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Scrutinisation of Chebyshev collocation method for mass transfer on a continuous flat plate moving in parallel to a free stream in the presence of a chemical reaction

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ABSTRACT

The paper presents the boundary layer flow of mass transfer on a continuous flat plate moving in parallel or reversely to a free stream with a chemical reaction. By using suitable similarity transformations, the boundary-layer equations are transformed into coupled nonlinear ordinary differential equations (NODEs) over semi-infinite interval. These equations have been analysed using a novel semi-numerical method, viz. spectral method. The dual solutions for velocity and concentration distributions are determined using Chebyshev collocation method (CCM) and the results are presented in the form of tables and graphs. The obtained spectral solutions are compared with previously published results and are comparable. Many interesting physical properties of the problem are observed and verified through both theoretical as well as semi-numerical approach. The derived quantities show that the mass transfer rate is established to be increased as the Schmidt number increases for the solution of the upper branch and reduces for the solution of lower branch.

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Boundary-layer flow; mass transfer; dual solutions; chemical reaction; spectral method; Chebyshev collocation method

Nomenclature

C	concentration kg/m^3
C_w	plate concentration kg/m^3
C_∞	concentration in the free stream kg/m^3
C_r^*	Shifted Chebyshev polynomial
D	diffusion coefficient m^2/s
f	non-dimensional stream function
f'	dimensionless velocity
L	reference length m
R	variable reaction rate 1/s
R_0	constant
Sc	Schmidt number
U	composite velocity m/s
U_w	plate velocity m/s
U_∞	free stream velocity m/s
u	velocity component in x direction m/s
v	velocity component in y direction m/s
x	distance along the plate m
y	distance normal to the plate m

Greek letters

β	reaction rate parameter
ε	velocity ratio parameter
η	similarity variable
μ	coefficient of fluid viscosity kg/(m.s)
υ	kinematic viscosity m^2/s
φ	non-dimensional concentration
ρ	fluid density kg/m^3
ψ	stream function

Subscripts

w	condition at the plate
∞	condition in the free stream
r	polynomial degree

1. Introduction

The study of two-dimensional incompressible viscous boundary layer flow fast a flat plate moving with parallel to a free stream in the appearance of a chemical reaction plays an important role in science and engineering fields. The first investigation of moving steady viscous incompressible laminar boundary layer fluid flow with a constant velocity of a flat plate was reported by Blasius (1908). Pohlhausen (1921) examined the same Blasius problem with heat transfer. Later, Howarth (1938) analysed the numerical solution with different aspects of Blasius problem. Abu-Sitta (1994) used the Weyl technique to analyse the existence of a solution for Blasius problem. Sakiadis (1961) investigated the equations, identical to those of Blasius with distinct boundary conditions by considering both laminar and turbulent boundary layer flow on a moving continuous flat plate moving with fixed velocity in a stationary fluid and compared the results with flat plate of finite length. Abdelhafez (1985) analysed the same kind of flow (Sakiadis 1961) but in parallel stream and confirmed that there are two important special cases of his problems, viz. Blasius and Sakiadis problems. Afzal, Badaruddin, and Elgarvi (1993) introduced the composite velocity $U = U_w + U_\infty$, where U_∞ and U_w are the free stream velocity and the velocity of moving plate, respectively, to develop a single set of boundary conditions and to avoid the separate consideration of U_∞ and U_w as concluded

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by Abdelhafez (1985). The influence of thermal radiation on laminar boundary layer flow was investigated by Bataller (2008) and Cortell (2008), which resulted in the extension of Blasius and Sakiadis problems, respectively. Blasius problem with nonstandard boundary conditions for upstream moving flat plate with uniform velocity was investigated by Hussaini, Lakin, and Nachman (1987). Lin, Wu, and Hoh (1993) reported problem arises with the motion of both ambient fluid and isothermal flat plate under the mixed convection condition with free stream in parallel or reversely. Sachdev, Bujurke, and Awati (2005) investigated various problems of boundary layer flows from stretching sheet through semi-numerical scheme. Afzal, Badaruddin, and Elgarvi (1993) work was extended by Ishak, Nazar, and Pop (2007a) and studied the same problem by considering the effects of suction and injection. The same kind of analysis in this area with various conditions were noted by many researchers (Ishak, Nazar, and Pop 2007b, 2007c, 2009a, 2009b; Weidman, Kubitschek, and Davis 2006; Ishak 2009; Mukhopadhyay, Bhattacharyya, and Layek 2011) and predicted the dual solutions. The investigation of dual solutions of various crucial boundary layer problems was discussed by (Wang 2008; Ishak, Nazar, and Pop 2008; Ishak, Lok, and Pop 2010; Bhattacharyya and Layek 2011a; Bhattacharyya, Mukhopadhyay, and Layek 2011; Rosali, Ishak, and Pop 2011; Bhattacharyya and Vajravelu 2012; Bachok, Ishak, and Pop 2012).

