




Computational assessment of hybrid and tri hybrid nanofluid influenced by slip flow and linear radiation

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Abstract A novel mathematical model for improving heat transfer using tri hybrid nanofluids is described in this paper. This study investigates the slip flow and radiative heat transfer over a moving sheet with polymer-based tri hybrid nanofluid. Under the imposed assumptions, equations governing the flow will be modeled. For nonlinear partial differential equations that cannot be solved accurately using similarity transformation, an ordinary differential equations system may be obtained. Numerical solutions to the streamlined equations will be found with the help of a BVP4C solver (MATLAB). The effects of the pertinent parameters on the velocity and temperature profiles along with the skin friction and heat transfer rates are discussed in detail through figures for both hybrid and tri hybrid nanofluid cases. It is reported that the Nusselt number, which measures the rate at which heat is transferred, is higher in tri hybrid nanofluid when compared to hybrid nanofluid flow.

List of symbols

u and v	Velocity components of the nanomaterial along the x and y direction
σ^*	Stefan–Boltzmann constant
k^*	Mean absorption coefficient
q_r	Radiative heat flux
μ_{hnf}	Viscosity of hybrid nanomaterial
k_{hnf}	Thermal conductivity of hybrid nanomaterial
ρ_{hnf}	Density of hybrid nanomaterial
$(\rho C_p)_{\text{hnf}}$	Heat capacity of hybrid nanomaterial
Re	Local Reynolds number
θ_w	Temperature ratio parameter
R	Radiation parameter
μ_{hnf}	Viscosity of hybrid nanomaterial
k_{hnf}	Thermal conductivity of hybrid nanomaterial
A	Velocity slip constant
T_∞	Ambient temperature
T_w	Surface temperature
ϕ_1, ϕ_2 and ϕ_3	Nanoparticle volume fraction
T	Temperature of nanomaterial
U_∞	Free stream
U_w	Constant velocity
μ_{thnf}	Viscosity of ternary hybrid nanomaterial
k_{thnf}	Thermal conductivity of ternary hybrid nanomaterial
ρ_{thnf}	Density of ternary hybrid nanomaterial
$(\rho C_p)_{\text{thnf}}$	Heat capacity of ternary hybrid nanomaterial
ρ_{hnf}	Density of hybrid nanomaterial
$(\rho C_p)_{\text{hnf}}$	Heat capacity of hybrid nanomaterial
δ	Slip parameter

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1 Introduction

When a combination of nanoparticles shorter than stutter stepping particles and a base fluid is described as having the potential to considerably boost thermal conductivity, it is said to be a nanofluid. Heat transfer qualities of nanofluids are found to be much higher than those of regular fluids, and the introduction of nanoparticles in the fluids themselves may improve the fluid's thermophysical properties. Even with fewer nanoparticles, the heat conductivity is higher than ever before, despite the decrease in particle number itself. In order to improve the thermal conductivity of the base fluid, Choi [1] invented the term "nano-fluids" which defines a liquid suspension of ultra-small particles (less than 50 nm in diameter) with the fast development of nanofabrication; various affordable combinations of fluid /particles are instantly accessible. Reddy et al. [2] studied the transverse magnetic flow over a Reiner–Philippoff nanofluid by considering solar radiation. Reddy et al. [3] studied the zero-mass flux and Cattaneo–Christov heat flux through a Prandtl non-Newtonian nanofluid in Darcy–Forchheimer porous space. Gurdal et al. [4] discussed the turbulent flow and heat transfer characteristics of ferro-nanofluid flowing in dimpled tube under magnetic field effect. Dzulkipli et al. [5] studied the flow and heat transfer over a stretching/shrinking sheet in nanofluid with slip velocity effect. Usafzai et al. [6] investigated the multiple solutions for nanofluids flow and heat transfer in porous medium with velocity slip and temperature jump.

Typically, it is considered that the physical characteristics of nanomaterials and hybrid nanomaterials are constant. Evidence suggests that nanomaterial and hybrid nanomaterial physical properties may shift as a function of temperature. Since the viscosity of a fluid is assumed to be constant, the temperature effect of heat transfer in fluids caused by internal friction is also constant. Layek and Mukhopadhyay [7] studied the effects of fluid radiation heat and variable viscosity on heat transfer and free convective flow via a porous stretched sheet. Nadeem et al. [8] investigated the second-grade fuzzy hybrid nanofluid flow and heat transfer over a permeable stretching/shrinking sheet. Waqas et al. [9] studied the heat transfer analysis of hybrid nanofluid flow with thermal radiation through a stretching sheet. Waini et al. [10] studied the MHD flow and heat transfer of a hybrid nanofluid past a nonlinear surface stretching/shrinking with effects of thermal radiation and suction. Reddy et al. [11] studied the hybrid dusty fluid flow through a Cattaneo–Christov heat flux model. Kumar et al. [12] studied an application of different hybrid nanofluids in convective heat transport of Carreau fluid. Reddy et al. [13] investigate the magnetohydrodynamic flow and heat transfer of a hybrid nanofluid over a rotating disk by considering Arrhenius energy.

