Journal of Porous Media, 23(8):751–765 (2020)

HEAT AND MASS TRANSFER ON MHD FLOW OF SECOND-GRADE FLUID THROUGH POROUS MEDIUM OVER A SEMI-INFINITE VERTICAL STRETCHING SHEET

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Original Manuscript Submitted: 9/15/2017; Final Draft Received: 8/17/2018

We have considered the MHD flow of an electrically conducting second-grade fluid through porous medium over a semi-infinite vertical stretching sheet. Thermophoresis, thermal radiation, and convective boundary conditions are taken into account. The governing equations reduced into a nondimensional form, making use of similarity transformations. The confined similarity equations are originated and solved using a shooting method together with a Runge–Kutta sixth-order system. The flow descriptions are discussed in detail through graphs and tables. The fluid velocity and temperature in the boundary-layer region become significantly higher with increasing values of the thermal radiation parameter. The chemical species concentration decreases in the presence of a thermophoretic parameter. The Nusselt number is enhanced with increasing surface convection parameter values. The rate of mass transfer increases with an increase in the thermophoretic parameter.

KEY WORDS: *convective boundary conditions, MHD flow, porous medium, second-grade fluid, thermal radiation, thermophoresis*

1. INTRODUCTION

The study of heat and mass transfer of non-Newtonian fluid has been increased due to their applications in many branches of science and engineering, such as metallurgical processes, polymer extrusion, glass blowing, crystal growing, and so on. The majority of the fluids dealt with by engineers and scientists, such as air, water, and oils, can be regarded as Newtonian under most conditions of interest. However, in many cases the assumption of Newtonian behavior is not valid and the rather more complex non-Newtonian response must be modeled. Such situations arise in the chemical processing industry and plastics processing industry. Non-Newtonian behavior is also encountered in the mining industry, where slurries and muds are often handled, and in applications such as lubrication and biomedical flows. The simulation of non-Newtonian fluid flow phenomena is therefore of importance to industry. The behavior of such liquids is often highly nonlinear and frequently difficult to predict by simple means. The introduction of computer-aided analysis and design software can have a large impact on these industries. Such an impact is plainly seen in the process of injection molding of thermoplastics, where sophisticated computer packages are available. This allows quite complex injection molds to be designed and analyzed, which significantly reduces the prototype testing phase. On the other hand, there are some embarrassing deficiencies in our ability to simulate the flow of non-

NOMENCLATURE

C	species concentration of the fluid (kg m^{-3})
C_f	coefficient of skin friction
C_p	specific heat at constant pressure p (J kg^{-1} K)
C_∞	species concentrations of the fluid in the free stream (kg m^{-3})
D	permeability parameter (Darcy parameter)
D_m	molecular diffusivity (m^2/s^{-1})
E	electric field (sm^{-1})
Ec	Eckert number
f_w	suction parameter
Gr	thermal Grashof number
Gm	solutal Grashof number
g	acceleration due to gravity (m/s^2)
K	thermal conductivity of the fluid (Wm^{-1} K^{-1})
k	permeability of the porous medium (m^2)
k^*	mean absorption coefficient (m^{-1})
K_r	rate of chemical reaction (mol L^{-1} s^{-1})
K_T	thermal diffusion ratio
K_1	thermophoretic coefficient
M	magnetic field parameter (Hartmann number)
Nr	thermal radiation parameter
Nu	Nusselt number
Pr	Prandtl number
p	fluid pressure (Nm^{-2})
q_r	radiative heat flux (kg/s^3)
Sc	Schmidt number
Sh	Sherwood number
Sr	Soret number
T	temperature of the fluid within the boundary layer (K)
t	time (s)
T_m	mean fluid temperature (K)
T_r	some reference temperature (K)
T_w	temperature at the plate (K)
T_∞	fluid temperature in the free stream (K)
$(u,\, v)$	velocity components along the directions x and y, respectively (ms^{-1})
V_T	thermophoretic velocity (ms^{-1})
v_0	suction/injection velocity (ms^{-1})
x, y	dimensionless coordinates (m)

Greek Symbols

α_1	material parameter
β	thermal expansion coefficient (K^{-1})
β^*	volumetric expansion coefficient (m^3 kg^{-1})
γ	the surface convection parameter
θ	the dimensionless temperature
ν	kinematic viscosity (m^2 s^{-1})
ρ	fluid density (kg m^{-3})
σ	electrical conductivity of the fluid (s m^{-1})
σ^*	Stefan–Boltzmann constant (Wm^{-2} K^{-4})
τ	the thermophoretic parameter
ϕ	the dimensionless concentration

Subscripts

w	conditions on the wall
∞	free stream conditions

Newtonian liquids, particularly for industrial processing operations. An outstanding example is the process of plastic extrusion. While we are quite able to model the viscous flow of the molten plastic material in the channels within the extrusion machine (as this is similar to the flow within an injection mold), the behavior of the stream of material after leaving the extrusion die (extrudate) is dominated by the elastic properties of the liquid and severe distortion of the extrudate can occur. This behavior is rather more difficult to model and has defied systematic treatment to date. However, considerable progress has been made in recent years with the introduction of new models and solution techniques. One application of non-Newtonian fluid could be a flexible military suit. Non-Newtonian fluids could be also used in shoe manufacturing. During standing, walking, or easy running, when pressures which act on the shoe are weak, the non-Newtonian fluid would remain in a liquid state and therefore the shoe interior would adapt to the position and shape of the foot. The boundary-layer flow of a non-Newtonian viscous fluid has drawn the attention of many researchers (Sakiadis, 1961; Vajravelu and Rollins, 2004; Hayat et al., 2008; Ahmad et al., 2009; Rashidi et al., 2010). Pal and Mondal (2011) discussed magnetohydrodynamic (MHD) non-Darcian mixed convection heat and mass transfer over a nonlinear stretching sheet. Heat and mass transfer past a stretching surface in a MHD micropolar fluid through a porous medium was studied by Pal and Chatterjee (2011). Pal and Mondal (2012b) extended their work (Pal and Mondal, 2011) by considering Soret and Dufour effects on non-Darcian MHD in presence of a nonuniform heat source/sink. Sheikholeslami (2014a) investigated the effect of a spatially variable magnetic field on ferrofluid flow. Sheikholeslami et al. (2015c) discussed the impact of nonuniform magnetic field on forced convection heat transfer of Fe_3O_4–water nanofluid. Sheikholeslami and Rashidi (2015) developed the work of Sheikholeslami et al. (2015b) by considering the space-dependent magnetic field. The effect of electric field on hydrothermal behavior of nanofluid in a complex geometry was investigated by Sheikholeslami et al. (2016b). Sheikholeslami et al. (2016c) also worked on forced convection heat transfer in a semiannulus under the influence of a variable magnetic field.