The advanced study of phenomenal chemical kinetics was highly influenced by mass transfer problems. In last few decades the combination of heat transfer in the presence of mass transfer has developed greater interest among the researchers for its immense applications in various fields of engineering, viz. chemical process, separation and enormous sub-disciplines of technological aspects. Chambre and Young (1958) considered the first order chemical reaction in the vicinity of flat plate of a boundary layer flow and analysed diffusion of a chemically reactive species. Gebhart and Pera (1971) demonstrated the free convection flow due to the interaction of gravitational force and density difference caused by the diffusion of chemical species and thermal energy. On the other hand, the researchers (Soundalgekar 1979; Soundalgekar, Birajdar, and Darwhekar 1984; Das, Deka, and Soundalgekar 1994; Muthucumaraswamy and Ganesan 2001) investigated the effects of mass transfer on the flow past of an infinite vertical plate, under several physical conditions. Fan, Shi, and Xu (1998) analysed mixed convection with chemical reaction and diffusion over a moving horizontal plate using similarity transformations. Anjali Devi and Kandasamy (2000) used R.K. Gill's method to analyse the influence of magnetic field on heat and mass transfer in MHD flow over a semi-infinite plate under the effect of chemical reaction. Recently, Awati (2017) demonstrated the series solution of boundary layer flow developed in water-based nanofluid over a semi-infinite flat plate moving with uniform velocity. Awati, Mahesh Kumar, and Wakif (2021) scrutinised the heat and mass transfer of convective boundary layer flow of a moving nanofluid on a nonlinearly stretching sheet, employing the simplest Haar wavelet method. The complete similarity solutions for forced convective MHD boundary layer flow and mass diffusion were examined in the presence of chemical reaction on a porous flat plate with suction/blowing was discussed by Bhattacharyya and Layek (2012). Awati and Mahesh Kumar (2021) used Haar wavelet collocation method to analyse water-based nanofluids for heat transfer and forced convection boundary layer flow past a semi-infinite flat plate for both static and moving cases. Recently, Bhatti et al. (2022) analysed the application of water-based hybrid nanofluid flow with diamond and silica nanoparticles in solar collector. Zhang et al. (2022) investigated mixed convection flow over a nonlinear elastic porous surface under viscous dissipation. The modern researchers (Andersson, Hansen, and Holmedal 1994; Chamkha, Aly, and Mansour 2010; Kandasamy, Periasamy, and Sivagnana Prabhu 2005; Bhattacharyya and Layek 2010, 2011b; Bhattacharyya 2011; Bhattacharyya, Mukhopadhyay, and Layek 2012) investigated the impact of stretching/shrinking sheet caused by chemical reaction on a flow.

The Chebyshev polynomials were probably first introduced by Chebyshev and these polynomials play an important role in the field of numerical computations from last few decades due to their rich properties like orthogonality, minimax property, etc. In advanced numerical computation worldwide, these polynomials signify their own impact in all aspects of numerical analysis. The many theories and applications of Chebyshev polynomials are presented by Fox and Parker (1968), Boyd (2000), Mason and Handscomb (2002). Clenshaw (1957) analysed the solution of linear ordinary differential equations (LODEs) by expressing it in terms of Chebyshev series. Sezer and Kaynak (1996) examined the solutions of LODEs using Chebyshev-matrix method, which involves the Chebyshev polynomials. Later, this method is improved and applied to higher order nonlinear ordinary differential equations (NODEs) with variable coefficients to obtain approximate solutions by using truncated Chebyshev series expansion (TCSE) as discussed by Akyüz-Daşcıoğlu and Çerdïk-Yaslan (2011). Kudenatti, Misbah, and Bharathi (2021) used Chebyshev collocation methods (CCM) for the solution of boundary-layer flow problems with power-law fluid over a moving wedge which is defined on a semi-infinite domain.

Inspired by modern technological applications, the present paper reflects the influence of chemical reaction on mass transfer over a flat plate moving continuously with uniform velocity moving parallel or opposite to uniform free stream. The similarity transformations convert the governing momentum and mass transfer equations into nonlinear self-similar ordinary differential equations. In this paper, we employ the CCM to obtain the solution of self-similar equations, which involves several parameters like the reaction rate parameter, velocity ratio parameter, Schmidt number and solute distributions. The obtained results are presented in terms of table and figures.

2. Mathematical formulation

Let us consider a two-dimensional steady, viscous incompressible laminar boundary layer flow and mass transfer with first order chemical reaction above a flat plate. The plate is advancing with U_w as a uniform velocity in the direction of or opposite to U_∞ a constant velocity in free stream. The x-direction is considered for the length of the plate and y-direction elongates across upmost, i.e. normal to the plate. The configuration of the governing problem is presented in Figure 1.

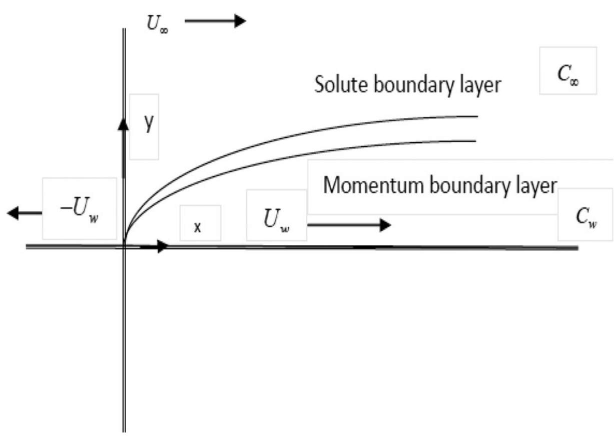

Figure 1. The schematic diagram of the physical problem.