Tri hybrid nanomaterials are relatively new concepts that demonstrate the simultaneous action of three nanoparticles in a single fluid. More research is devoted to this recently created tri hybrid mixture model since it outperforms both the hybrid and nanomaterial variants. Tri hybrid nanomaterial research is prominent in heat transport issues and has numerous practical implications in other areas of physics. In their discussion of the use of a tri hybrid nanomaterial based on water for advanced thermal applications in a radiator, Arif et al. [14] emphasized and provided some context for the dispersion of nanoparticles of varying sizes and shapes. A similar novel correlation for tri hybrid nanomaterials with heat transfer applications was established by Sahoo and Kumar [15]. In order to improve thermal performance, Xuan et al. [16] studied the sensitivity analysis of tri hybrid nano-liquids. Lately, Ahmad et al. [17] conducted study on heat transmission using tri hybrid nano-liquids with applications. The fluid containing three nanoparticles was analyzed and synthesized by Adun et al. [18] using a combination of Fe₃O₄, Al₂O₃, and ZnO in a water base fluid; they also determined the influence of ϕ , on the fluid's temperature. Furthermore, a hybrid machine for prediction was constructed in this research. Regenerative evaporative coolers utilizing a combination of three nanoparticles were analyzed by Kashyap et al. [19], who determined that the cooler's thermal performance and temperature might be improved. Using mathematics, Kumar and Sahoo [20] analyzed the effects of three variables on the fluid and its uses in an air heat exchanger. Prakasha et al. [21] discussed the comparative study of hybrid and ternary hybrid nanomaterials over a moving plate.

The passage of heat energy through the particles of a liquid may be described as a sort of heat transfer known as thermal radiation. It is a condition that must be met in a wide variety of applications, such as the planning of propulsion devices for space spacecraft, nuclear power plants, and gas turbines. In space applications, where devices are required to operate at high temperature to get the required thermal efficiency, the impact of thermal radiation on controlling the heat transfer in certain applications and on estimating the thermal effects for processes with high temperature is unavoidable. This is because thermal radiation has an impact on both controlling the heat transfer in certain applications and estimating the thermal effects for processes with high temperature. Recently, Bakar and Soid [22] studied the MHD stagnation-point flow and heat transfer over an exponentially stretching/shrinking vertical sheet in a micropolar fluid with a buoyancy effect. Ali et al. [23] analyzed the flow and heat transfer over stretching/shrinking and porous surfaces. Azam et al. [24] discussed the transient bioconvection and activation energy impacts on Casson nanofluid with gyrotactic microorganisms and nonlinear radiation. Reddy et al. [25] studied the unsteady absorption flow and dissipation heat transfer over a non-Newtonian fluid. Gnanaswara et al. [26] investigated the effect of thermal conductivity on Blasius–Rayleigh–Stokes flow and heat transfer over a moving plate by considering magnetic dipole moment.

Here, we want to analyze the impacts of flow and heat transfer across a sheet surface with linear radiation and velocity slip condition. It is emphasized that we proposed the idea of modified tri hybrid nanofluid, from which the cases of hybrid nanofluid and tri hybrid nanofluid can be retrieved as special cases. In addition, graphene / magnesium oxide is mixed to form a hybrid nanomaterial, and ethylene glycol is employed as the solvent. Tri hybrid nanofluid consists of graphene/zirconium oxide/magnesium oxide dissolved in ethylene glycol. The governing equations (equation of motion and energy equation) are transformed into a system of nonlinear ODE's that can be solved numerically in MATLAB by applying the appropriate similarity transformations. The impacts

of the controlling parameters are illustrated graphically here in terms of the velocities and temperatures profiles. Analyses of the similarities and differences between hybrid nanofluid and tri hybrid nanofluid are investigated.

2 Mathematical formulation

Consider a steady flow and heat transfer of hybrid and Tri hybrid nanomaterial over a moving frame. The plate is same or opposite direction to the free stream U_∞ and constant velocity U_w . The flow is confined to $y > 0$, and sheet coincides with the plane $y = 0$. The ambient fluid temperature is a constant T_∞ . The effect of linear radiation is considered in the heat equation. In addition, the hybrid nanomaterial, graphene, and magnesium oxide are combined and ethylene glycol is used as the working liquid. In the ternary nanomaterial, graphene, zirconium oxide, and magnesium oxide are all combined, and ethylene glycol is used as the working liquid.