MHD flow problems in the presence thermal radiation have become more important in industry at high temperature. So the knowledge of the radiation heat transfer becomes very important. Cogley et al. (1968) observed that in the optically thin limit the fluid does not absorb its own emitted radiation but absorbs radiation emitted by the boundaries. The effect of thermal radiation on heat transfer problems has been studied by Makinde (2005), Ibrahim et al. (2008), Pal and Chatterjee (2010), Olajuwon (2011), and Zheng et al. (2012). Pal and Mondal (2012a) examined the effect of chemical reaction and thermal radiation on mixed convection heat and mass transfer over a stretching sheet in Darcian porous medium. Sheikholeslami et al. (2016a) numerically examined MHD free convection of Al_2O_3–water nanofluid in the presence of thermal radiation. Ferrofluid flow and heat transfer in a semiannulus enclosure in the presence of thermal radiation was studied by Sheikholeslami et al. (2015a). Later Sheikholeslami et al. (2015b) considered the effect of thermal radiation on a two-phase model of nanofluid flow and heat transfer. Thermophoresis is a physical phenomenon in which aerosol particles move from a hot to cold surface, has attracted considerable attention regarding the collection of submicrometer and nanometer particles. The force experienced by the suspended particles due to the temperature gradient is termed thermophoretic force, which is used in commercial precipitators. In this occurrence, the repulsion of particles from hot objects takes place and so a layer is obtained around hot bodies which is particle free (Goldsmith and May, 1966). This phenomenon has many applications, for example, to remove small particles from gas-particle trajectories from combustion devices and to study particulate material deposition turbine blades. The effect of thermophoresis particle deposition on boundary-layer flow under different situations was discussed by many researchers (Selim et al., 2003; Chamkha and Pop, 2004; Chamkha et al., 2006; Zueco et al., 2011). Pal and Mondal (2013) discussed the effect of thermophoresis on MHD heat and mass transfer over a nonisothermal wedge. KKL correlation for simulation of nanofluid flow and heat transfer in a permeable channel was examined by Sheikholeslami (2014b). Sheikholeslami and Ganji (2015) studied nanofluid flow and heat transfer between parallel plates using differential transform method. In the study of boundary-layer flow problems, the boundary conditions are either a specified surface temperature or a specified surface heat flux. Mondal and Mukherjee (2013) investigated the effect of viscous dissipation on shear driven flow between two parallel plates with constant heat flux boundary conditions. But there are many problems in which surface heat transfer depends on the surface temperature. Newtonian heating arises in the situation where the heat is supplied to the convective fluid through a bounding surface with a finite heat capacity. Boundary-layer heat transfer problems concerning a convective boundary condition were investigated by Makinde and Aziz (2010), Ishak (2010), and Rahman (2011). Recently Das (2014) studied the effect of chemical reaction on MHD mixed convection second-grade fluid flow passing through a semi-infinite stretching sheet. Das et al. (2016) studied the second-grade fluid flow passing through a semi- infinite stretching sheet with convective surface heat flux. Veera Krishna and Prakash (2015) discussed Hall effects on the unsteady flow of an incompressible viscous fluid in a rotating parallel-plate channel bounded on one side by a porous bed under the influence of a uniform transverse magnetic field making use of the Darcy–Lapwood model. Krishna et al. (2009) discussed the unsteady hydromagnetic flow of an incompressible viscous fluid in a rotating parallel-plate channel with porous lining under the influence of a uniform transverse magnetic field normal to the channel, and it was extended by Veera Krishna et al. (2010a). Veera Krishna et al. (2009) studied the steady hydromagnetic flow of a couple stress fluid through a composite medium in a rotating parallel-plate channel with a porous bed on the lower half subjected normal to the channel and extended the problem by taking the Hall current into account in Veera Krishna et al. (2010b). Veera Krishna and Malashetty (2012) discussed the unsteady flow of an in compressible electrically conducting second-grade fluid in a rigidly rotating parallel-plate channel bounded below by a sparsely packed porous bed subjected to normal to the channel and extended the problem by taking the Hall current into account (Veera Krishna and Malashetty, 2011). Recently, Veera Krishna and Gangadhar Reddy (2016) discussed MHD free convective rotating flow of viscoelastic fluid past an infinite vertical oscillating plate. Veera Krishna and Subba Reddy (2016) explored unsteady MHD convective flow of second-grade fluid through a porous medium in a rotating parallel-plate channel with a temperature-dependent source. Recently Sheikholeslami et al. (2016, 2017a,b) discussed the numerical investigation of MHD nanofluid through a porous medium with different configurations. Fumei et al. (2017) discussed heat transfer enhancement in a pipe filled with porous media by an axisymmetric TLB model. El-Kabeir et al. (2015, 2016), Hossam et al. (2015, 2017), and Mallikarjuna et al. (2016) discussed the heat and mass transfer by unsteady natural convection in different phenomenon. Aminreza et al. (2011, 2013), Ali et al. (2017), Malathy et al. (2017), and Shahnazari et al. (2017) discussed numerical methods of MHD flows of non-Newtonian fluid through some configurations.

Veera Krishna et al. (2018b) discussed heat and mass transfer of unsteady MHD oscillatory flow of blood through porous arteriole. The effects of radiation and Hall current on an unsteady MHD free convective flow in a vertical channel filled with a porous medium have been studied by Veera Krishna et al. (2018d). The heat generation/absorption and thermodiffusion on an unsteady free convective MHD flow of radiating and chemically reactive second-grade fluid near an infinite vertical plate through a porous medium and taking the Hall current into account have been studied by Veera Krishna and Chamkha (2018). Veera Krishna et al. (2018c) discussed the effects of heat and mass transfer on unsteady, MHD oscillatory flow of second-grade fluid through a porous medium between two vertical plates under the influence of a fluctuating heat source/sink and chemical reaction. Veera Krishna et al. (2018a) discussed heat and mass transfer effects on an unsteady flow of a chemically reacting micropolar fluid over an infinite vertical porous plate in the presence of an inclined magnetic field, Hall current effect with thermal radiation taken into account.

Veera Krishna et al. (2019a) investigated the heat and mass transfer on MHD free convective flow over an infinite nonconducting vertical flat porous plate. Veera Krishna and Jyothi (2019) discussed the effect of heat and mass transfer on free convective rotating flow of a viscoelastic incompressible electrically conducting fluid past a vertical porous plate with time-dependent oscillatory permeability and suction in the presence of a uniform transverse magnetic field and heat source. Veera Krishna and Subba Reddy (2019) investigated the transient MHD flow of a reactive second-grade fluid through a porous medium between two infinitely long horizontal parallel plates. Veera Krishna et al. (2019b) discussed Hall effects on unsteady hydromagnetic natural convective rotating flow of second-grade fluid past an impulsively moving vertical plate entrenched in a fluid-inundated porous medium while the temperature of the plate had a temporarily ramped profile. Veera Krishna et al. (2019c) discussed Hall effects on MHD peristaltic flow of Jeffrey fluid through a porous medium in a vertical stratum. The effects of heat and mass transfer on free convective flow of micropolar fluid were studied over an infinite vertical porous plate in the presence of an inclined magnetic field with a constant suction velocity taking Hall current into account was also discussed by Veera Krishna et al. (2019d). Veera Krishna and Chamkha (2019a) discussed the systematic solution of time-dependent mean velocity on MHD peristaltic rotating flow of an electrically conducting couple stress fluid in a uniform elastic porous channel. Veera Krishna and Chamkha (2019b) discussed the MHD squeezing flow of a water-based nanofluid through a saturated porous medium between two parallel disks taking the Hall current into account. Veera Krishna and Chamkha (2019c) investigated the diffusion-thermo, radiation-absorption, and Hall and ion-slip effects on MHD free convective rotating flow of nanofluids (Ag and TiO_2) past a semi-infinite permeable moving plate with constant heat source. Veera Krishna et al. (2019e) discussed the Soret and Joule effects on MHD mixed convective flow of an incompressible and electrically conducting viscous fluid past an infinite vertical porous plate taking Hall effects into account.

Hall and ion-slip effects on unsteady MHD convective rotating flow of nanofluids was discussed by Veera Krishna and Chamkha (2020a). Veera Krishna (2020) investigated the heat transport on steady MHD flow of copper and alumina nanofluids past a stretching porous surface. Veera Krishna et al. (2020a) investigated the Hall and ion-slip effects on unsteady MHD free convective rotating flow through a porous medium past an exponentially accelerated inclined plate. The combined effects of Hall and ion slip on MHD rotating flow of ciliary propulsion of microscopic organisms through a porous medium was studied by Veera Krishna et al. (2020b). Veera Krishna and Chamkha (2020b) investigated the Hall and ion-slip effects on MHD convective flow of elastoviscous fluid through a porous medium between two rigidly rotating parallel plates with a time-fluctuating sinusoidal pressure gradient.

Keeping the above-mentioned facts in mind, this paper discusses the effects of heat and mass transfer on MHD flow of an electrically conducting second-grade fluid over a semi-infinite vertical stretching sheet. The influence of thermophoresis and thermal radiation on the flow is included in the present model.