The governing motion of steady state two-dimensional incompressible boundary layer flow equations with mass transfer can be written as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}, \tag{2}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}-R(C-C_\infty), \tag{3}$$

where u and v are components of velocity in x and y axis, $\upsilon(=\mu/\rho)$ be the kinematic viscosity, μ be the coefficient of fluid viscosity, ρ be the density of the fluid, C be the concentration, D be the diffusion coefficient, C_∞ be the value of the concentration in the free stream, $R(x)=LR_0/x$ be the variable reaction rate, L be the reference length and R_0 is constant. The relevant boundary conditions for components of velocity and concentrations are given as

$$u=U_w,\ v=0 \text{ at } y=0;\ u\to U_\infty \text{ as } y\to\infty, \tag{4}$$

$$\text{and } C=C_w \text{ at } y=0;\ C\to C_\infty \text{ as } y\to\infty, \tag{5}$$

where C_w be the plate concentration. The stream function $\psi(x,y)$ is defined as

$$u=\frac{\partial\psi}{\partial y} \text{ and } v=-\frac{\partial\psi}{\partial x}. \tag{6}$$

By using Equation (6), continuity Equation (1) is automatically satisfied, the momentum Equation (2) and the concentration Equation (3) takes the following forms

$$\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial x\partial y}-\frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y^2}=\upsilon\frac{\partial^3\psi}{\partial y^3}, \tag{7}$$

and

$$\frac{\partial\psi}{\partial y}\frac{\partial C}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}-R(C-C_\infty). \tag{8}$$

Using Equation (6), the components of velocity in the boundary conditions (4) reduces to

$$\frac{\partial\psi}{\partial y}=U_w,\ \frac{\partial\psi}{\partial x}=0 \text{ at } y=0;\ \frac{\partial\psi}{\partial y}\to U_\infty \text{ as } y\to\infty. \tag{9}$$

Introducing non-dimensional variables for ψ and C as

$$\psi=\sqrt{U\upsilon x}f(\eta) \text{ and } C=C_\infty+(C_w-C_\infty)\varphi(\eta), \tag{10}$$

where $\eta=y\sqrt{\frac{U}{x\upsilon}}$ be the similarity transformation and $U=U_w+U_\infty$ be the combined velocity (Afzal, Badaruddin, and Elgarvi 1993). Substituting Equation (10) into Equations (7) and (8), we get self-similar equations as

$$f'''+\frac{1}{2}ff''=0, \tag{11}$$

$$\text{and } \varphi''+\frac{1}{2}Scf\varphi'-Sc\beta\varphi=0, \tag{12}$$

where $Sc=\upsilon/D$ be the Schmidt number and $\beta=LR_0/U$ be the reaction rate parameter. Remark that the chemical reaction is constructive if $\beta<0$ and it is destructive if $\beta>0$. The reduced conditions of Equations (9) and (5) become

$$f(\eta)=0,\ f'(\eta)=\varepsilon \text{ at } \eta=0;\ f'(\eta)\to 1-\varepsilon \text{ as } \eta\to\infty, \tag{13}$$

$$\text{and } \varphi(\eta)=1, \text{ at } \eta=0;\ \varphi(\eta)\to 0 \text{ as } \eta\to\infty, \tag{14}$$

where $\varepsilon=U_w/U$ be the ratio of velocity parameter.

3. Method of solution

We seek a CCM solution of Equations (11) and (12) with relevant boundary conditions (13) and (14), for this first transform the domain of the problem from semi-infinite domain into shifted Chebyshev polynomial domain, i.e. semi-infinite domain can be converted into finite interval [0, 1] by using the following transformations (Kudenatti, Misbah, and Bharathi 2021)

$$\bar\eta=\frac{\eta}{\eta_\infty}, \tag{15}$$

$$f(\eta)=\eta_\infty f(\bar\eta) \text{ and } \varphi(\eta)=\eta_\infty\varphi(\bar\eta), \tag{16}$$

where η_∞ be the thickness of the boundary layer. Using Equations (15) and (16) into Equations (11) and (14), we get the following system of equations in the form

$$f'''+\frac{\eta_\infty^2}{2}ff''=0, \tag{17}$$

$$\text{and } \varphi''+\frac{1}{2}Sc\eta_\infty^2 f\varphi'-Sc\beta\eta_\infty^2\varphi=0 \tag{18}$$

The transformed boundary conditions become

$$f(\bar\eta)=0,\ f'(\bar\eta)=\varepsilon \text{ at } \bar\eta=0;\ f'(\bar\eta)=1-\varepsilon \text{ at } \bar\eta=1 \tag{19}$$

$$\text{and } \varphi(\bar\eta)=\frac{1}{\eta_\infty} \text{ at } \bar\eta=0;\ \varphi(\bar\eta)=0;\text{at } \bar\eta=1 \tag{20}$$

where prime indicates the differentiation with respect to $\bar\eta$. Suppose the solution of Equation (17) and its derivative can be represented in terms of truncated shifted Chebyshev series expansion (TSCSE) as (Akyüz-Daşcıoğlu and Çerdİk-Yaslan 2011)

$$f(\bar\eta)=\sum_{r=0}^{N}a_rC_r^*(\bar\eta) \tag{21}$$

$$\text{and}\quad f^{(s)}(\bar{\eta}) = \sum_{r=0}^{N} a_r^{(s)} C_r^*(\bar{\eta}) \tag{22}$$

where a_r and $a_r^{(s)}$, $0 \le r \le N$ are the unknown Chebyshev coefficients of $f(\bar{\eta})$ and its derivatives, which are to be determined, $C_r^*(\bar{\eta})$ denotes the shifted Chebyshev polynomial of degree r defined on the interval $[0,\ 1]$ and N is any non-negative integer such that $3 \le N$.