Governing equations of hybrid nanofluid are defined below:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\mu_{hnf}}{\rho_{thnf}} \frac{\partial^2 u}{\partial y^2}, \tag{2}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k_{hnf}}{(\rho c_p)_{hnf}} \frac{\partial^2 T}{\partial y^2} + \frac{1}{(\rho c_p)_{hnf}} \frac{16\sigma^* T_\infty^3}{3k^*} \frac{\partial^2 T}{\partial y^2}. \tag{3}$$

Governing equations of tri hybrid nanofluid are defined below:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{4}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\mu_{thnf}}{\rho_{thnf}} \frac{\partial^2 u}{\partial y^2}, \tag{5}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k_{thnf}}{(\rho c_p)_{thnf}} \frac{\partial^2 T}{\partial y^2} + \frac{1}{(\rho c_p)_{thnf}} \frac{16\sigma^* T_\infty^3}{3k^*} \frac{\partial^2 T}{\partial y^2}. \tag{6}$$

where u and v denote the velocity components of the nanomaterial along the x and y directions commonly, σ^* - Stefan–Boltzmann constant, and k^* - mean absorption coefficient.

The thermophysical properties μ_{hnf} , k_{hnf} , $(\rho c_p)_{hnf}$ and ρ_{hnf} for hybrid nanomaterial (Graphene + Magnesium oxide) are defined as:

The hybrid nanomaterial density model by:

$$\frac{\rho_{hnf}}{\rho_f} = (1 - \phi_2)[(1 - \phi_1)\rho_f + \phi_1\rho_{s1}] + \phi_2\rho_{s2},$$

The hybrid nanofluid specific heat model by:

$$(\rho c_p)_{hnf} = (1 - \phi_2)\left[(1 - \phi_1)(\rho c_p)_f + \phi_1(\rho c_p)_{s1}\right] + \phi_2(\rho c_p)_{s2}.$$

The hybrid nanofluid viscosity model by:

$$\frac{\mu_{hnf}}{\mu_f} = \frac{1}{(1 - \phi_1)^{2.5}(1 - \phi_2)^{2.5}}.$$

The hybrid nanofluid thermal conduction model by:

$$\frac{k_{hnf}}{k_{bf}} = \frac{k_2 + 2k_{bnf} - 2\phi_2(k_{bnf} - k_2)}{k_2 + 2k_{bnf} + \phi_2(k_{bnf} - k_2)},$$

where $\frac{k_{bf}}{k_f} = \frac{k_1 + 2k_f - 2\phi_1(k_f - k_1)}{k_1 + 2k_f + \phi_1(k_f - k_1)}$.

The expression for $\mu_{thnf}(\beta)$, k_{thnf} , $(\rho c_p)_{thnf}$ and ρ_{thnf} of the ternary hybrid nanomaterial (Graphene + Zirconium oxide + Magnesium oxide) is as follows:

The tri hybrid nanofluid viscosity model by:

$$\frac{\mu_{thnf}}{\mu_f} = \frac{1}{(1 - \phi_1)^{2.5}(1 - \phi_2)^{2.5}(1 - \phi_3)^{2.5}},$$

The tri hybrid nanofluid density model by:

$$\frac{\rho_{thnf}}{\rho_f} = (1 - \phi_1) \left\{ (1 - \phi_2) \left[(1 - \phi_3) + \frac{\phi_3(\rho c_p)_{s3}}{\rho_f} \right] + \frac{\phi_2(\rho c_p)_{s2}}{\rho_f} \right\} + \frac{\phi_1(\rho c_p)_{s1}}{\rho_f},$$

The tri hybrid nanofluid specific heat model by:

$$\frac{\rho_{thnf}}{(\rho C_p)_f} = \frac{\phi_1(\rho C_p)_{s1}}{\rho_f} + (1 - \phi_1) \left\{ (1 - \phi_2) \left[(1 - \phi_3) + \frac{\phi_3(\rho C_p)_{s3}}{\rho_f} \right] + \frac{\phi_2(\rho C_p)_{s2}}{\rho_f} \right\},$$

The tri hybrid nanofluid thermal conduction model by:

$$\frac{k_{thnf}}{k_{hnf}} = \frac{k_3 + 2k_{hnf} - 2\phi_3(k_{hnf} - k_3)}{k_3 + 2k_{hnf} + \phi_3(k_{hnf} - k_3)}, \quad \frac{k_{hnf}}{k_{bf}} = \frac{k_2 + 2k_{bnf} - 2\phi_2(k_{bnf} - k_2)}{k_2 + 2k_{bnf} + \phi_2(k_{bnf} - k_2)},$$

$$\frac{k_{bf}}{k_f} = \frac{k_1 + 2k_f - 2\phi_1(k_f - k_1)}{k_1 + 2k_f + \phi_1(k_f - k_1)}$$

Interrelated boundary conditions are accustomed by:

$$u = U_w + Av_f \frac{\partial u}{\partial y}, \quad v = 0, \quad T = T_w \quad \text{at } y = 0,$$

$$u \rightarrow 0, \quad T \rightarrow T_\infty \quad \text{as } y \rightarrow \infty. \tag{7}$$

