

Hydromagnetic Coupled Heat and Mass Transfer by Natural Convection from a Permeable Constant Heat Flux Surface in Porous Media

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by natural convection in a porous medium for various geometries. Bejan and Khair (1985) reported on the natural convection boundary-layer flow in a saturated porous medium with combined heat and mass transfer. The coupled heat and mass buoyancy-induced inclined boundary layer in a porous medium was studied by Jang and Chang (1988). Later, Lai and Kulacki (1991) extended the problem of Bejan and Khair (1985) to include wall fluid injection effects. Early studies that considered coupled heat and mass transfer without the presence of porous media include the works of Gebhart and Pera (1971) on vertical plate, Pera and Gebhart (1972), and Chen and Yuh (1980) on inclined plates. Recently, Lai (1991) and Yih (1997) studied coupled heat and mass transfer by mixed convection from a vertical plate embedded in a fluid-saturated porous medium. The purpose of this work is to consider hydromagnetic and wall mass transfer effects on coupled heat and mass transfer by natural convection from a semiinfinite plate maintained at a constant heat flux.

PROBLEM FORMULATION

Consider steady, laminar, hydromagnetic coupled heat and mass transfer by natural convection flow along a semiinfinite vertical plate embedded in a fluid-saturated porous medium as shown in Fig. 1. The surface of the plate has a constant heat flux condition and a constant or variable concentration. A magnetic field of uniform strength B_0 is applied in the y -direction that is normal to the plate. A constant fluid suction or blowing is imposed at the plate surface. The fluid is assumed to be Newtonian, electrically conducting, and has constant properties except the density

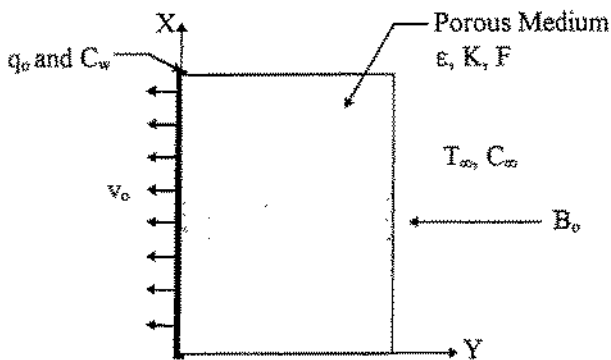


Figure 1. Vertical plate embedded in a fluid-saturated porous medium.

in the buoyancy term of the balance of momentum equation. Also, the porosity and the permeability of the porous medium are assumed to be constant.

The magnetic Reynolds number is assumed to be small so that the induced magnetic field can be neglected. In addition, there is no applied electric field and both of the Hall effect and viscous dissipation are neglected. By invoking the Boussinesq and boundary layer approximations, the governing equations for this problem can be written as

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{1}$$

$$U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = \nu \frac{\partial^2 U}{\partial Y^2} + \beta_T g(T - T_\infty) + \beta_c g(C - C_\infty) - \frac{\sigma B_0^2}{\rho} U - \frac{\nu E}{K} U - F \epsilon^2 U^2 \tag{2}$$

$$U \frac{\partial T}{\partial X} + V \frac{\partial T}{\partial Y} = \alpha_c \frac{\partial^2 T}{\partial Y^2} \tag{3}$$

$$U \frac{\partial C}{\partial X} + V \frac{\partial C}{\partial Y} = D \frac{\partial^2 C}{\partial Y^2} \tag{4}$$

where the parameters are defined in the nomenclature list.