Let $\bar{\eta}_k$ be the Chebyshev collocation points in natural ordering and is defined as

$$\bar{\eta}_k = \frac{1}{2}\left[1 - \cos\left(\frac{k\pi}{N}\right)\right], k = 0,1,2,3,\ldots\ldots N. \tag{23}$$

Now, to evaluate the function defined in Equation (21) at each Chebyshev collocation points, the matrix representation of this function is of the form

$$\mathbf{f}(\bar{\boldsymbol{\eta}}) = \mathbf{C}^*(\bar{\eta})\mathbf{U} \tag{24}$$

where $\mathbf{f}(\bar{\boldsymbol{\eta}}) = [f(\bar{\eta}_0), f(\bar{\eta}_1), \ldots\ldots, f(\bar{\eta}_N)]^T$, $\mathbf{C}^*(\bar{\boldsymbol{\eta}}) = [C_{i,j}]^T$; $C_{i,j} = C_{i-1}^*(\bar{\eta}_{j-1})$; $i,j = 1,2,\ldots, N+1$, and $\mathbf{U} = [a_0,\ a_1,\ldots\ldots\ldots, a_N]^T$. In the similar manner, Equation (22) can be written in matrix form as

$$\mathbf{f}^{(s)}(\bar{\boldsymbol{\eta}}) = \mathbf{C}^*(\bar{\eta})\mathbf{U}^{(s)} \tag{25}$$

The relation between $\mathbf{U}$ of Equation (24) and $\mathbf{U}^{(s)}$ of Equation (25) is given by (Sezer and Kaynak 1996)

$$\mathbf{U}^{(s)} = 4^{(s)}\mathbf{M}^{(s)}\mathbf{U} \tag{26}$$

Then, Equation (25) takes the form

$$\mathbf{f}^{(s)}(\bar{\boldsymbol{\eta}}) = 4^{(s)}\mathbf{C}^*(\bar{\eta})\mathbf{M}^{(s)}\mathbf{U}, \tag{27}$$

where,

$$\mathbf{M} = \begin{pmatrix} 0 & 1/2 & 0 & 3/2 & 0 & 5/2 & \cdots & m_1 \\ 0 & 0 & 2 & 0 & 4 & 0 & \cdots & m_2 \\ 0 & 0 & 0 & 3 & 0 & 5 & \cdots & m_3 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots & N \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \end{pmatrix}_{(N+1)\times(N+1)}$$

for N is even; $m_1 = 0$, $m_2 = N$, $m_3 = 0$, and N is odd; $m_1 = \frac{N}{2}$, $m_2 = 0$, $m_3 = N$.

Substituting, Equations (24) and (27) into Equation (17), we get

$$4^3\mathbf{C}^*\mathbf{M}^3\mathbf{U} + \frac{(4\eta_\infty)^2}{2}\widehat{\mathbf{C}}\widehat{\mathbf{U}}\mathbf{C}^*\mathbf{M}^2\mathbf{U} = \mathbf{0} \tag{28}$$

$$\text{where,}\quad \widehat{\mathbf{C}} = \begin{pmatrix} \mathbf{C}^*(\bar{\eta}_0) & 0 & \cdots & 0 \\ 0 & \mathbf{C}^*(\bar{\eta}_1) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \mathbf{C}^*(\bar{\eta}_N) \end{pmatrix}_{(N+1)\times(N+1)}$$

$$\text{and}\quad \widehat{\mathbf{U}} = \begin{pmatrix} \mathbf{U} & 0 & \cdots & 0 \\ 0 & \mathbf{U} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \mathbf{U} \end{pmatrix}_{(N+1)\times(N+1)}$$

Now, Equation (28) can be written in the form of

$$\mathbf{AU} = \mathbf{0} \tag{29}$$

where, $\mathbf{A} = 4^3\mathbf{C}^*\mathbf{M}^3 + \frac{(4\eta_\infty)^2}{2}\widehat{\mathbf{C}}\widehat{\mathbf{U}}\mathbf{C}^*\mathbf{M}^2$ is a matrix of order $(N+1)\times(N+1)$ and $\mathbf{0}$ is column vector of order $(N+1)\times 1$. Equation (29) is a nonlinear system of $(N+1)$ equations in $(N+1)$ unknown Chebyshev coefficients a_r, $0 \le r \le N$ and Equation (19) determines non-homogeneous systems of equations in $a_r, r = 1,2,3,\ldots\ldots, N$. Using Equations (19) and (27), the system of equations can be written in terms of matrix form as

$$\widehat{\mathbf{A}}\mathbf{U} = \mathbf{B} \tag{30}$$

where, $\mathbf{B} = [0,\ \varepsilon,\ 1-\varepsilon]^T$.