Now introduce the similarity transformation as:

$$u = Uf'(\eta), \quad v = \sqrt{\frac{v_f U}{2x}} (\eta f'(\eta) - f(\eta)), \quad \theta(\eta) = \frac{T - T_w}{T_w - T_\infty}, \quad \eta = \sqrt{\frac{U}{2v_f x}} y. \tag{8}$$

where $U = U_w + U_\infty$,

Making use of Eq. (5), Eq. (1) is identically satisfied and Eqs. (2) and (3) take the form

2.1 For hybrid nanomaterial case

$$\frac{\mu_{hnf}}{\mu_f} f''' + \frac{\rho_{hnf}}{\rho_f} f f'' - f'^2 = 0, \tag{9}$$

$$\left(\frac{k_{hnf}}{k_f} + \frac{4}{3} R \right) \theta'' + Pr f \theta' \left[\begin{array}{l} (1 - \phi_2)[(1 - \phi_1) \\ + \phi_1 \left(\frac{(\rho C_p)_{s1}}{(\rho C_p)_f} \right)] \\ + \phi_2 \left(\frac{(\rho C_p)_{s2}}{(\rho C_p)_f} \right) \end{array} \right] = 0. \tag{10}$$

2.2 For ternary hybrid nanomaterial case

$$\frac{\mu_{thnf}}{\mu_f} f''' + \frac{\rho_{thnf}}{\rho_f} f f'' - f'^2 = 0, \tag{11}$$

$$\left(\frac{k_{thnf}}{k_f} + \frac{4}{3} R \right) \theta'' + Pr f \theta' \left[\frac{\phi_1(\rho C_p)_{s1}}{\rho_f} + (1 - \phi_1) \left\{ (1 - \phi_2) \left[\frac{(1 - \phi_3)}{\rho_f} + \frac{\phi_3(\rho C_p)_{s3}}{\rho_f} \right] + \frac{\phi_2(\rho C_p)_{s2}}{\rho_f} \right\} \right] = 0. \tag{12}$$

Interrelated conditions are in the form:

$$f(0) = 0, \quad f'(0) = 1 + \frac{\delta}{(1 + \phi)^{2.5}} f''(0), \quad \theta(0) = 1, \quad f'(\infty) = 0, \quad \theta(\infty) = 0, \tag{13}$$

where $\delta = A \frac{U_w}{v_f}$, $Pr = \frac{(\mu c_p)_f}{k_f}$, and $R = \frac{16\sigma^* T_\infty^3}{3k_{nf} k^*}$.

2.3 Engineering quantities of interest

The C_{fx} and local Nu_x are prescribed by

$$\sqrt{Re_x} C_{fx} = \frac{\mu_{thnf}}{\mu_f} f''(0), \quad \frac{Nu_x}{\sqrt{Re_x}} = - \left(\frac{k_{thnf}}{k_f} + \frac{4}{3} R \right) \theta'(0),$$

where $Re = \frac{U_x}{v_f}$ is local Reynolds number.

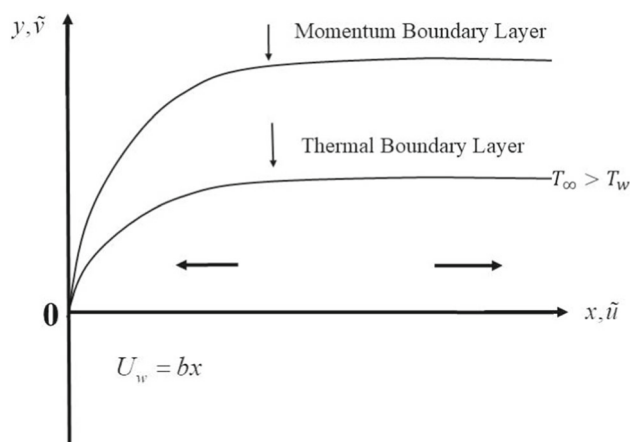
Table 1 Thermophysical properties of a hybrid (graphene + magnesium oxide) and ternary hybrid nanomaterial (graphene + zirconium oxide + magnesium oxide)

Thermophysical property	Fluid base	Hybrid nanofluid		Tri hybrid nanofluid		
	EG	Graphene	Magnesium oxide	Graphene	Zirconium oxide	Magnesium oxide
ρ (kg/m ³)	1114.0	2200	3560	2200	5680	3560
ρC_p (j/kgK)	2415	5000	955	5000	502	955
k (w/mk)	0.2520	790	45	790	1.7	45

Table 2 Comparison of present results

Pr	Ghadikolaei et al. [27]	Hosseinzadeh et al. [28]	Present results
0.7	0.4538	0.4541	0.45415
2.0	0.9113	0.9114	0.91133
7.0	1.8954	1.8954	1.89545