Nondimensionalization of the above equations is obtained by using

$$x = \frac{X}{L}, \quad y = \frac{Y}{L}, \quad u = \frac{UL}{\alpha_c}, \quad v = \frac{VL}{\alpha_c} \tag{5}$$

$$R = \frac{T - T_\infty}{\alpha_c \nu / (g \beta_T L^3)}, \quad c = \frac{C - C_\infty}{\alpha_c \nu / (g \beta_c L^3)}$$

(where L is a characteristic plate length) to give

$$\frac{1}{Pr} \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) + \frac{\epsilon L^2}{K} u + \frac{F \epsilon^2 L}{Pr} u^2 = R + c + \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2 L^2}{\rho \nu} u \tag{6}$$

$$u \frac{\partial R}{\partial x} + v \frac{\partial R}{\partial y} = \frac{\partial^2 R}{\partial y^2} \tag{7}$$

$$u \frac{\partial c}{\partial x} + v \frac{\partial c}{\partial y} = \frac{1}{Le} \frac{\partial^2 c}{\partial y^2} \tag{8}$$

where

$$Pr = \frac{\nu}{\alpha_c} \quad \text{and} \quad Le = \frac{\alpha_c}{D} \tag{9}$$

are the Prandtl number and the Lewis number, respectively.

The appropriate boundary conditions for the problem under consideration are

$$\begin{aligned}
 U(X, 0) = 0, \quad U(X, \infty) = 0, \quad V(X, 0) = -v_0 \\
 -k_e \frac{\partial T(X, 0)}{\partial Y} = q_0, \quad T(X, \infty) = T_\infty, \quad (10) \\
 C(X, 0) = C_w(X), \quad C(X, \infty) = C_\infty
 \end{aligned}$$

where q_0 and $C_w(X)$ are the heat flux at the wall (a constant) and the wall concentration which is allowed to be a function of X so as to avoid singularities at $X = 0$, respectively.

In their work on possible transformation for natural convection over a vertical plate embedded in porous media for prescribed heat flux, Kou and Huang (1996) suggested the following transformations:

$$\begin{aligned}
 \eta = \frac{Q_0^{1/5} y}{x^{1/5}}, \quad \xi = x, \quad f(\xi, \eta) = \frac{\psi(x, y)}{Q_0^{1/5} x^{4/5}} \\
 \theta(\xi, \eta) = \frac{R(x, y)}{Q_0^{2/5} x^{1/5}} \quad (11)
 \end{aligned}$$

With $\zeta = (C - C_\infty)/(C_w - C_\infty)$, and using Eqs. (11), the non-similar equations governing this case become

$$\begin{aligned}
 f''' + \frac{1}{Pr} \left[\frac{4}{5} f f'' - \frac{3}{5} (f')^2 \right. \\
 \left. - \xi \left(f' \frac{\partial f'}{\partial \xi} - f'' \frac{\partial f}{\partial \xi} \right) \right] + \theta + e \zeta \\
 = (A + M) \xi^{2/5} f' + \frac{B}{Pr} \xi (f')^2 \quad (12)
 \end{aligned}$$

$$\theta'' + \frac{4}{5} f \theta' - \frac{1}{5} f' \theta = \xi \left(f' \frac{\partial \theta}{\partial \xi} - \theta' \frac{\partial f}{\partial \xi} \right) \quad (13)$$

$$\frac{1}{Le} \zeta'' + \frac{4}{5} f \zeta' - \frac{1}{5} f' \zeta = \xi \left(f' \frac{\partial \zeta}{\partial \xi} - \zeta' \frac{\partial f}{\partial \xi} \right) \quad (14)$$

with the concentration at the wall being in the following form

$$\frac{C_w - C_\infty}{C_{w0} - C_\infty} = \left(\frac{X}{L} \right)^{1/5} = x^{1/5} \quad (15)$$

for a constant e ($e = c_{w0}/Q_0^{4/5}$) where C_{w0} is a reference value for concentration at the wall. In Eq. (12) through (14),

$$\begin{aligned}
 Q_0 = \frac{q_0}{(k_e \alpha_c \nu)/(g \beta_T L^4)}, \quad c_{w0} = \frac{C_{w0} - C_\infty}{(\nu \alpha_c)/(g \beta_c L^3)} \\
 A = \frac{\epsilon L^2}{K Q_0^{2/5}}, \quad B = F \epsilon^2 L, \quad M = \frac{\sigma B_0^2 L^2}{\rho \nu Q_0^{2/5}} \quad (16)
 \end{aligned}$$

