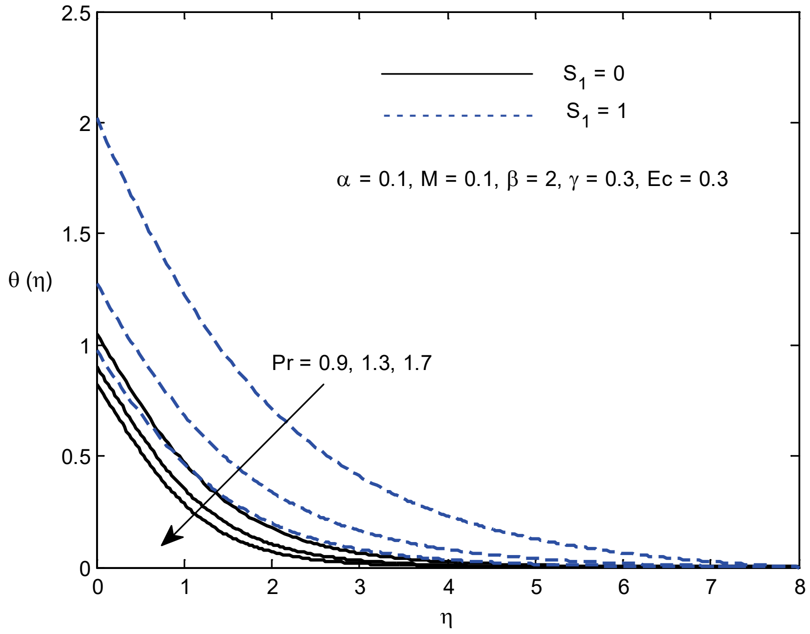

Figure 6 The temperature $\theta(\eta)$ for distinct values of Pr.

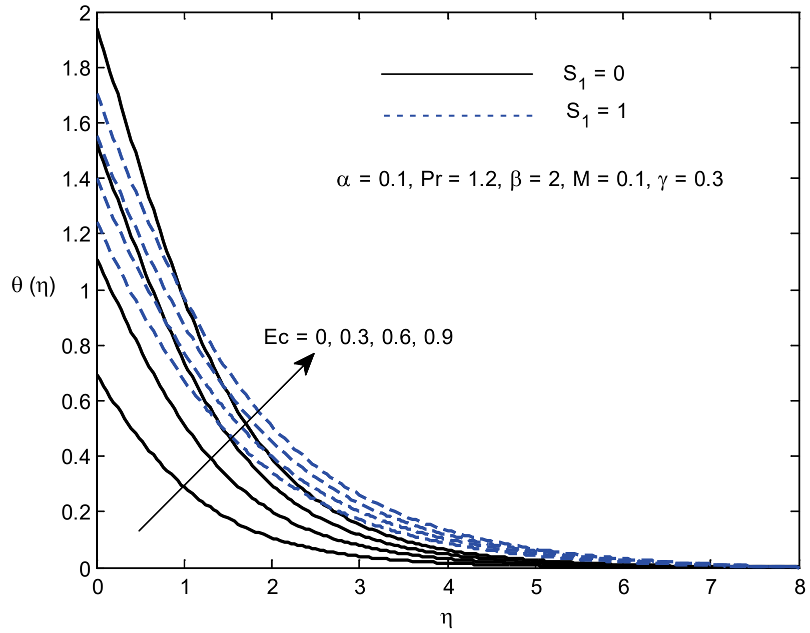

Figure 7 The temperature $\theta(\eta)$ for distinct values of Ec.

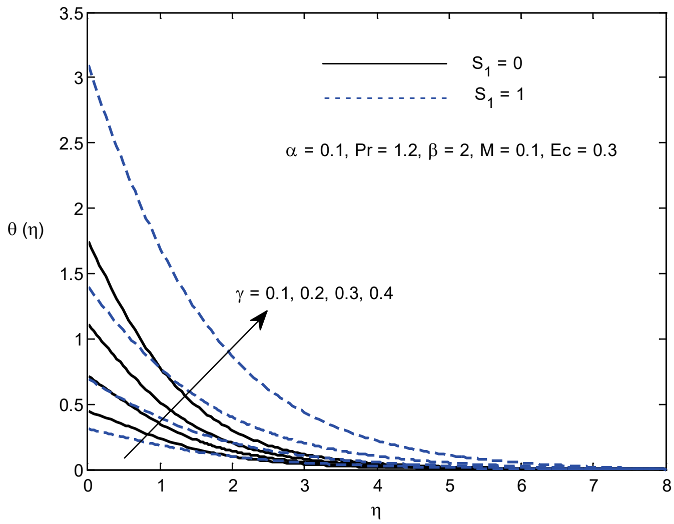

Figure 8 The temperature $\theta(\eta)$ for distinct values of γ.