2. FORMULATION AND SOLUTION OF THE PROBLEM

Consider the boundary-layer flow of second-grade fluid through a porous medium over a stretching sheet coinciding with the plane $y = 0$ and the flow being confined to $y > 0$ in the presence of viscous dissipation and Joule heating, as depicted in Fig. 1. The flow is generated due to the stretching of the sheet caused by the simultaneous action of two equal and opposite forces along the x axis. The sheet is then stretched with a velocity $u_w(x) = ax$, where a is a constant and x is the coordinate measured along the stretching surface from the slit. The thermal radiation is taking place in the flow, and the effect of thermophoresis is being taken into account to help in understanding of the mass deposition variation on the surface. A uniform transverse magnetic field of strength B_0 is applied parallel to the y axis. The applied magnetic field and magnetic Reynolds number are assumed to be very small so that the induced magnetic field is negligible. Under these assumptions, the governing boundary-layer equations for a second-grade fluid flow can be written as (Olajuwon, 2011; Das, 2014)

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$\begin{aligned} u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + \frac{\alpha_1}{\rho}\left[\frac{\partial}{\partial x}\left(u\frac{\partial^2 u}{\partial y^2}\right) - \frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x \partial y} + \nu\frac{\partial^3 u}{\partial y^3}\right] \\ - \frac{\sigma B_0^2}{\rho}u - \frac{\nu}{k}u + g\beta\left(T - T_\infty\right) + g\beta^*\left(C - C_\infty\right), \end{aligned} \tag{2}$$

$$\rho c_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = K\frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y} + \mu\left(\frac{\partial u}{\partial y}\right)^2 + \alpha_1\frac{\partial u}{\partial y}\left[\frac{\partial}{\partial y}\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right)\right] + \left(\sigma B_0^2 + \frac{\nu}{k}\right)u^2, \tag{3}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_m\frac{\partial^2 C}{\partial y^2} + \frac{D_m K_T}{T_m}\frac{\partial^2 T}{\partial y^2} - \frac{\partial\left(V_T C\right)}{\partial y} - K_r\left(C - C_\infty\right), \tag{4}$$

where $V_T = -[(K_1\nu)/T_r](\partial T/\partial y)$ is the thermophoretic velocity. We assume the bottom surface of the plate is heated by convection from a hot fluid at temperature T_w, which provides a heat transfer coefficient h_w. The boundary conditions of the present model are (Olajuwon, 2011; Ishak, 2010; Das, 2014)

$$u = u_w, \quad v = v_0, \quad -K\frac{\partial T}{\partial y} = h_w\left(T_w - T\right), \quad C = C_w \quad \text{at} \quad y = 0, \tag{5}$$

$$u \to 0, \quad \frac{\partial u}{\partial x} \to 0, \quad T \to T_\infty, \quad C \to C_\infty \quad \text{at} \quad y \to \infty. \tag{6}$$

Using the Rosseland approximation, the radiative heat flux term is given by (Ishak, 2010)

$$q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y}. \tag{7}$$

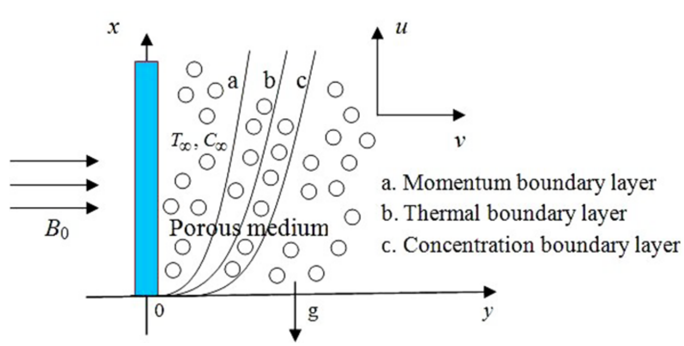

FIG. 1: Physical configuration of the problem

Assuming that the differences in the temperature within the flow are such that T^4 can be expressed as a linear combination of the temperature, we expand T^4 in Taylor's series about T_∞, and neglecting higher-order terms we get (Ishak, 2010)

$$T^4 = 4T_\infty^4 T - 3T_\infty^4. \tag{8}$$

Thus we have

$$\frac{\partial q_r}{\partial y} = -\frac{16T_\infty^3 \sigma^*}{3k^*}\frac{\partial^2 T}{\partial y^2}. \tag{9}$$

Following the lines of Olajuwon (2011), the similarity transformations as given below are introduced:

$$\eta = \left(\frac{c}{\nu}\right)^{1/2} y, \quad u = cxf'(\eta), \quad v = -(c\nu)^{1/2} f(\eta), \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}. \tag{10}$$

Equation (1) is automatically satisfied. Using Eq. (9) in Eq. (3) and applying transformation (10), Eqs. (2)–(4) reduce to the following ordinary differential equations:

$$f'^2 - ff'' - f''' - \lambda_1\left(2f'''f' - ff^{iv} - f''^2\right) + \left(M^2 + \frac{1}{D}\right)f' - Gr\theta - Gm\phi = 0, \tag{11}$$

$$(1 + Nr)\,\theta'' + \mathrm{Pr}f\theta' + \mathrm{PrEc}\left[f''^2 + \left(M^2 + \frac{1}{D}\right)f'^2 + \lambda_1 f''\left(f'f'' - ff'''\right)\right] = 0, \tag{12}$$

$$\phi'' + \mathrm{Sc}\,(f - \tau\theta')\,\phi' + \mathrm{Sc}\,(\mathrm{Sr} - \tau\phi)\,\theta'' - \mathrm{Sc}\,Kr\phi = 0. \tag{13}$$

The boundary conditions (5) and (6) then turn into

$$f = f_w, \quad f' = 1, \quad \theta' = -\gamma\,(1 - \theta), \quad \phi = 1 \quad \text{at} \quad \eta = 0, \tag{14}$$

$$f' \to 0, \quad f'' \to 0, \quad \theta \to 0, \quad \phi \to 0 \quad \text{at} \quad \eta \to \infty. \tag{15}$$

Here prime denotes differentiation with respect to η, where $\lambda_1 = (\alpha_1 a)/(\rho\nu)$ is the second-grade fluid parameter, $M^2 = (\sigma B_0^2)/(\rho a)$ is the magnetic field parameter, $D = (kv_0^2)/\nu^2$ is the permeability parameter (Darcy parameter), $f_w = v_0/(a\nu)^{1/2}$ is the suction parameter, $Nr = (16T_\infty^3\sigma^*)/(3k^*K)$ is the thermal radiation parameter, $\mathrm{Pr} = (\mu C_p)/K$ is the Prandtl number, $\mathrm{Ec} = u_w^2/[C_p(T_w - T_p)]$ is the Eckert number, $\tau = -[k(T_w - T_p)]/T_r$ is the thermophoretic parameter, $\mathrm{Sc} = \nu/D_m$ is the Schmidt number, $\mathrm{Sr} = [D_m K_T(T_w - T_\infty)]/[T_m\nu(C_w - C_\infty)]$ is the Soret number, $Gr = [g\beta(T_w - T_\infty)]/a$ is the thermal Grashof number, $Gm = [g\beta^*(C_w - C_\infty)]/a$ is the solutal Grashof number, $Kr = (K_r\nu)/(aD_m)$ is the chemical reaction parameter, and $\gamma = h_w/[K(\nu/a)^{1/2}]$ is the surface convection parameter.

The physical quantities of practical and engineering primary interest are the skin friction coefficient, Nusselt number, and Sherwood number. The equation defining the wall shear stress is

$$\tau_w = \left[\mu\frac{\partial u}{\partial y} + \rho\alpha_1\left(2\frac{\partial u}{\partial x}\frac{\partial u}{\partial y} + u\frac{\partial^2 u}{\partial x\partial y}\right)\right]_{y=0}. \tag{16}$$

The local dimensionless skin friction coefficient is given by

$$C_f = 2\mathrm{Re}^{-1/2}\left[1 + 3\lambda_1 f'(0)\right] f''(0), \tag{17}$$

or

$$C_f^* = \left[1 + 3\lambda_1 f'(0)\right] f''(0), \tag{18}$$

where $C_f^* = (1/2)\mathrm{Re}^{1/2} C_f$.

Knowing the temperature field, it is interesting to study the effect of the free convection and thermal radiation on the rate of heat transfer q_w given by

$$q_w = -K\left(\frac{\partial T}{\partial y}\right)_{y=0} - \frac{16T_\infty^3\sigma^*}{3k^*}\left(\frac{\partial^2 T}{\partial y^2}\right)_{y=0}. \tag{19}$$

So the rate of heat transfer in terms of dimensionless Nusselt number (Nu) is defined as follows:

$$\text{Nu} = -\text{Re}^{1/2}\,(1+Nr)\,\theta'(0), \tag{20}$$

or

$$\text{Nu}^* = -\,(1+Nr)\,\theta'(0), \tag{21}$$

where $\text{Nu}^* = \text{Re}^{-1/2}\text{Nu}$.

The rate of mass transfer in terms of dimensionless Sherwood number (Sh) is given by

$$\text{Sh} = -\text{Re}^{1/2}\phi'(0), \tag{22}$$

or

$$\text{Sh}^* = -\phi'(0), \tag{23}$$

where $\text{Sh}^* = \text{Re}^{-1/2}\text{Sh}$.