As Equation (17) satisfies the boundary conditions (19), using Equation (30) in Equation (29), the final version of non-linear system of algebraic equation is written in matrix form as

$$\bar{\mathbf{A}}\mathbf{U} = \bar{\mathbf{B}} \tag{31}$$

Solving Equation (31), gives the unknown Chebyshev coefficients $a_r, r = 1,2,3,\ldots\ldots, N$ and using these coefficients in Equation (21) yields an approximate solution of Equation (17) and it matches Equation (19) approximately.

Further, use these solutions in Equation (18) and we explore the solution of Equation (18) in terms of

$$\varphi(\bar{\eta}) = \sum_{r=0}^{N} b_r C_r^*(\bar{\eta}) \tag{32}$$

and its derivative can be written in the form

$$\varphi^{(s)}(\bar{\eta}) = \sum_{r=0}^{N} b_r^{(s)} C_r^*(\bar{\eta}) \tag{33}$$

Substituting Equations (32) and (33) in Equation (18) and using Equation (23), it can be transformed to compact matrix as

$$4^2\mathbf{C}^*\mathbf{M}^2\mathbf{V} + 2\eta_\infty^2 Sc\widehat{\mathbf{F}}\mathbf{C}^*\mathbf{MV} - Sc\beta\eta_\infty^2\mathbf{C}^*\mathbf{V} = \mathbf{0} \tag{34}$$

where, $\widehat{F} = diag\,(d_j)$; $d_j = f(\bar{\eta}_{j-1})$, $j = 1,2,\ldots, N+1$ and $\mathbf{V} = [b_0, b_1,\ldots\ldots, b_N]^T$.

Now, Equation (34) can be written as

$$\mathbf{WV} = \mathbf{0} \tag{35}$$

where $\mathbf{W} = 4^2\mathbf{C}^*\mathbf{M}^2 + 2\eta_\infty^2 Sc\hat{\mathbf{F}}\mathbf{C}^*\mathbf{M} - Sc\beta\eta_\infty^2\mathbf{C}^*$ and $\mathbf{0}$ is column vector of order $(N+1)\times 1$. Similar procedure as adopted in Equation (30) and using Equations (20) and (27), the two non-homogeneous systems of equations in $b_r, r = 1,2,3,\ldots\ldots, N$ can be written as

$$\widehat{\mathbf{W}}\mathbf{V} = \mathbf{R} \tag{36}$$

where, $\mathbf{R} = [1/\eta_\infty,\ 0]^T$. The same procedure is employed in Equations (35) and (36), as in the case of Equations (29) and (30), which will generate $(N+1)$ system of equations in $(N+1)$ unknown Chebyshev coefficients in terms of

$$\bar{\mathbf{W}}\mathbf{V} = \bar{\mathbf{R}} \tag{37}$$

Now, solving Equation (37) yields an approximate value of Chebyshev coefficients $b_r, r = 1,2,3,\ldots\ldots, N$ and these values are substituted in Equation (32) gives the function $\varphi(\bar{\eta})$ in terms of TSCSE. Finally, using Equations (15) and (16), the results are obtained in terms of original variable η.

4. Programming algorithm

(i) Input N (integer)
(ii) Generate system of equations $\bar{\mathbf{A}}\mathbf{U} = \bar{\mathbf{B}}$ (using CCM)
(iii) Input $\mathbf{U}_{old} = \mathbf{U_0}$, ($\mathbf{U_0}$ is an initial approximation)
(iv) By applying Newton's method, $\mathbf{U}_{new}$ is found.
(v) If $|\mathbf{U}_{new} - \mathbf{U}_{old}| < tol$ then $\mathbf{U}_{new} = \mathbf{U}$, break (The program is finished)
(vi) Else then $\mathbf{U}_{old} = \mathbf{U}_{new}$ and repeat step (i) to (v).

5. Results and discussion

The determination of self-similar Equations (11) and (12) are obtained by using the semi-numerical scheme described above for various values of parameters, viz. the velocity ratio parameter ε, the reaction rate parameter β and Schmidt number Sc. To illustrate the effects of these parameters on the concentration of the fluid, which are demonstrated in terms of graphs and that reflects CCM results.

The system of Equations (31) and (37) are solved by using optimisation technique with the help of MATLAB solver that in turn uses Newton's method. The good choice of initial approximation is required to obtain the solution, and it produces desirable solutions of the system of equations. For various values of ε, the semi-numerical results obtained here for the skin-friction coefficients $f''(0)$ which agree with results obtained previously by Blasius (1908), Sakiadis (1961), Ishak, Nazar, and Pop (2009a) and Bhattacharyya (2012) as demonstrated in Table 1. There are mainly four different cases considered, depending on the values of ε. In the first case ε lies in the interval (0, 1), i.e. plate and fluid move in identical directions and for the second case ε is identically zero, i.e. for fixed plate reported by Blasius (1908). In third case $\varepsilon < 0$ or $\varepsilon > 1$, i.e. the plate and the fluid move in reverse directions and in the final case $\varepsilon = 1$ i.e. for moving plate with zero fluid velocity as discussed by Sakiadis (1961).