Fig. 1 Physical model and coordinate system



2.4 Numerical method

A numerical method With the assistance of boundary limitations, the nonlinear system of ODEs that is solved with RKF-45 using MATLAB (see Eqs. (9)–(12)), may be solved using the method that is given in (13). This section presents a graphical analysis of the physical influence that a number of different body forces and control factors have on a velocity and temperature profile. There is a tabular representation of the computational values of $Sh_x(Re)^{-1/2}$ and $Nu_x Re_x^{-1/2}$. When the following conditions are $= 6, R = 0.6, Ec = 0.5, \theta_w = 1.2,$ and $A = 0.5$. Graphical and numerical results for the relevant variables are produced.

3 Result and discussion

In this part, the results of the modulating velocity and heat distributions for various parameters are shown visually as an output to highlight the underlying physical significance of the data. In addition, exemplary results for the friction factor ($C_{f,x} Re_x^{-1/2}$) and Nusselt number ($Nu_x Re_x^{-1/2}$) have been documented via graphs for a variety of parameter values for both hybrid and tri hybrid nanofluids. Table 1 mentions the thermophysical properties of hybrid and tri hybrid nanofluid. We analyze the effects of the physical parameters on the velocity, and thermal fields through figures by using the fixed parametric values are $\phi_1 = \phi_2 = \phi_3 = 10%, \delta = 0.4, Pr = 6$ and $R = 0.5$. Additionally, the results of the comparison of the Prandtl number to previously published studies are shown in Table 2. The conclusion that can be drawn from this is that the current findings are in excellent accord with the research that came before them (Fig. 1).

The stream line consequence of the parameter A is studied in Fig. 2a–b and 3a–b for both hybrid and ternary hybrid nanomaterials, respectively. It is shown in Fig. 2a–b that the augmentation of heat transmission is greater (where $A = 0.5$) in the ternary nanomaterial scenario when compared with the hybrid nanomaterial situation. Figure 3a–b demonstrates that for the value of $A (= 1)$, the greater fluid temperature is shown. This can be noticed by comparing the two figures. In addition, the performance of the fluid temperature is greater in the case of the ternary hybrid fluid than in the case of the hybrid nanomaterial.

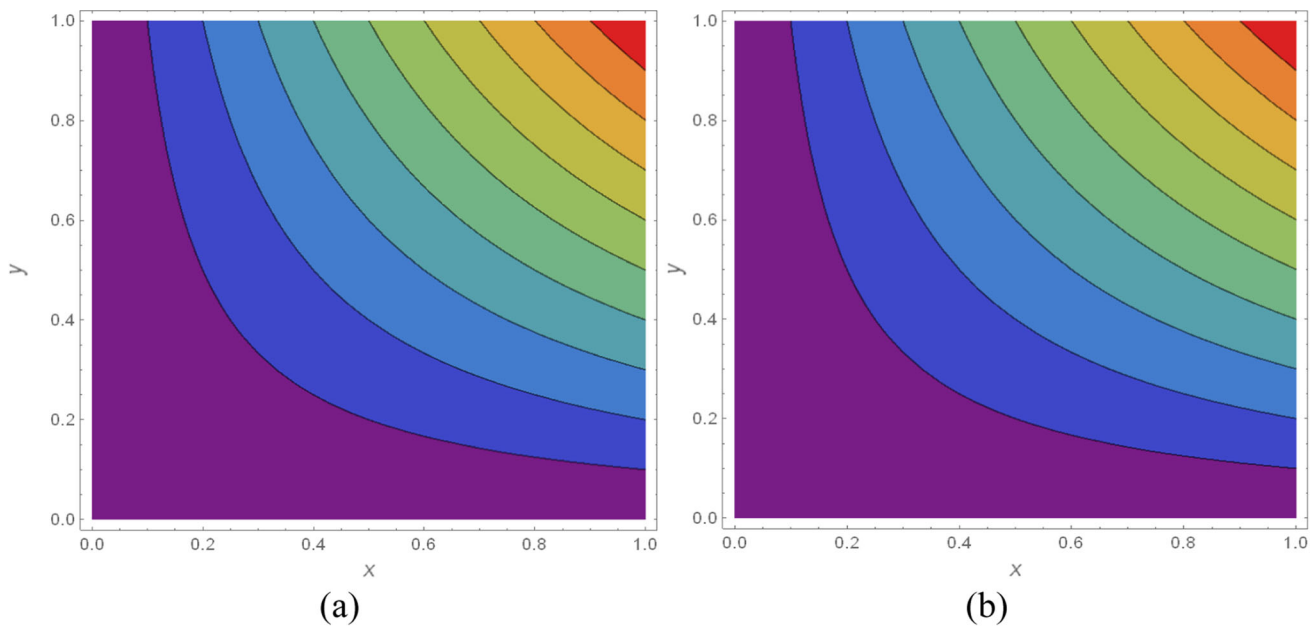


Fig. 2 **a** Stream lines for hybrid nanofluid $A = 0.5$, **b** stream lines for ternary hybrid nanofluid $A = 0.5$