The set of Eqs. (11)–(13) under the boundary conditions (14) and (15) are solved numerically by applying the Nachtsheim and Swigert shooting iteration technique together with a Runge–Kutta sixth-order integration scheme. To ensure the validity of the numerical cipher, the values of $-\theta'(0)$ have been calculated for $\tau = \text{Ec} = Kr = \gamma = 0$ and for different values of thermal radiation parameter Nr in Table 1. The numerical methods are described in detail referring to Nachtsheim and Swigert (1965). For the accuracy of the numerical results, the present investigation is compared with the previous investigations by Olajuwon (2011), Das et al. (2014, 2016), and Reddy et al. (2011). It is observed that the present result is in good agreement with that of Das et al. (2016). This favorable comparison lends confidence to the numerical results to be reported in the next sections.

3. RESULTS AND DISCUSSION

We have considered the MHD flow of an electrically conducting second-grade fluid through a porous medium over a semi-infinite vertical stretching sheet. Thermophoresis, thermal radiation, and convective boundary conditions are taken into account. The governing equations reduced into a nondimensional form making use of similarity transformations. The confined similarity equations are originated and solved using the shooting method together with a Runge–Kutta sixth-order system. The computational results are demonstrated graphically in Figs. 2–4 and are mentioned in Table 2. The default values of nondimensional parameters are considered in the simulation as $M = 2$, $D = 1$, $\text{Sr} = 0.5$, $\text{Sc} = 0.64$, $\lambda_1 = 1.5$, $Kr = 0.2$, $\gamma = 0.1$, $Gr = 5$, $Gm = 5$, $\text{Pr} = 0.71$, $Nr = 0.4$, $\text{Ec} = 0.02$, and $\tau = 0.2$ unless otherwise specified.

TABLE 1: Comparison of $-\theta'(0)$ with Nr

Nr	**Olajuwon (2011)**	**Das (2014)**	**Das et al. (2016)**	**Present results**
0.2	1.595708	1.588825	1.595721	1.602254
0.5	1.170509	1.720674	1.170500	1.172145
0.7	0.373504	0.373817	0.373516	0.374558
2.0	2.673501	0.598306	2.675600	2.684582
5.0	2.354506	2.348611	2.354533	2.365569