are the dimensionless wall heat flux, dimensionless wall concentration, inverse Darcy number, dimensionless porous medium inertia coefficient, and square of the Hartmann number, respectively. The transformed form of the boundary conditions becomes

$$\begin{aligned}
 f'(\xi, 0) = 0, \quad f(\xi, 0) = V_0 \xi^{1/5}, \\
 \theta'(\xi, 0) = -1, \quad \zeta(\xi, 0) = 1 \\
 f'(\xi, \infty) = 0, \quad \theta(\xi, \infty) = 0, \quad \zeta(\xi, \infty) = 0 \quad (17)
 \end{aligned}$$

where $V_0 = (\nu_0 L)/(\alpha_c Q_0^{1/5})$ is the dimensionless wall mass transfer. Setting e , V_0 , and M to zero and ignoring Eq. (14) produces the transformed equations reported by Kou and Huang (1996) for the problem of natural convection from a vertical impermeable plate maintained at uniform heat flux and embedded in a porous medium.

The skin friction parameter, Nusselt number, and the Sherwood number are important physical parameters. These are given by

$$SFP = \frac{\tau_w}{\mu(L^2/\alpha_c)Q_0^{3/5}} = \xi^{2/5} f''(\xi, 0) \quad (18)$$

$$Nu = \frac{hx}{k_e} = \xi^{4/5} Q_0^{1/5} \frac{1}{\theta(\xi, 0)} \quad (19)$$

$$Sh_s = \frac{h_m x}{D} = -\xi^{4/5} Q_0^{1/5} \zeta'(\xi, 0) \quad (20)$$

The average convective heat transfer and the average mass diffusion coefficient can be computed from the following:

$$\bar{h} = \frac{\int h dx}{\int dx}, \quad \bar{h}_m = \frac{\int h_m dx}{\int dx} \quad (21)$$

Accordingly, Nu_{AVG} and Sh_{AVG} take on the forms

$$Nu_{AVG} = \frac{\bar{h}L}{k_e}, \quad Sh_{AVG} = \frac{\bar{h}_m L}{D} \quad (22)$$

NUMERICAL METHOD

The resulting nonsimilar equations are transformed into system of nonlinear coupled second-order differential equations and then discretized and solved by iterative finite difference implicit technique as discussed by Blottner (1970). The trapezoidal rule is used in between for finding the dependent variable f in the momentum equation. Because rapid changes in the dependent variables are expected near the wall, variable step sizes in the η direction are used with

a starting value of 0.001 at the wall and a growth factor of 1.03 such that $\delta\eta_{n+1} = 1.03\delta\eta_n$. Moreover, a constant step of 0.005 in the ξ direction was selected after performing many trials to assess grid independence. These values produced accurate results with minimum computational efforts. The convergence criterion for this problem required that the difference between the current and the previous iterations be 10^{-8} . A representative set of graphical results is presented in Figs. 2–11 to show the influence of the physical parameters on the solutions.

It should be mentioned here that the above numerical method was employed to solve the nonsimilar equations excluding all of the magnetic and the porous media terms and ignoring the mass diffusion equation. The results of f' , θ , and Nu were found to be in good agreement with the solution of laminar natural convection boundary layer flow along a vertical wall with constant wall heat flux reported by Sparrow (1955), respectively. These comparisons lend confidence in the adequacy and accuracy of the numerical method.

RESULTS AND DISCUSSION

Figures 2 and 3 present representative velocity and temperature profiles at $\xi = 1$ for various values of the square of the Hartmann number M , respectively. It is a known fact that application of a transverse magnetic field normal to the flow direction results in a flow-resistive force called the Lorentz force which acts in the opposite direction of flow. This force has the effect of slowing the motion of the fluid and increasing its temperature as can be seen from

Figs. 2 and 3, respectively. It is also seen from Fig. 3 that the wall temperature increases as the square of Hartmann number increases.