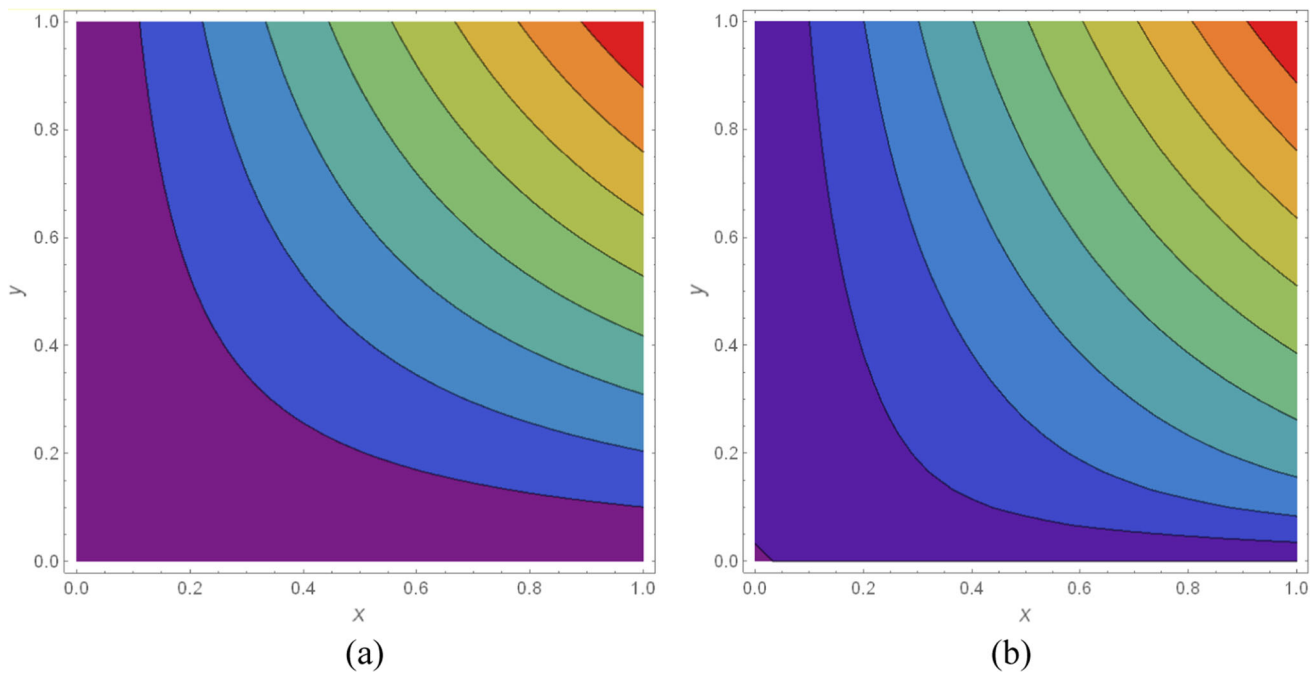


Fig. 3 **a** Stream lines of hybrid nanofluid for $A = 1$. **b**. Stream lines of ternary hybrid nanofluid for $A = 1$

Figure 4 displays how the friction factor is working to varying slip parameter (δ) versus Prandtl number (Pr) parameter values. The friction factor enhances for higher estimates of slip parameter (δ) versus Prandtl number (Pr) parameter. Additionally, friction of the fluid enhances for rising values of slip parameter (δ) versus Prandtl number (Pr) parameter. Furthermore, performance of fluid velocity is more comfortable in ternary hybrid case when treated with hybrid nanomaterial case. Characteristics of major parameters Prandtl number (Pr) and radiation parameter (R) on the Nusselt number ($Nu_x Re_x^{-1/2}$) curve exhibit variance in Fig. 5, as Nusselt number ($Nu_x Re_x^{-1/2}$) drops with rising Prandtl number (Pr) but reverses with larger radiation parameter ($R > 1$) values.