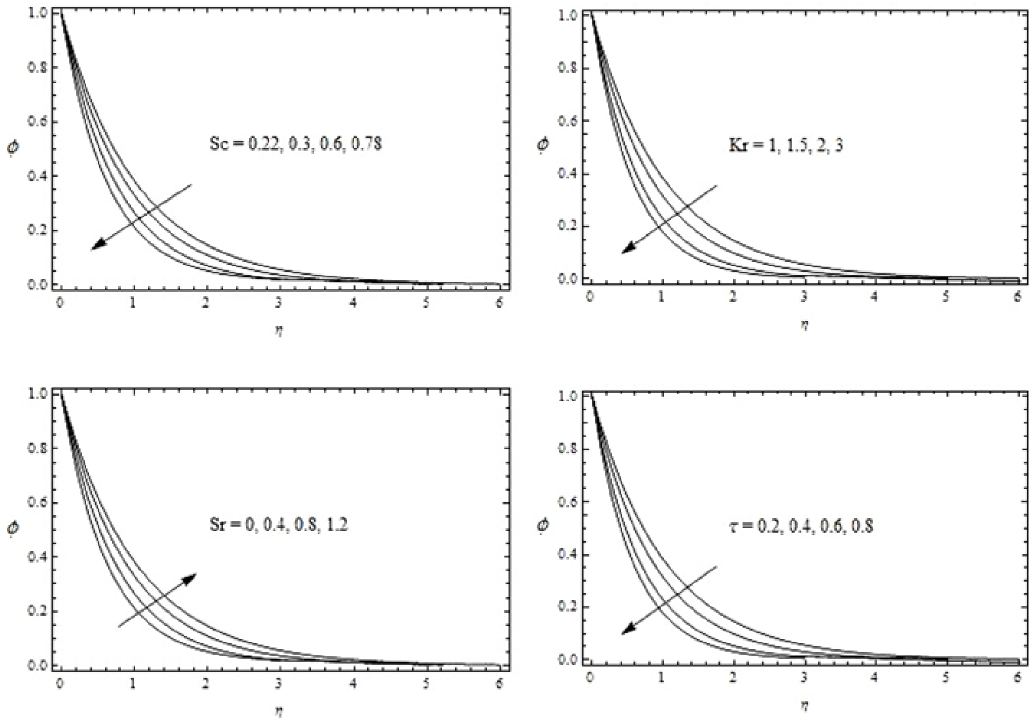

FIG. 2: Concentration profiles versus Sc, Kr, Sr, and τ

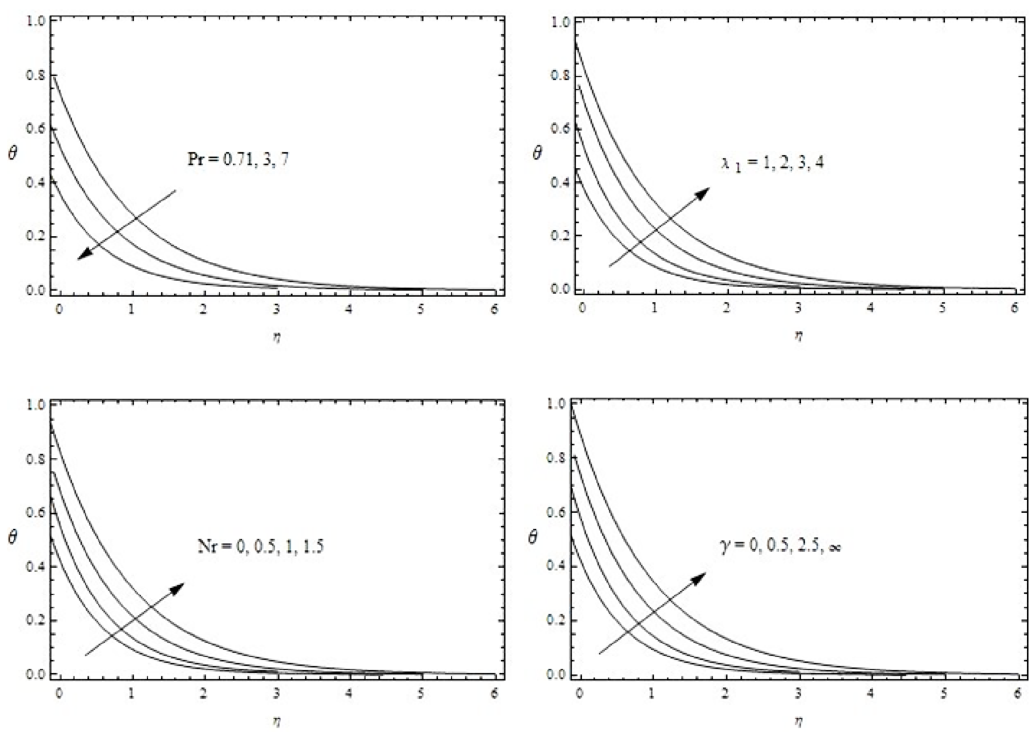

FIG. 3: Temperature profil1es versus Pr, λ_1, Nr, and γ

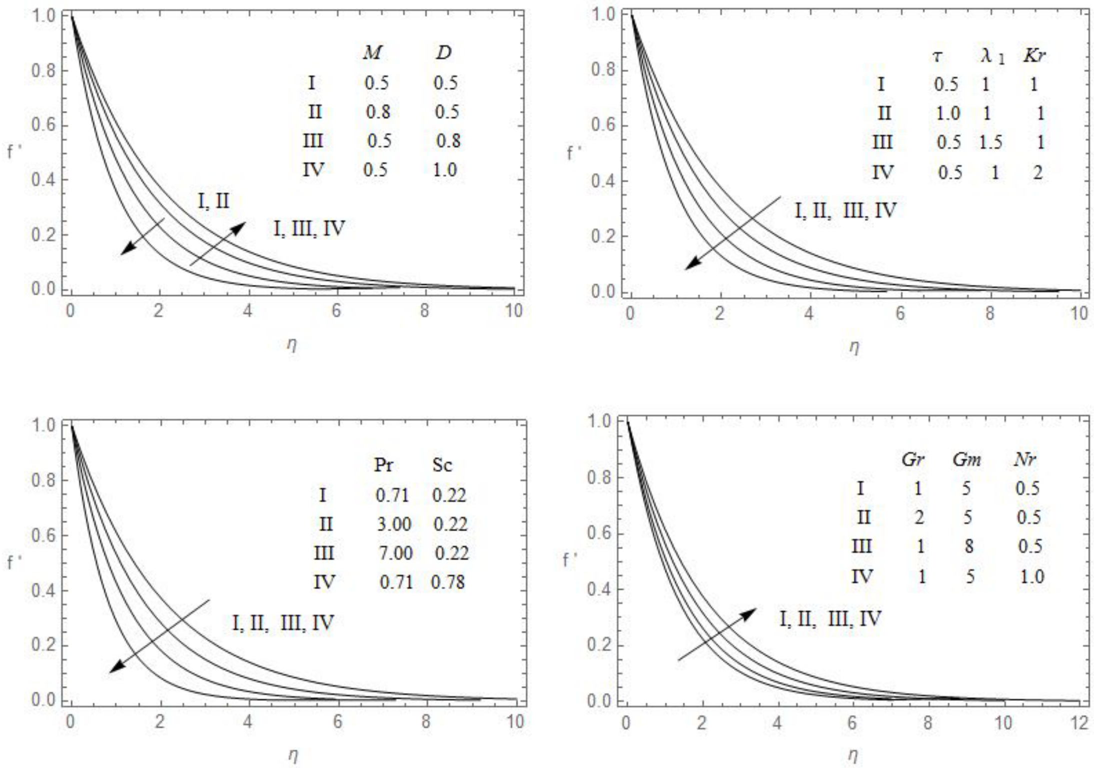

FIG. 4: Velocity profiles versus M, D, τ, λ_1, Kr, Pr, Sc, Gr, Gm, and Nr

TABLE 2: Effects of various parameters on C_f^*, Nu^*, and Sh^*

M	D	Nr	γ	τ	λ_1	Kr	C_f^*	Nu*	Sh*
2	**1**	**0.5**	**0.5**	**0.5**	**1**	**1**	−2.587745	0.058554	0.690552
3	1	0.5	0.5	0.5	1	1	−2.568854	0.041788	—
4	1	0.5	0.5	0.5	1	1	−2.510052	0.032256	—
2	**2**	0.5	0.5	0.5	1	1	−2.601887	0.085547	—
2	**3**	0.5	0.5	0.5	1	1	−2.855475	0.099885	—
2	1	**0.8**	0.5	0.5	1	1	−1.554412	0.144527	—
2	1	**1.0**	0.5	0.5	1	1	−1.112548	0.255478	—
2	1	0.5	**1.0**	0.5	1	1	−3.588456	0.322562	—
2	1	0.5	**1.5**	0.5	1	1	−4.188759	0.855479	—
2	1	0.5	0.5	**1.0**	1	1	−5.887497	—	0.785549
2	1	0.5	0.5	**1.5**	1	1	−8.785478	—	0.850479
2	1	0.5	0.5	0.5	**2**	1	−1.588704	0.036085	0.924458
2	1	0.5	0.5	0.5	**3**	1	−0.122058	0.014415	1.114758
2	1	0.5	0.5	0.5	1	**2**	−0.885478	—	0.855479
2	1	0.5	0.5	0.5	1	**3**	−0.041255	—	0.980256

In the boundary-layer region, the concentration of the fluid decreases with increasing the values of Schmidt number Sc, chemical reaction parameter Kr, and thermophoretic parameter τ, and increases with an increase in Soret parameter Sr as presented in Fig. 2. So the thermophoretic parameter is expected to amend the concentration boundary layer. The Schmidt numbers are chosen as Sc $= 0.22$, 0.3, 0.6, and 0.78, corresponding to H_2, He, H_2O vapor, and NH_3, accordingly. The Schmidt number is defined as the ratio of momentum diffusivity (kinematic viscosity) to the mass diffusivity. It is used to characterize fluid flows in which there are simultaneous momentum and mass diffusion convection processes. The Schmidt number Sc quantizes the virtual efficiency of momentum and mass transportation through distribution in the momentum and concentration boundary layers. This was acquired to increase the Schmidt number, which induces the absorption of particles and causes the boundary-layer thickness to diminish extensively. Its similar nature is identified with enlarging the chemical reaction parameter Kr. An enlargement in the chemical reaction parameter Kr reduces the concentration distribution quickly. Because the number of solute particles experiencing by the chemical reaction effect and is increased with increasing in the chemical reaction parameter, and therefore the concentration field is diminished throughout the fluid region. Hence, the concentration boundary layer thickness lessens significantly with increasing the chemical reaction parameter.

From Fig. 3 it is seen that the temperature increases uniformly with increasing Nr. Thus by escalating Nr the thermal boundary-layer thickness is enhanced. The impact of γ on temperature in the presence of thermal radiation is demonstrated in Fig. 3. The temperature increases with increasing γ in the boundary-layer region. For large values of γ, i.e., $\gamma - 1$, the solution reduces to the solution for constant surface temperature. From the boundary condition (14), it can be seen that $\theta(0) = 1$ as $\gamma - 1$, which coincides with the numerical results obtained in the present study. It is observed that with the increase in the surface convection parameter γ or the second-grade fluid parameter λ_1, the temperature profiles increase and hence thickness of the thermal boundary layer increases. From the same figure, the temperature reduces with an increase in Pr and hence the thickness of the thermal boundary layer decreases.

Figure 4 displays the behavior of the velocity distribution for various values of thermal radiation parameter Nr and Darcy parameter D. An increase in Darcy parameter or radiation parameter tends to increase the fluid velocity in the boundary-layer region. Lower the permeability lesser the fluid speed is observed in the entire fluid region. The physics behind the results is that the thermal radiation increases the thickness of momentum boundary layer, which ultimately enhances the velocity. The effect of the second-grade parameter λ_1 on the fluid velocity distribution is illustrated in Fig. 4. The velocity component across the boundary layer reduces with an increase in the second-grade parameter and also decreases asymptotically to zero at the edge of the hydrodynamic boundary layer. The effect of surface convection parameter γ on the streamwise velocity component is shown in Fig. 4. As the value of γ increases the flow rate is enhanced, thereby giving rise to an increase in the velocity profiles. Figure 4 illustrates the variation of the velocity distribution for various values of thermophoretic parameter τ. The fluid velocity decreases with an increase in the thermophoretic parameter and so the momentum boundary-layer thickness decreases. The fluid velocity reduces with increasing the intensity of the magnetic field parameter M. It is since of classical consequence—the Lorentz force emerges owing to the submission to a magnetic field just before an electrical conducting fluid and provides a resistive-type force. Appropriate to this strength, the movement of fluid flow and momentum boundary-layer thickness starts to slow down. Similar behavior is observed with increasing Prandtl number or Schmidt number Sc. It is perceived that the enlargement of Pr made the fluid flow slow down for primary velocity, whereas it is enhanced for secondary velocity. Therefore the velocity is reduced with an augment in Pr throughout the region occupied by the fluid. This is due to the fact that, the fluid with the highest Prandtl number has high viscosity, this made to the fluid has considerable thickness. Likewise, as Sc increases the velocity is decreased and hence the momentum boundary-layer thickness is reduced. An increase in the chemical reaction parameter Kr diminishes the velocity profile, as shown in the same figure. Thus for a heavier diffusing species, an increasing Schmidt number as well as increasing the rate of chemical reaction causes a lessening in the velocity. Furthermore, the heavier species through disparaging reaction causes a deceleration within the velocity delivery. Likewise, the magnitude of the velocity is enhanced with increasing the thermal Grashof number Gr or mass Grashof number Gm in the boundary-layer region. The thermal Grashof number is an important outcome of the thermal buoyancy force to the viscous force acting on the fluid through the boundary layer; at the same time, the mass Grashof number establishes the proportion of the concentration buoyancy force to the viscous force. The fluid velocity increases by virtue of the strengthening of heat and solute buoyancy forces. The velocity quickly increases next to the porous surface and later declines easily for the gratuitous torrent quantity. The momentum velocity transverse to the boundary layer enlarges through an increase in Gr or Gm. Hence the boundary-layer thickness is augmented with an increase in Gr or Gm.

It is found from Table 2 that an increase in τ leads to increasing values of the wall shear stress (in magnitude), Nusselt number, and Sherwood number. It is observed that with increasing values of λ_1, the reduced skin friction coefficient (in absolute sense), the Nusselt number diminishes whereas the Sherwood number increases. It can be seen from Table 2 that the magnitude of the reduced skin friction coefficient decreases with an increase in the radiation parameter Nr, whereas the thermal radiation increases the rate of heat transfer. From Table 2, it is observed that the heat transfer rate at the plate increases with increasing values of γ, whereas the effect is opposite for the wall shear stress (in magnitude) at the plate, i.e., the reduced skin friction coefficient (in magnitude) decreases with increasing surface convection parameter. The skin friction coefficient reduces with Kr and enhances with increasing M or D.