Table 1 Comparative analysis for $-\left(1+\frac{1}{\beta}\right)f''(0)$ with Hayat *et al.* [2] and Fang *et al.* [21] for distinct values of M when $\beta \to \infty, \alpha = 0 = S_1$.

M	$-f''(0)$ Fang *et al.* [21]	$-f''(0)$ Hayat *et al.*[2]	Present results ($\beta \to \infty, \alpha = 0 = S_1$)
0.0		1.00000	1.0000
0.2		1.01980	1.0198
0.5	1.1180	1.11803	1.1180
0.8		1.26063	1.2606
1.0		1.41421	1.4142

Table 2 Numeric values of $\left(Nu\,\mathrm{Re}_x^{-1/2}\right)$ for distinct values of α, M, Ec, γ and Pr when $\beta = 2, S_1 = 0$.

α	M	Pr	Ec	γ	$Nu\,\mathrm{Re}_x^{-1/2}$ Hayat *et al.*[2]	$Nu\,\mathrm{Re}_x^{-1/2}$ Present result
0.0	0.1	1.2	0.3		0.5504	0.5504
0.1					0.5630	0.5630
0.2					0.5753	0.5753
0.1	0.0				0.5649	0.5649
	0.2				0.5565	0.5565
	0.4				0.5341	0.5341
	0.1	1.3			0.5777	0.5777
		1.4			0.5909	0.5909
		1.5			0.6028	0.6028
		1.2	0.0		0.7257	0.7257
			0.4		0.5330	0.5330
			0.8		0.4604	0.4604
			0.3	0.4	0.6225	0.6225
				0.5	0.6649	0.6649
				0.6	0.6969	0.6969

Conclusions

This research discloses the nature of slip and Newtonian heating phenomenon in magnetohydrodynamics Casson fluid flow generated by the dissipated cylinder. Both no-slip and slip circumstances have been considered and evaluated. The numeric solutions of obtained model are derived with help of Runge-Kutta methodology together with shooting scheme. We revealed that the liquid velocity $f'(\eta)$ is retarded for both no-slip and slip circumstances with an increment in Casson constraint. Here $f'(\eta)$ at wall is less in case of slip factor presence in comparative to no-slip situation. Temperature $\theta(\eta)$ is boosted for the increasing M for both no-slip and slip cases. The presence of Newtonian heating constraint corresponds to stronger temperature. The temperature at wall is more significantly enhanced due to higher values of constraint of Newtonian heating. The numeric values of $Nu\,\mathrm{Re}_x^{-1/2}$ are risen for larger constraint of curvature.

References

[1] T. Hayat, Z. Hussain, A. Alsaedi, M.Farooq, MHD flow by a stretching cylinder Newtonian heating and homogeneous-heterogeneous reactions. PLOS ONE 11(6) (2016) 1-23.

[2] T. Hayat, M. Waqas, M.I.Khan, A.Alsaedi, M. Tamoor, MHD flow of Casson fluid over a stretching cylinder. Results. Phys 7 (2017) 98-502.

[3] K. Rehman, A. Qaiser, M.Y. Malik Ali U, Numerical communication for MHD thermally stratified dual convection flow of Casson fluid yields by stretching cylinder Chin. J. Phys 55(4) (2017) 1605-1614.

[4] S. Jain, A. Parmar, Radiation effect on MHD Williamson fluid flow over stretching cylinder through porous medium with heat source, Appl. Flud Dyns, (2017) 61-78.

[5] O.D. Makinde, V. Nagendramma, Leelarathnam, Effects of Cattaneo-Christov heat flux on Casson nanofluid flow past a stretching cylinder, Def. Diff. Forum, 378 (2017) 28-38.

[6] M. Ramzan, M. Bilal, J.D. Chung, Radiative flow of Powell Eyring magneto nanofluid stretching cylinder with chemical reaction and double stratification near a stagnation point, PLOS ONE. 12(1) (2017) 1-19.