In Fig. 6, you can see the effect of radiation parameter (R) on the temperature profile ($\theta(\eta)$) of interest for both hybrid and tri hybrid nanofluid cases. It indicates that the greater the value of radiation parameter, the more intense the heat provided to the liquid, resulting in an enhancement in thermal layer thickness for both hybrid and tri hybrid nanofluid cases. With a higher of radiation

Fig. 4 Effect of slip parameter (δ) and Prandtl number (Pr) on skin friction coefficient

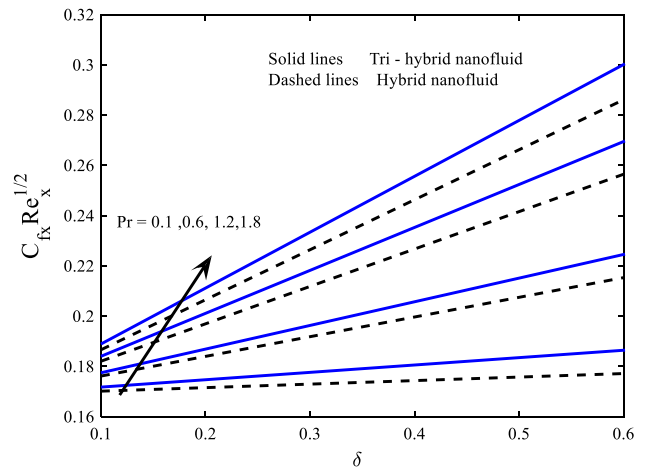


Fig. 5 Effect of Prandtl number (Pr) and radiation parameter (R) on Nusselt number

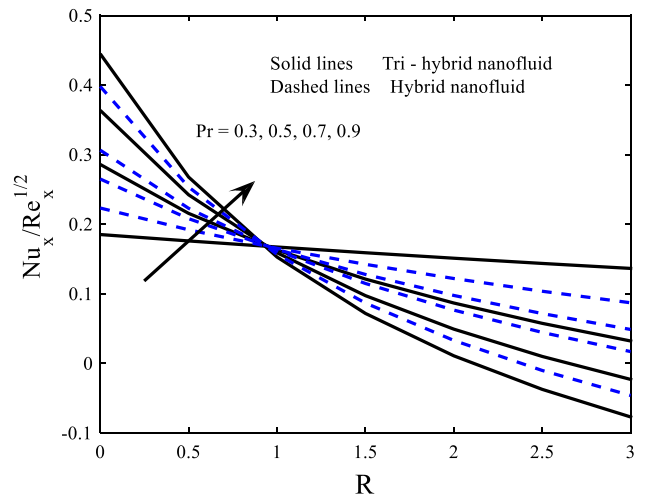
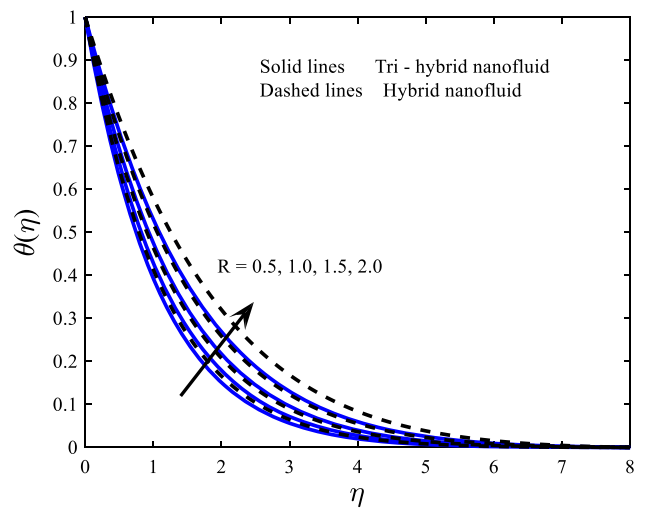


Fig. 6 Effect of radiation parameter (R) on temperature profile ($\theta(\eta)$)



parameter, the heat distribution expands and the environment becomes more of a factor. Furthermore, higher radiation parameter values result in a substantial amount of heat being transferred to the fluids.

For both hybrid and ternary hybrid nanomaterial situations, the impact of the Prandtl number (Pr) versus the thermal profile ($\theta(\eta)$) is shown in Fig. 7. For both the hybrid and ternary hybrid nanomaterial instances, the temperature of the liquid lowers dramatically as the Prandtl number increases. This is because when Prandtl number increases, the liquid's thermal diffusivity reduces, and the temperature drops. In addition, tri hybrid nanofluid provides a greater fluid temperature advantage than hybrid nanofluid. For the

Fig. 7 Effect of Prandtl number on temperature profile ($\theta(\eta)$)

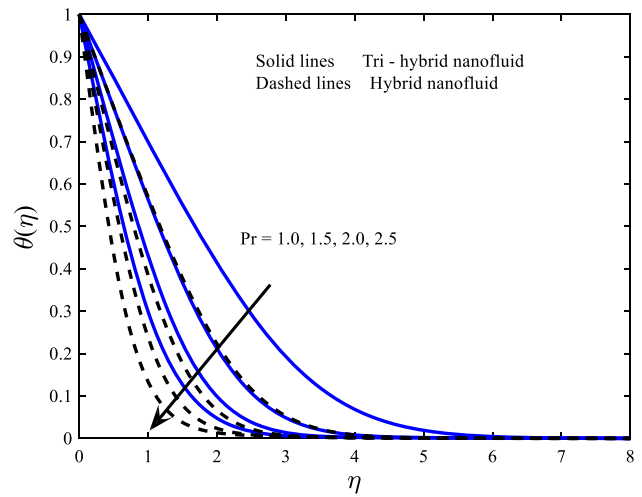


Fig. 8 Effect of nanoparticle volume friction ϕ_1 on velocity profile ($f'(\eta)$)