Figures 4 and 5 illustrate the influence of the magnetic parameter (square of the Hartmann number) M on the development of the local skin-friction parameter SFP and the local Nusselt number $Nu^Q (= NuQ_w^{-1/5})$ along the plate, respectively. As a result of the slowing motion of the fluid caused by the presence of the Lorentz force, the wall slope of the velocity profile decreases. This has the direct effect of reducing the wall shear stress represented by SFP. However, the wall temperature increases as M increases. This produces lower Nusselt number values at every point along the plate. These behaviors are depicted by the decreases in the values of SFP and Nu^Q as M increases displayed in Figs. 4 and 5, respectively. It should be mentioned that similar to the Nusselt number, the Sherwood number decreases as M increases. In general, the effect of the porous medium on the velocity and temperature profiles is similar to the effect of magnetic field, as both effects add resistance against the fluid motion.

Figures 6 and 7 illustrate the influence of the suction or injection parameter V_0 on the values of SFP and Nu^Q , respectively. In general, imposition of fluid suction at the wall has a tendency to decrease the thermal boundary layer. This causes the wall temperature to decrease. This results in enhancement of the wall heat transfer represented by increases in the Nusselt number as shown in Fig. 7. From Fig. 6, one observes that the values of the skin-friction parameter SFP increases as the suction parameter increases.

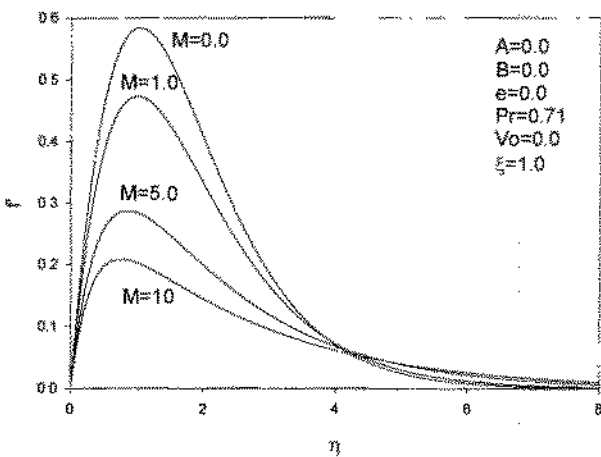


Figure 2. Effects of M on tangential velocity profiles.

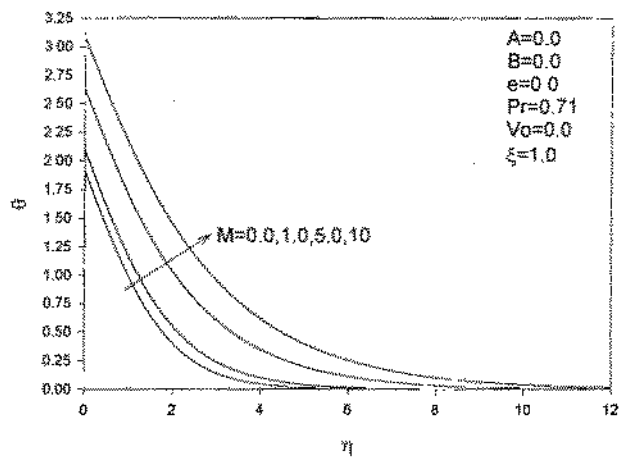


Figure 3. Effects of M on temperature profiles.

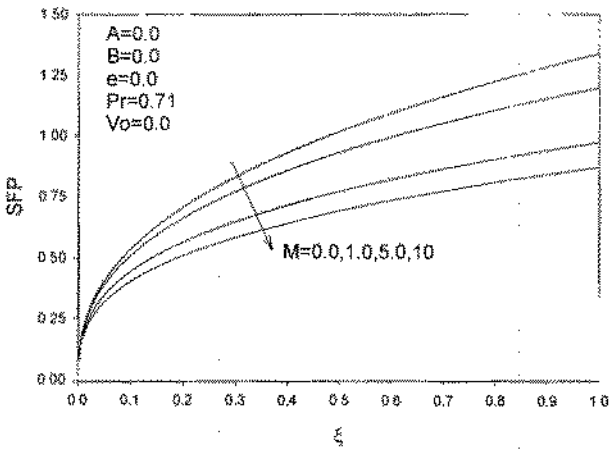


Figure 4. Effects of M on the local skin friction parameter.