To obtain CCM results in the present study, minimum 32 terms are required in TSCSE (i.e. $N = 31$). As $N = 34$, the time taken by algorithm to obtain the required values of skin-friction $f''(0)$ for upper branch solution is approximately 45.57 s and for lower branch solution it is approximately 23.8 s. To analyse the convergence of CCM results, attempt is made to increase the degree of polynomial, i.e. by taking sufficiently large number of terms in Equation (21) (i.e. approximately $N = 45$). In all the computations 10^{-6} is the default error tolerance set for this algorithm. The accurate solutions of $f''(0)$ are obtained for different values of ε and are presented in Table 1 along with the number of iterations. For this, the algorithm performs approximately same number of iterations, viz. $N = 34$, but time consumed for upper branch solution is approximately 147.07 s and lower branch solution it requires 71.75 s. This increase in time is acceptable as the number of unknown Chebyshev coefficients is increased from 35 to 46. Also, the number of equations in the system (31) and (37) increases accordingly.

In the present analysis $\varepsilon \le 1$ is considered and it is observed that unique solution is obtained for $0 < \varepsilon$, there exists two solutions for $-0.548257 \le \varepsilon \le 0$ and no boundary layer solution occur for $\varepsilon < -0.548257$, in this case, the surface of the plate is separated from boundary layer. The observations are similar to Ishak, Nazar, and Pop (2009a) and Bhattacharyya (2012). It demonstrates that more accurate value of ε for the existence of solution is obtained as compared to earlier findings (i.e.$-0.548257 \le \varepsilon \le 0$).

Figure 2 presents the profile of skin friction coefficient $f''(0)$ for different values of ε and guarantees the existence of dual solutions. For $0 \le \varepsilon \le 1$, it is concluded that the solution of $f''(0)$ strictly decreases for increasing ε. When $-0.548257 \le \varepsilon \le 0$, for the lower branch solution $f''(0)$ strictly decreases as an increase in ε and for upper branch solution $f''(0)$ increases initially with ε, eventually it decreases with ε (after some negative values).

Figures 3 and 4 show the concentration gradient at the plate $-\varphi'(0)$ for various values of Schmidt number Sc and reaction rate parameter β, it predicts that concentration gradient is proportional to mass transfer rate. Also, it is observed from the figures which exhibit dual character of concentration gradient in velocity field. The value of $-\varphi'(0)$ increase with increase in Sc and ε for the solution of upper branch and strictly decreases for the solution of lower branch with decrease in Sc and increase in ε. In present investigation, the nature of solution is studied by increasing value of Schmidt number up to 3 ($Sc = 3$) as compared to Bhattacharyya (2012) and noticed that the behaviour of concentration gradient is unaltered. For some values of ε, the constructive ($\beta < 0$) chemical reaction the mass absorption takes place and for lower branch solution, it is at high rate with ascending values of ε. Besides, in the solution of upper branch again mass absorption occurs when ε is close to 1 for the constructive chemical reaction. In the case of destructive chemical

Table 1. The value of skin-friction $f''(0)$ for different values of ε.

			Ishak, Nazar, and Pop (2009a)		Bhattacharyya (2012)		CCM			
ε	Blasius (1908)	Sakiadis (1961)	Upper branch	Lower branch	Upper branch	Lower branch	Upper branch	Number of Iterations	Lower branch	Number of Iterations
−0.5	–	–	0.3990	0.1710	0.39895	0.17103	0.39785	51	0.17103	09
−0.4	–	–	0.4357	0.0834	0.43566	0.08336	0.43560	51	0.08336	22
−0.3	–	–	0.4339	0.0367	0.43387	0.03672	0.43387	52	0.03672	09
−0.2	–	–	0.4124	0.0114	0.412437	0.01143	0.41237	74	0.01143	12
−0.1	–	–	0.3774	0.0010	0.37739	0.00105	0.37741	69	0.00104	22
0	0.332	–	0.3321	–	0.33206	–	0.33206	56	–	
0.5	–	–	0	–	0	–	0	1	–	
1	–	−0.4438	−0.4438	–	−0.44375	–	−0.44375	59	–	

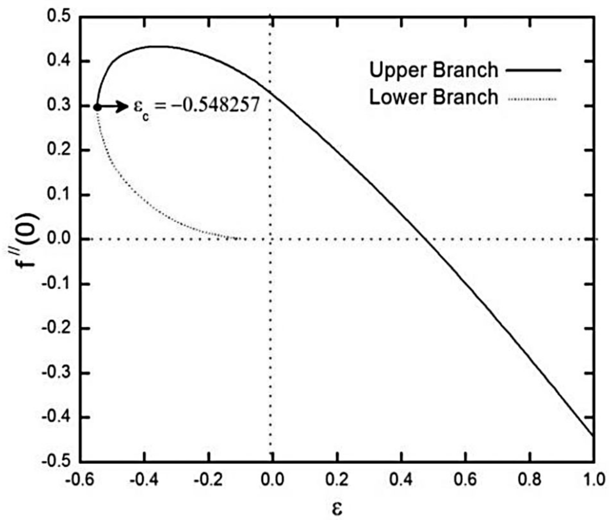

Figure 2. Skin friction coefficient $f''(0)$ for various values of ε.