4. CONCLUSIONS

We have considered the MHD flow of an electrically conducting second-grade fluid through porous medium over a semi-infinite vertical stretching sheet. Thermophoresis, convective boundary conditions, and thermal radiation are taken into account. The conclusions are made as follows:

1. The velocity in the boundary-layer region increases for an increasing thermal radiation parameter and surface convection parameter, but the effect is reversed for the Hartmann number, Darcy parameter, thermophoretic parameter, and second-grade fluid parameter.

2. The temperature profile is enhanced with an increase in the thermal radiation parameter, second-grade fluid parameter, and surface convection parameter.

3. The chemical species concentration decreases in the presence of thermophoresis. Consequently, the rate of mass transfer increases with the thermophoretic parameter.

4. The skin friction coefficient (in magnitude) decreases with increase of thermal radiation parameter, second-grade fluid parameter, and surface convection parameter, but the effect is reverse for the thermophoretic parameter.

5. The rate of heat transfer increases for increasing values of the surface convection parameter and thermal radiation parameter, while it decreases with an increase in the values of the second-grade fluid parameter.

REFERENCES

Ahmad, I., Sajid, M., and Hayat, T., Heat Transfer in Unsteady Axi-Symmetric Second Grade Fluid, *Appl. Math. Comput.*, vol. **215**, pp. 1685–1695, 2009.

Ali, N., Sajid, M., and Abbas, Z., Slip Effects in the Hydromagnetic Flow of a Viscoelastic Fluid through Porous Medium over a Porous Oscillatory Stretching Sheet, *J. Porous Media*, vol. **20**, no. 3, pp. 249–262, 2017.

Aminreza, N., Rashid, P., and Ghalambaz, M., Effect of Partial Slip Boundary Condition on the Flow and Heat Transfer of Nanofluids past Stretching Sheet Prescribed Constant Wall Temperature, *Int. J. Therm. Sci.*, vol. **54**, pp. 253–261, 2011.

Aminreza, N., Saffarian, M.R., Rashid, P., and Ghalambaz, M., Entropy Analysis for Nanofluid Flow over a Stretching Sheet in the Presence of Heat Generation/Absorption and Partial Slip, *J. Mech. Sci. Technol.*, vol. **27**, no. 3, pp. 927–937, 2013.

Chamkha, A.J., Al-Mudhaf, A.F., and Pop, I., Effects of Heat Generation or Absorption on Thermophoretic Free Convection Boundary Layer from a Vertical Flat Plate Embedded in a Porous Medium, *Int. Commun. Heat Mass Transf.*, vol. **33**, pp. 1096–1102, 2006.

Chamkha, A.J. and Pop, I., Effects of Thermophoresis Particle Deposition in Free Convection Boundary Layer from a Vertical Flat Plate Embedded in a Porous Medium, *Int. Commun. Heat Mass Transf.*, vol. **31**, pp. 421–430, 2004.

Cogley, A.C., Vincenty, W.E., and Gilles, S.E., Differential Approximation for Radiation in a Non-Gray Gas near Equilibrium, *Am. Inst. Aeronaut. Astronaut. J.*, vol. **6**, pp. 551–553, 1968.

Das, K., Influence of Chemical Reaction and Viscous Dissipation on MHD Mixed Convection Flow, *J. Mech. Sci. Technol.*, vol. **28**, no. 5, pp. 1881–1885, 2014.

Das, K., Sharma, R.P., and Sarkar, A., Heat and Mass Transfer of a Second Grade Magneto Hydrodynamic Fluid over a Convectively Heated Stretching Sheet, *J. Comput. Design Eng.*, vol. **3**, pp. 330–336, 2016.

El-Kabeir, S.M.M., Modather, M., and Rashad, A.M., Heat and Mass Transfer by Unsteady Natural Convection over a Moving Vertical Plate Embedded in a Saturated Porous Medium with Chemical Reaction, Soret and Dufour Effects, *J. Appl. Fluid Mech.*, vol. **8**, no. 3, pp. 453–463, 2015.

El-Kabeir, S.M.M., El-Zahar, E.R., and Rashad, A.M., Effect of Chemical Reaction on Heat and Mass Transfer by Mixed Convection Flow of Casson Fluid about a Sphere with Partial Slip, *J. Comput. Theor. Nanosci.*, vol. **13**, no. 8, pp. 5218–5226, 2016.

Fumei, R., Zhang, W., Shi, B., and Guo, Z., Numerical Study of Heat Transfer Enhancement in a Pipe Filled with Porous Media by Axisymmetric TLB Model based on GPU, *Int. J. Heat Mass Transf.*, vol. **111**, pp. 1039–1049, 2017.

Goldsmith, P. and May, F.G., Diffusiophoresis and Thermophoresis in Water Vapour Systems, in *Aerosol Science*, C.N. Davies, Ed., London: Academic Press, pp. 163–194, 1966.

Hayat, T., Saif, S., and Abbas, Z., The Influence of Heat Transfer in a MHD Second Grade Fluid Film over an Unsteady Stretching Sheet, *Phys. Lett. A*, vol. **372**, pp. 5037–5045, 2008.

Hossam, A.N., El-Kabeir, S.M.M., and Rashad, A.M., Lie Group Analysis of Effects of Radiation and Chemical Reaction on Heat and Mass Transfer by Unsteady Slip Flow from a Non-Isothermal Stretching Sheet Immersed in a Porous Medium, *J. Comput. Theor. Nanosci.*, vol. **12**, no. 11, pp. 4056–4062, 2015.

Hossam, A.N., Boumazgour, M., and Rashad, A.M., Group Method Analysis of Mixed Convection Stagnation-Point Flow of Non-Newtonian Nanofluid over a Vertical Stretching Surface, *Indian J. Phys.*, vol. **91**, no. 7, pp. 731–742, 2017.

Ibrahim, F.S., Elaiw, A.M., and Bakr, A.A., Influence of Viscous Dissipation and Radiation on Unsteady MHD Mixed Convection Flow of Micropolar Fluids, *Appl. Math. Inf. Sci.*, vol. **2**, pp. 143–162, 2008.

Ishak, A., Similarity Solutions for Flow and Heat Transfer over a Permeable Surface with Convective Boundary Condition, *Appl. Math. Comput.*, vol. **217**, pp. 837–842, 2010.

Krishna, D.V., Prasad, R.S., and Krishna, M.V., Unsteady Hydro Magnetic Flow of an Incompressible Viscous Fluid in a Rotating Parallel Plate Channel with Porous Lining, *J. Pure Appl. Phys.*, vol. **21**, pp. 131–137, 2009.

Makinde, O.D. and Aziz, A., MHD Mixed Convection from a Vertical Plate Embedded in a Porous Medium with a Convective Boundary Condition, *Int. J. Therm. Sci.*, vol. **49**, pp. 1813–1820, 2010.

Makinde, O.D., Free Convection Flow with Thermal Radiation and Mass Transfer past a Moving Vertical Porous Plate, *Int. Commun. Heat Mass Transf.*, vol. **32**, pp. 1411–1419, 2005.

Malathy, T., Srinivas, S., and Reddy, A.S., Chemical Reaction and Radiation Effects on MHD Pulsatile Flow of an Oldroyd-B Fluid in a Porous Medium with Slip and Convective Boundary Conditions, *J. Porous Media*, vol. **20**, no. 4, pp. 287–301, 2017.

Mallikarjuna, M., Rashad, A.M., Hussein, A.K., and Raju, S.H., Transpiration and Thermophoresis Effects on Non-Darcy Convective Flow past a Rotating Cone with Thermal Radiation, *Arabian J. Sci. Eng.*, vol. **41**, no. 11, pp. 4691–4700, 2016.

Mondal, P.K. and Mukherjee, S., An Analytical Approach to the Effect of Viscous Dissipation on Shear-Driven Flow between Two Parallel Plates with Constant Heat Flux Boundary Conditions, *Int. J. Eng.*, vol. **26**, no. 5, pp. 533–542, 2013.

Nachtsheim, P.R. and Swigert, P., Satisfaction of Asymptotic Boundary Conditions in Numerical Solution of Systems of Nonlinear Equations of Boundary-Layer Type, NASA Technical Note D-3004, NASA Scientific and Technical Publications, Washington, DC, pp. 1–34, 1965.