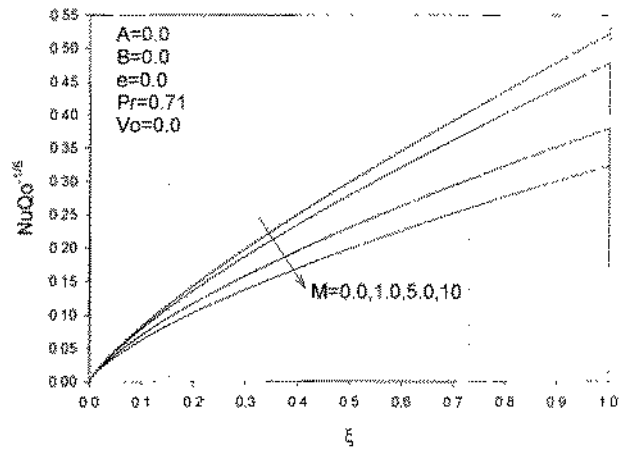


Figure 5. Effects of M on the local Nusselt number.

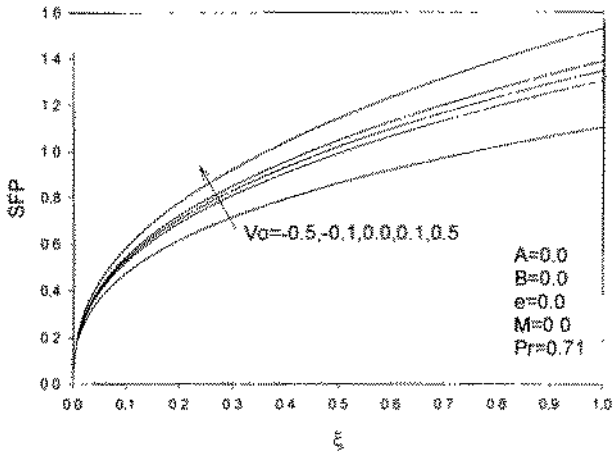


Figure 6. Effects of V_0 on the local skin friction parameter.

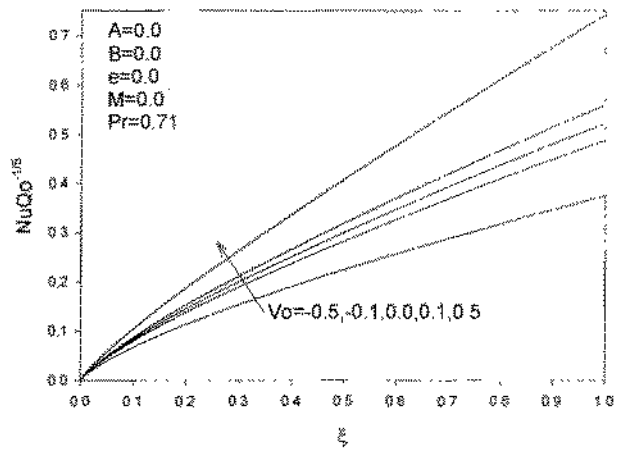


Figure 7. Effects of V_0 on the local Nusselt number.

Figures 8–10 depict the behavior of the local skin-friction parameter SFP , the local Nusselt number Nu^Q , and the local Sherwood number Sh^Q for various values of the concentration to thermal buoyancies ratio e , respectively. As expected, increases in the values of e cause increases in the total buoyancy effect and therefore increases in the flow induced by this effect. This is done at the expense of both the fluid temperature and concentration. As a result of this and as explained previously, the values of SFP , Nu^Q and Sh^Q tend to increase as is clear from Figs. 8–10.