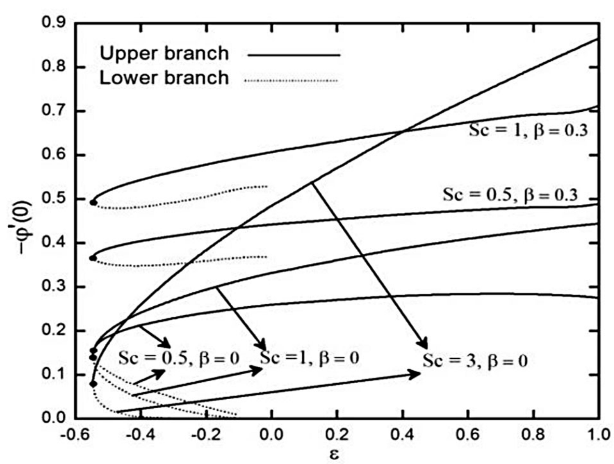

Figure 3. Concentration gradient at the sheet $-\varphi'(0)$ vs. ε for various values of Sc and β.

reaction ($\beta > 0$) the increase mass transfer rate, while in case of constructive it decreases for both solutions. It is important to obtain a stable solution in practice and is noticed that, the solution of upper branch are the only solutions that are physically stable. The lower branch solutions are not found physically stable. Blasius (1908) and Sakiadis (1961) were obtained for the upper branch solution. Also, the upper branch solutions are unique for ε in (0, 1]. To achieve the accuracy in the numerical computations of $-\varphi'(0)$ for different values of Sc and β more than 40 terms are required for TSCSE.

The dual velocity and concentration profiles under the effect of velocity ratio parameter (i.e. $\varepsilon < 0$) are depicted in Figures 5 and 6. The thickness of the momentum boundary layer decreases in the solution of lower branch and increases for the solution upper branch with increasing magnitude of velocity ratio parameter ε as noticed from dual velocity profiles and is plotted in Figure 5. On the other hand, at a fixed point, the concentration profiles increase for upper branch and decrease for the lower branch with an increase in the magnitude of ε. It is noticed that the solute thickness of boundary layer increases

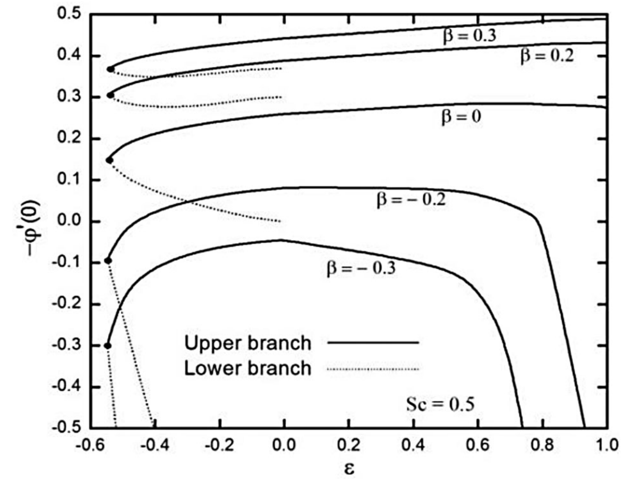

Figure 4. Concentration gradient at the sheet $-\varphi'(0)$ vs. ε for various values of β.

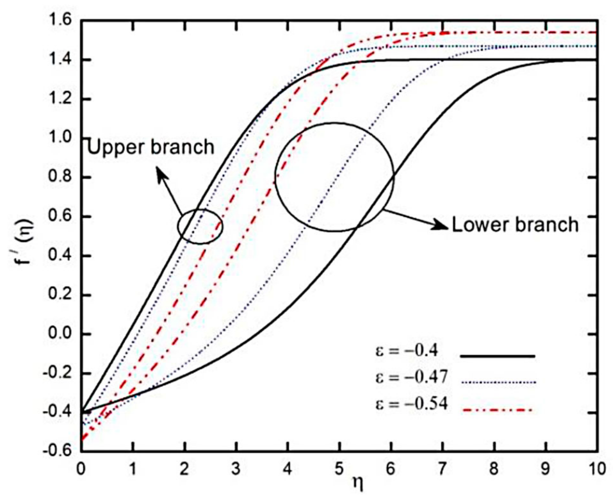

Figure 5. Velocity profiles $f'(\eta)$ for various values of ε.