Olajuwon, B.I., Convection Heat and Mass Transfer in a Hydromagnetic Flow of a Second Grade Fluid in the Presence of Thermal Radiation and Thermal Diffusion, *Int. Commun. Heat Mass Transf.*, vol. **38**, pp. 377–382, 2011.

Pal, D. and Chatterjee, S., Heat and Mass Transfer in MHD Non-Darcian Flow of a Micropolar Fluid over a Stretching Sheet Embedded in a Porous Media with Non-Uniform Heat Source and Thermal Radiation, *Commun. Nonlinear Sci. Numer. Simul.*, vol. **15**, pp. 1843–1857, 2010.

Pal, D. and Chatterjee, S., Mixed Convection Magnetohydrodynamic Heat and Mass Transfer past a Stretching Surface in a Micropolar Fluid-Saturated Porous Medium under the Influence of Ohmic Heating, Soret and Dufour Effects, *Commun. Nonlinear Sci. Numer. Simul.*, vol. **16**, pp. 1329–1346, 2011.

Pal, D. and Mondal, H., MHD Non-Darcian Mixed Convection Heat and Mass Transfer over a Non-Linear Stretching Sheet with

Soret–Dufour Effects and Chemical Reaction, *Int. Commun. Heat Mass Transf.*, vol. **38**, pp. 463–467, 2011.

Pal, D. and Mondal, H., Influence of Chemical Reaction and Thermal Radiation on Mixed Convection Heat and Mass Transfer over a Stretching Sheet in Darcian Porous Medium with Soret and Dufour Effects, *Energy Convers. Manage.*, vol. **62**, pp. 102–108, 2012a.

Pal, D. and Mondal, H., Soret and Dufour Effects on MHD Non-Darcian Mixed Convection Heat and Mass Transfer over Stretching Sheet with Non-Uniform Heat Source/Sink, *Physica B*, vol. **407**, pp. 642–651, 2012b.

Pal, D. and Mondal, H., Influence of Thermophoresis and Soret-Dufour on Magneto Hydro Dynamic Heat and Mass Transfer over a Non-Isothermal Wedge with Thermal Radiation and Ohmic Dissipation, *J. Magn. Magn. Mater.*, vol. **331**, pp. 250–255, 2013.

Rahman, M.M., Locally Similar Solutions for Hydromagnetic and Thermal Slip Flow Boundary Layers over a Flat Plate with Variable Fluid Properties and Convective Surface Boundary Condition, *Meccanica*, vol. **46**, no. 5, pp. 1127–1143, 2011.

Rashidi, M.M., Siddiqui, A.M., and Asadi, M.O., Application of Homotopy Analysis Method to the Unsteady Squeezing Flow of a Second Grade Fluid between Circular Plates, *Math. Probl. Eng.*, vol. **2010**, pp. 1–18, 2010.

Reddy, M.G. and Reddy, N.B., Mass Transfer and Heat Generation Effects on MHD Free Convection Flow past an Inclined Vertical Surface in a Porous Medium, *J. Appl. Fluid Mech.*, vol. **4**, no. 3, pp. 7–11, 2011.

Sakiadis, B.C., Boundary Layer Behaviour on Continuous Solid Surface: I. Boundary Layer on a Continuous Flat Surface, *Am. Inst. Chem. Eng. J.*, vol. **7**, pp. 213–215, 1961.

Selim, A., Hossain, M.A., and Rees, D.A.S., The Effect of Surface Mass Transfer on Mixed Convection Flow past a Heated Vertical Flat Permeable Plate with Thermophoresis, *Int. J. Therm. Sci.*, vol. **42**, pp. 973–982, 2003.

Shahnazari, M.R., Ahmadi, Z., and Masooleh, L.S., Perturbation Analysis of Heat Transfer and a Novel Method for Changing the Third Kind Boundary Condition into the First Kind, *J. Porous Media*, vol. **20**, no. 5, pp. 449–460, 2017.

Sheikholeslami, M., Effect of Spatially Variable Magnetic Field on Ferro Fluid Flow and Heat Transfer Considering Constant Heat Flux Boundary Condition, *Euro. Phys. J.*, vol. **129**, p. 248, 2014a.

Sheikholeslami, M., KKL Correlation for Simulation of Nanofluid Flow and Heat Transfer in a Permeable Channel, *Phys. Lett. A*, vol. **378**, no. 45, pp. 3331–3339, 2014b.

Sheikholeslami, M., Influence of Magnetic Field on Nanofluid Free Convection in an Open Porous Cavity by Means of Lattice Boltzmann Method, *J. Mol. Liq.*, vol. **234**, pp. 364–374, 2016.

Sheikholeslami, M., Magnetohydrodynamic Nanofluid Forced Convection in a Porous Lid Driven Cubic Cavity Using Lattice Boltzmann Method, *J. Mol. Liq.*, vol. **231**, pp. 555–565, 2017a.

Sheikholeslami, M., Magnetic Field Influence on Nanofluid Thermal Radiation in a Cavity with Tilted Elliptic Inner Cylinder, *J. Mol. Liq.*, vol. **229**, pp. 137–147, 2017b.

Sheikholeslami, M. and Ganji, D.D., Nanofluid Flow and Heat Transfer between Parallel Plates Considering Brownian Motion Using DTM, *Comput. Methods Appl. Mech. Eng.*, vol. **283**, pp. 651–663, 2015.

Sheikholeslami, M. and Rashidi, M.M., Effect of Space Dependent Magnetic Field on Free Convection of Fe_3O_4–Water Nanofluid, *J. Taiwan Inst. Chem. Eng.*, vol. **56**, pp. 6–15, 2015.

Sheikholeslami, M., Ganji, D.D., and Rashidi, M.M., Ferro Fluid Flow and Heat Transfer in a Semi Annulus Enclosure in the Presence of Magnetic Source Considering Thermal Radiation, *J. Taiwan Inst. Chem. Eng.*, vol. **47**, pp. 6–17, 2015a.

Sheikholeslami, M., Ganji, D.D., Javed, M.Y., and Ellahi, R., Effect of Thermal Radiation on Magnetohydrodynamics Nanofluid Flow and Heat Transfer by Means of Two Phase Model, *J. Magn. Magn. Mater.*, vol. **374**, pp. 36–43, 2015b.

Sheikholeslami, M., Rashidi, M.M., and Ganji, D.D., Effect of Non-Uniform Magnetic Field on Forced Convection Heat Transfer of Fe_3O_4–Water Nanofluid, *Comput. Methods Appl. Mech. Eng.*, vol. **294**, pp. 299–312, 2015c.

Sheikholeslami, M., Hayat, T., and Alsaedi, A., MHD Free Convection of Al_2O_3–Water Nanofluid Considering Thermal Radiation: A Numerical Study, *Int. J. Heat Mass Transf.*, vol. **96**, pp. 513–524, 2016a.

Sheikholeslami, M., Soleimani, S., and Ganji, D.D., Effect of Electric Field on Hydro Thermal Behaviour of Nanofluid in a Complex Geometry, *J. Mol. Liq.*, vol. **213**, pp. 153–161, 2016b.

Sheikholeslami, M., Vajravelu, K., and Rashidi, M.M., Forced Convection Heat Transfer in a Semi Annulus under the Influence of a Variable Magnetic Field, *Int. J. Heat Mass Transf.*, vol. **92**, pp. 339–348, 2016c.

Vajravelu, K. and Rollins, D., Hydromagnetic Flow of a Second Grade Fluid over a Stretching Sheet, *Appl. Math. Comput.*, vol. **148**, pp. 783–791, 2004.

Veera Krishna, M. and Chamkha, A.J., Hall Effects on Unsteady MHD Flow of Second Grade Fluid through Porous Medium with Ramped Wall Temperature and Ramped Surface Concentration, *Phys. Fluids*, vol. **30**, no. 5, p. 053101, 2018.

Veera Krishna, M. and Chamkha, A.J., MHD Peristaltic Rotating Flow of a Couple Stress Fluid through a Porous Medium with Wall and Slip Effects, *Special Topics Rev. Porous Media – Int. J.*, vol. **10**, no. 3, pp. 245–258, 2019a.

Veera Krishna, M. and Chamkha, A.J., Hall Effects on MHD Squeezing Flow of a Water based Nano Fluid between Two Parallel Disks, *J. Porous Media*, vol. **22**, no. 2, pp. 209–223, 2019b.