In Fig. 11, a parametric study illustrates the effects of the Darcy number A , inertia coefficient B , buoyancy ratio e , Prandtl number Pr , and the wall mass transfer parameter

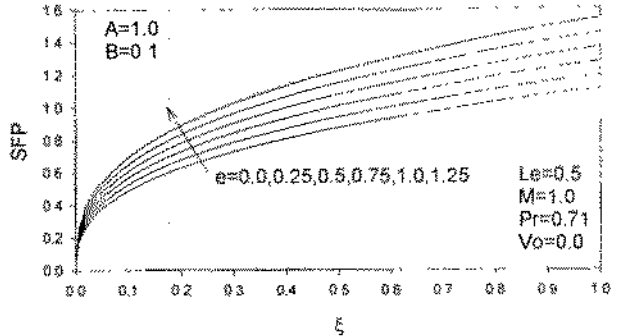


Figure 8. Effects of e on the local skin friction parameter.

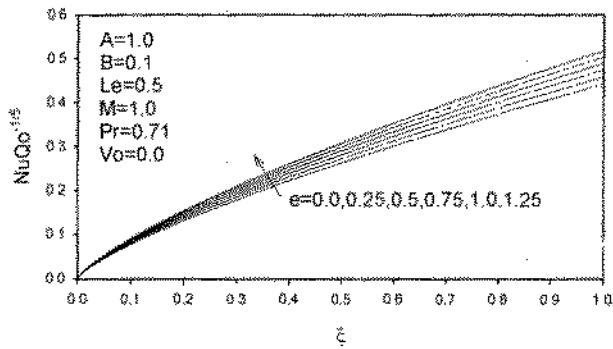


Figure 9. Effects of e on the local Nusselt number.

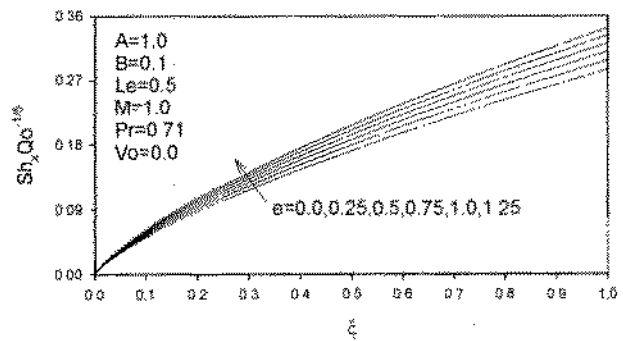


Figure 10. Effects of e on the local Sherwood number.