[7] K. Rehman, M.Y. Malik, Application of shooting method on MHD thermally stratified mixed convection flow of non-Newtonian fluid over an inclined stretching cylinder, J. Phys: Conf. Ser. 822(1) (2017) 1-13.

[8] I. Tlili, W.A. Khan, I. Khan, Multiple slip effects on MHD SA-Al_2O_3 and SA-Cu non Newtonian nanofluids flow over a stretching cylinder in porous medium with radiation and chemical reaction, Res. in Phys 8 (2017) 213-222.

[9] G.K. Ramesh, K.G. Kumar, S.A. Shehzad, B.J. Gireesha, Enhancement of radiation on hydro magnetic Casson fluid flow towards a stretched cylinder with suspension of liquid particles, Can. J. Phys. 96(1) (2017) 1-7.

[10] C.S.K. Raju, R.V.M.S.S.K. Kumar, S.V.K. Varma, A.G. Madaki, P.D. Prasad, Transpiration effects on MHD flow over a stretched cylinder with Cattaneo-Christove heat flux with suction/injection, Arab. J. Sci. Eng. 43(5) (2018) 1-8.

[11] K. Rehman, A. A. Khan, M. Y. Malik, Ali U, Temperature and concentration stratification effects on non Newtonian fluid flow past a cylindrical surface, Res. in Phys.7 (2017) 3659-3667.

[12] T. Hayat, R. Sajjad, R. Ellahi, A. Alsaedi, T. Muhammad, Homogeneous heterogeneous reactions in MHD flow of micropolar fluid by a curved stretching surface, J. Mol. Lids. 240 (2017) 209-220.

[13] M. Awais, M. Y. Malik, T. Salahuddin, MHD flow of Sisko fluid near the axisymmetric stagnation point towards a stretching cylinder, Res. Phys, 7 (2017) 49-56.

[14] S.E. Ahmed, M. A. Mansour, A. Mahdy, Heat transfer and entropy generation due to a nanofluid over stretching cylinder: effects of thermal stratification, Comp. Therml. Sci: Int. J. 9(1) (2017) 29-47.

[15] M. Reddy, P. Kumari, Vijaya, Padma, Effect of thermal radiation on MHD Casson nanofluid over a cylinder, J. Nanofluids, 7(3) (2017) 428-438

[16] M. I. Khan, M. Tamoor, T. Hayat, A. Alsaedi, MHD boundary layer thermal slip flow by nonlinearly stretching cylinder with suction/blowing and radiation, Res. Phys., 7 (2017) 1207-1211.

[17] S. Mukhopadhyay, A. Ishak, Mixed convection flow along a stretching cylinder in a thermally stratified medium, J. Appl. Math., 2012 (2012) 1-9.

[18] S. Mukhopadhyay, MHD boundary layer slip flow along a stretching cylinder, Ain Shams Eng. J., 4 (2013) 317-324.

[19] T. Hayat, M. Farooq, A. Alsaedi, Thermally stratified stagnation point flow of Casson fluid with slip conditions. Int. J. Num. Method. Heat. Flud. Flow, 25(4) (2015) 724-748.

[20] M. I. Khan, M. Waqas, T. Hayat, A. Alsaedi, A comparative study of Casson fluid with homogeneous-heterogeneous reactions, J. Colloid. Inter. Sci., 498 (2017) 85-90.

[21] T. Fang, J. Zhang, S. Yao, Slip MHD viscous flow over a stretching sheet an exact solution, Comm. Nonlinear Sci. Num. Simul, 14(11) (2009) 3731-3737.

[22] B. Ramadevi, V. Sugunamma, J.V. Ramana Reddy, Cross-Diffusion Effects on MHD Cattaneo-Christov Flow of Casson Fluid Past a Convective Linear/Non-Linear Stretching Sheet, Def. Diff. Forum, 388 (2018) 96-113.

[23] K. Anantha Kumar, V. Sugunamma, N. Sandeep, Ramana J Reddy, Impact of brownian motion and thermophoresis on bioconvective flow of nanoliquids past a variable thickness surface with slip effects, Mul. Mod. Mat. Strs., 15(1) (2019) 103-132.

[24] K. Bhagya Lakshmi, K. Anantha Kumar, Ramana J Reddy, V. Sugunamma, Influence of nonlinear radiation and cross diffusion on MHD flow of Casson and walters-B nanofluids past a variable thickness sheet, J. Nanofluids, 8(1) (2019) 73-83.