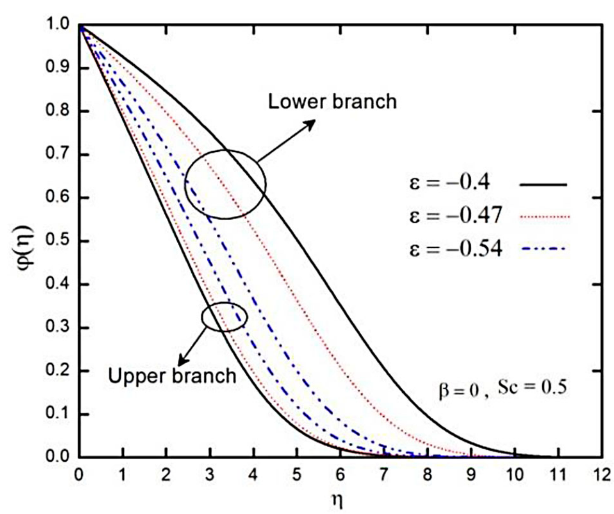

Figure 6. Concentration profiles $\varphi(\eta)$ for various values of ε.

for upper branch and decreases in the case of lower branch, as the magnitude of the velocity ratio parameter ε increases. The results obtained are in agreement with the analysis made by

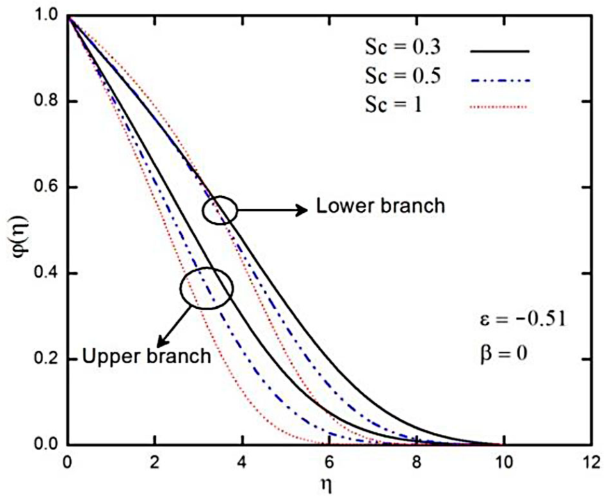

Figure 7. Concentration profiles $\varphi(\eta)$ for various values of *Sc*.

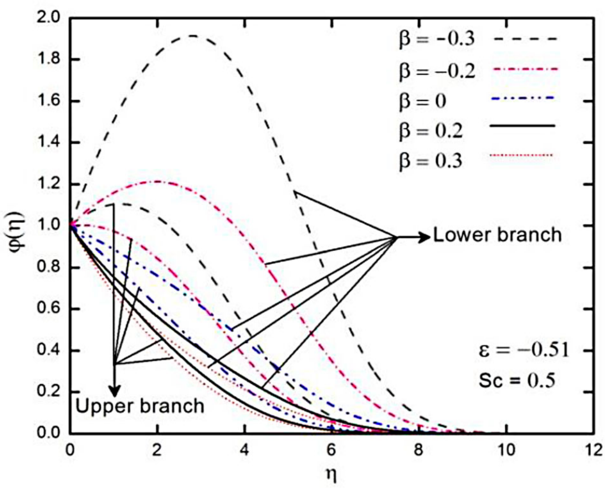

Figure 8. Concentration profiles $\varphi(\eta)$ for various values of β.

Bhattacharyya (2012). Also, the thickness of the boundary layer for a lower branch is thicker compared to the solution upper branch at all time (momentum and solute are considered to be a boundary layer thickness).

Figure 7 presents various values of Schmidt number *Sc* and concentration profiles. It is seen that; the appearance of dual concentration profiles indicates that nondimensional concentration $\varphi(\eta)$ for the lower branch rises initially with *Sc* and it decreases for large η. On the other side, upper branch solution does not change its nature, i.e. it strictly decreases for any value of *Sc*. It is worth mentioning that the concentration boundary layer thickness decreases with an increase of Schmidt number *Sc* for both upper and lower solutions. Also, predict that solute boundary layer thickness gets thinner with an increase in *Sc* due to that its diffusion coefficient reduces.

Figure 8 demonstrates the effect of different numerical values of reaction rate parameter β, the nondimensional concentration profiles $\varphi(\eta)$. It depicts that both destructive and constructive chemical reactions are exhibited in it and also, the range of reaction rate parameter is increased (i.e. $-0.3 \le \beta \le 0.3$) and CCM results indicates the similar nature that are reported by Bhattacharyya (2012) i.e. the boundary layer thickness for both solutions enhances for $\beta < 0$ (constructive chemical reaction) and it reduces for $\beta > 0$ (destructive chemical reaction). The other aspects of these solutions with respect to constructive chemical reactions, the concentration profiles increase initially and gradually decreases along η, it means that mass absorption takes place (i.e. transformation of mass from fluid to the plate is occurred). On the other aspect, for destructive chemical reaction, the concentration profiles decrease at any point. Also, the negative values of $-\varphi'(0)$ for constructive chemical reaction are authenticated in particular cases.

6. Conclusion

The semi-numerical analysis of mass transfer in the presence of chemical reaction on a continuous flat plate moving in parallel or in a reverse manner to a free stream is exhibited. The nonlinear self-similar ordinary differential equations are obtained from the governing equations with the help of similarity transformations on an infinite interval and these equations are converted onto a finite domain. The solution of transformed equations is obtained by using CCM. In the present investigation, solution exhibits the dual nature for particular values of velocity ratio parameters in both concentration and velocity profiles. Intensified reaction rate parameter and Schmidt number show the reduction in thickness of the concentration boundary layer. Moreover, mass absorption near the plate is accounted for in constructive reaction.

Disclosure statement

No potential conflict of interest was reported by the author(s).

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