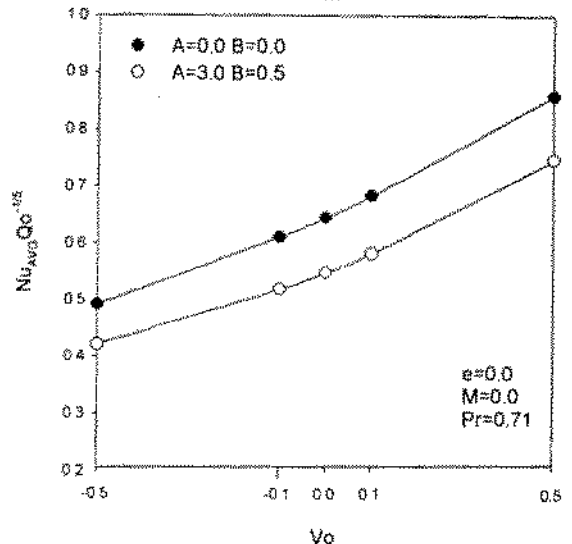
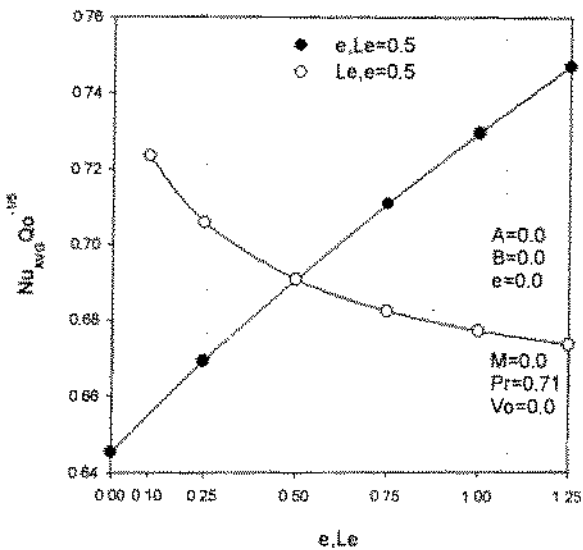
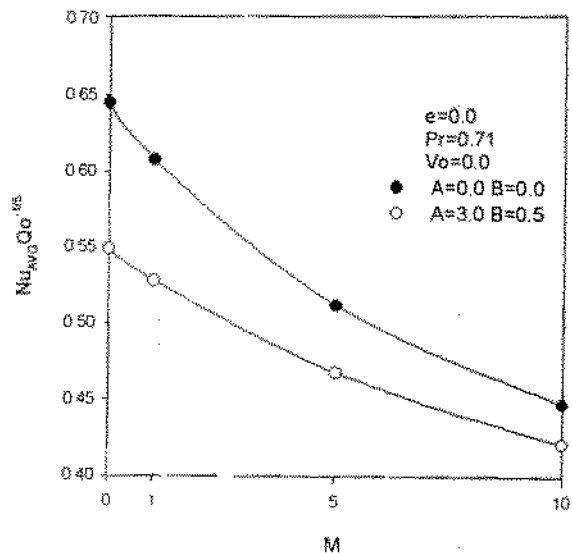
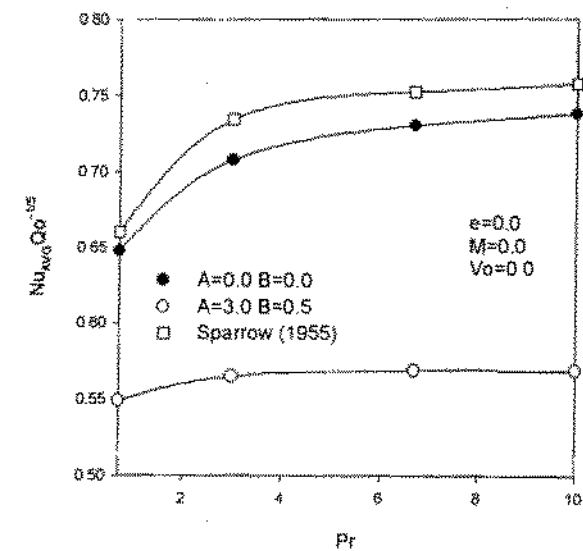


Figure 11. Effects of $A, B, e, Le, M, Pr,$ and V_0 on average Nusselt number.

V_0 on the average Nusselt number. With fluids of high Prandtl number, the thermal boundary layer becomes thinner and as a result an increase in Nu_{AVG} is expected. A good agreement with the results reported by Sparrow (1955) on the buoyancy-induced flow over a vertical impermeable plate is observed in the first part of Fig. 11. The differences in the results of Sparrow (1955) are due to his using an approximate integral method. From previous discussion and knowing that Nu_{AVG} is the integral of the local Nusselt number over the plate length, the Nu_{AVG} decreases as either A , B , or M increases and increases as V_0 or ϵ increases. Because the Lewis number is different for variable combinations of fluid and diffused species, if the species have a higher tendency to diffuse into the fluid, the Lewis number will be lower. Thus, as Le decreases, Nu_{AVG} increases.

CONCLUSION

The problem of steady, laminar, hydromagnetic heat and mass buoyancy-induced natural convection boundary-layer flow of an electrically conducting along an isoflux vertical and permeable semiinfinite surface embedded in a uniform porous medium was considered. The obtained results for a special case of the problem were compared with previously published work and found to be in good agreement. It was found that while the skin-friction parameter, Nusselt number, and the Sherwood number decreased as a result of the presence of either the magnetic field or the porous medium, they increased due to imposition of fluid suction at the plate surface. Furthermore, increasing the ratio of concentration to thermal buoyancies was found to cause enhancements in the values of the skin-friction parameter, Nusselt number, and the Sherwood number for the two studied thermal cases. It is hoped that the present work will serve as a vehicle for understanding more complex problems involving the various physical effects investigated in the present problem.

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