Veera Krishna, M. and Chamkha, A.J., Hall and Ion Slip Effects on MHD Rotating Boundary Layer Flow of Nanofluid past an Infinite Vertical Plate Embedded in a Porous Medium, *Results Phys.*, **15**, p. 102652, 2019c.

Veera Krishna, M. and Chamkha, A.J., Hall and Ion Slip Effects on Unsteady MHD Convective Rotating Flow of Nanofluids – Application in Biomedical Engineering, *J. Egypt. Math. Soc.*, vol. **28**, no. 1, pp. 1–14, 2020a.

Veera Krishna, M. and Chamkha, A.J., Hall and Ion Slip Effects on MHD Rotating Flow of Elastico-Viscous Fluid through Porous Medium, *Int. Commun. Heat Mass Transf.*, vol. **113**, p. 104494, 2020b.

Veera Krishna, M. and Gangadhar Reddy, M., MHD Free Convective Rotating Flow of Visco-Elastic Fluid past an Infinite Vertical Oscillating Porous Plate with Chemical Reaction, *IOP Conf. Ser.: Mater. Sci. Eng.*, vol. **149**, p. 012217, 2016.

Veera Krishna, M. and Jyothi, K., Heat and Mass Transfer on MHD Rotating Flow of a Visco-Elastic Fluid through Porous Medium with Time Dependent Oscillatory Permeability, *J. Anal.*, vol. **27**, no. 1, pp. 643–662, 2019.

Veera Krishna, M. and Malashetty, S.G., Hall Effects on Unsteady Flow of an Incompressible Electrically Conducting Second Grade Fluid through a Porous Medium in a Rotating Parallel Plate Channel Bounded by a Porous Bed, *Int. J. Adv. Sci. Technol.*, vol. **2**, pp. 87–102, 2011.

Veera Krishna, M. and Malashetty, S.G., Unsteady Flow of an Incompressible Electrically Conducting Second Grade Fluid through a Porous Medium in a Rotating Parallel Plate Channel Bounded by a Porous Bed, *Int. J. Appl. Math. Mech.*, vol. **8**, pp. 61–84, 2012.

Veera Krishna, M. and Prakash, J., Hall Effects on Unsteady MHD Flow of Rotating Parallel Plate Channel Bounded by Porous Bed on the Lower Half – Darcy Lapwood Model, *J. Fluid Dyn.*, vol. **5**, pp. 275–294, 2015.

Veera Krishna, M. and Subba Reddy, G., Unsteady MHD Convective Flow of Second Grade Fluid through a Porous Medium in a Rotating Parallel Plate Channel with Temperature Dependent Source, *IOP Conf. Ser.: Mater. Sci. Eng.*, vol. **149**, p. 012216, 2016.

Veera Krishna, M. and Subba Reddy, G., Unsteady MHD Reactive Flow of Second Grade Fluid through Porous Medium in a Rotating Parallel Plate Channel, *J. Anal.*, vol. **27**, no. 1, pp. 103–120, 2019.

Veera Krishna, M., Suneetha, S.V., and Siva Prasad, R., Steady Hydro Magnetic Flow of a Couple Stress Fluid through a Composite Medium in a Rotating Parallel Plate Channel with Porous Bed on the Lower Half, *Proc. of 54th Congress of Indian Society of Theoretical and Applied Mechanics*, New Delhi, India, pp. 140–149, December 18–21, 2009.

Veera Krishna, M., Suneetha, S.V., and Siva Prasad, R., Hall Effects on Unsteady Hydro Magnetic Flow of an Incompressible Viscous Fluid in a Rotating Parallel Plate Channel with Porous Lining, *J. Ultra Sci. Phys. Sci.*, vol. **22**, pp. 95–106, 2010a.

Veera Krishna, M., Siva Prasad, R., and Krishna, D.V., Hall Effects on Steady Hydro Magnetic Flow of a Couple Stress Fluid through a Composite Medium in a Rotating Parallel Plate Channel with Porous Bed on the Lower Half, *Adv. Studies Contemp. Math.*, vol. **20**, no. 2, pp. 457–468, 2010b.

Veera Krishna, M., Swarnalathamma, B.V., and Chamkha, A.J., Heat and Mass Transfer on Magnetohydrodynamic Chemically Reacting Flow of Micropolar Fluid through a Porous Medium with Hall Effects, *Special Topics Rev. Porous Media – Int. J.*, vol. **9**, no. 4, pp. 347–364, 2018a.

Veera Krishna, M., Swarnalathamma, B.V., and Prakash, J., Heat and Mass Transfer on Unsteady MHD Oscillatory Flow of Blood through Porous Arteriole, *Appl. Fluid Dyn.*, *Lecture Notes Mech. Eng.*, vol. **XXII**, pp. 207–224, 2018b.

Veera Krishna, M., Subba Reddy, G., and Chamkha, A.J., Hall Effects on Unsteady MHD Oscillatory Free Convective Flow of Second Grade Fluid through Porous Medium between Two Vertical Plates, *Phys. Fluids*, vol. **30**, no. 2, p. 023106, 2018c.

Veera Krishna, M., Jyothi, K., and Chamkha, A.J., Heat and Mass Transfer on Unsteady, Magnetohydrodynamic, Oscillatory Flow of Second-Grade Fluid through a Porous Medium between Two Vertical Plates, under the Influence of Fluctuating Heat Source/Sink, and Chemical Reaction, *Int. J. Fluid Mech. Res*., vol. **45**, no. 5, pp. 459–477, 2018d.

Veera Krishna, M., Gangadhara Reddy, M., and Chamkha, A.J., Heat and Mass Transfer on MHD Free Convective Flow over an Infinite Non-Conducting Vertical Flat Porous Plate, *Int. J. Fluid Mech. Res*., vol. **46**, no. 1, pp. 1–25, 2019a.

Veera Krishna, M., Gangadhar Reddy, M., and Chamkha, A.J., Heat and Mass Transfer on MHD Rotating Flow of Second Grade Fluid past an Infinite Vertical Plate Embedded in Uniform Porous Medium with Hall Effects, *Appl. Math. Sci. Comput.*, *Trends Math.*, vol. **1**, pp. 417–427, 2019b.

Veera Krishna, M., Bharathi, K., and Chamkha, A.J., Hall Effects on MHD Peristaltic Flow of Jeffrey Fluid through Porous Medium in a Vertical Stratum, *Interfacial Phenom. Heat Transf.*, vol. **6**, no. 3, pp. 253–268, 2019c.

Veera Krishna, M., Anand, P.V.S., and Chamkha, A.J., Heat and Mass Transfer on Free Convective Flow of a Micropolar Fluid through a Porous Surface with Inclined Magnetic Field and Hall Effects, *Special Topics Rev. Porous Media – Int. J.*, vol. **10**, no. 3, pp. 203–223, 2019d.

Veera Krishna, M., Swarnalathamma, B.V., and Chamkha, A.J., Investigations of Soret, Joule and Hall Effects on MHD Rotating Mixed Convective Flow past an Infinite Vertical Porous Plate, *J. Ocean Eng. Sci.*, vol. **4**, no. 3, pp. 263–275, 2019e.

Veera Krishna, M., Heat Transport on Steady MHD Flow of Copper and Alumina Nanofluids past a Stretching Porous Surface, *Heat Transf.*, vol. **49**, no. 3, pp. 1374–1385, 2020.

Veera Krishna, M., Ameer Ahamad, N., and Chamkha, A.J., Hall and Ion Slip Effects on Unsteady MHD Free Convective Rotating Flow through a Saturated Porous Medium over an Exponential Accelerated Plate, *Alexandria Eng. J.*, vol. **59**, pp. 565–577, 2020a.

Veera Krishna, M., Sravanthi, C.S., and Gorla, R.S.R., Hall and Ion Slip Effects on MHD Rotating Flow of Ciliary Propulsion of Microscopic Organism through Porous Media, *Int. Commun. Heat Mass Transf.*, vol. **112**, p. 104500, 2020b.

Zheng, L., Wang, L., Zhang, X., and Ma, L., Unsteady Flow and Heat Transfer over a Stretching Sheet with Radiation in the Presence of a Magnetic Field, *Chem. Eng. Commun.*, vol. **99**, no. 1, pp. 1–16, 2012.

Zueco, J., Beg, O.A., and Lopez-Ochoa, L.M., Effects of Thermophoresis Particle Deposition and of the Thermal Conductivity in a Porous Plate with Dissipative Heat and Mass Transfer, *Acta Mech. Sin.*, vol. **27**, pp. 389–398, 2011.