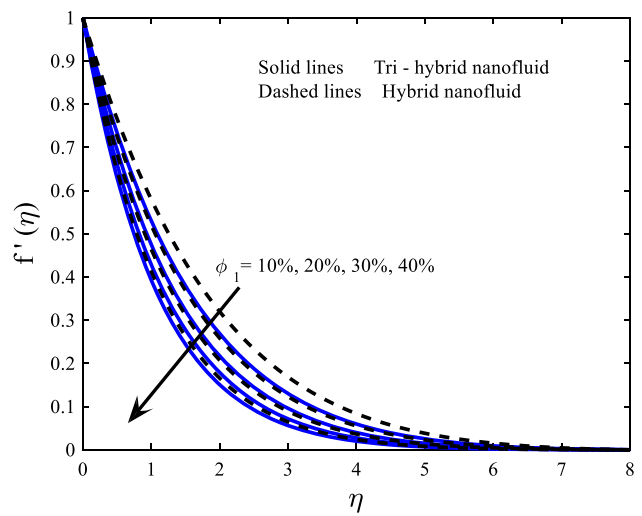
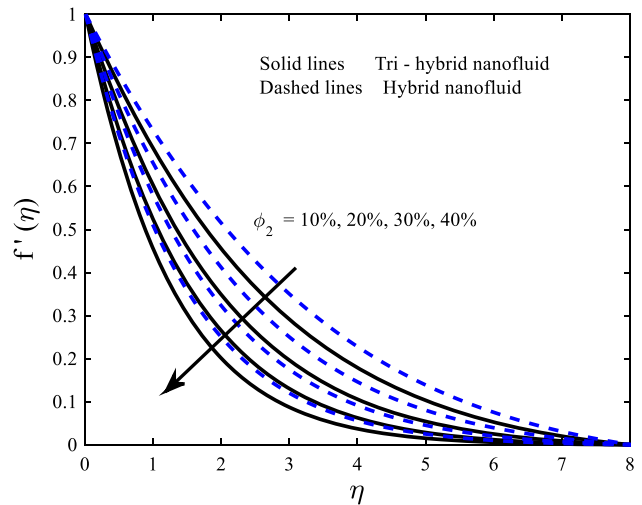


Fig. 9 Effect of nanoparticle volume friction ϕ_2 on velocity profile ($f'(\eta)$)



hybrid and tri hybrid nanofluid examples, the impact of nanoparticles volume friction parameter (ϕ_1 , ϕ_2 and ϕ_3) on the $f'(\eta)$ profile is shown in Fig. 8, 9 and 10. In both the hybrid and the tri hybrid nanofluid cases, the velocity curve accelerates as a result of the addition of solid nanoparticles to the nanofluid. Collisions between widely scattered nanoparticles are responsible for this result. In addition, for both nanofluid cases this is shown in Figs. 11 and 12; it is important to notice that the values of nanoparticles volume friction parameter (ϕ_1 , ϕ_2 and ϕ_2) have increased in order to decrease the appropriate layer thickness.

Fig. 10 Effect of nanoparticle volume friction ϕ_3 on velocity profile ($f'(\eta)$)

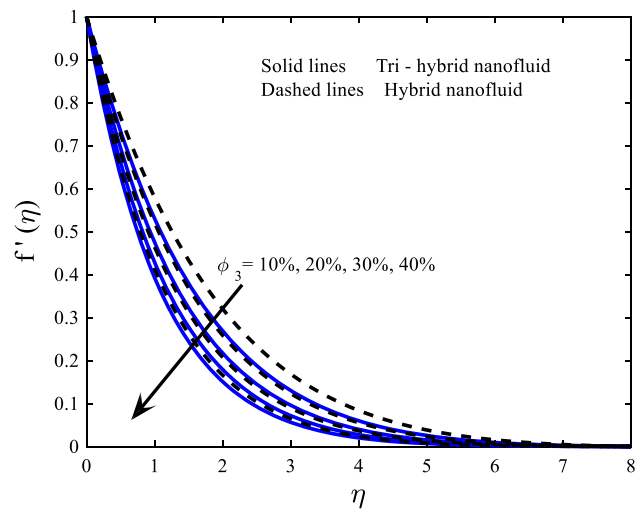


Fig. 11 Effect of nanoparticle volume friction ϕ_1 on temperature profile ($\theta(\eta)$)

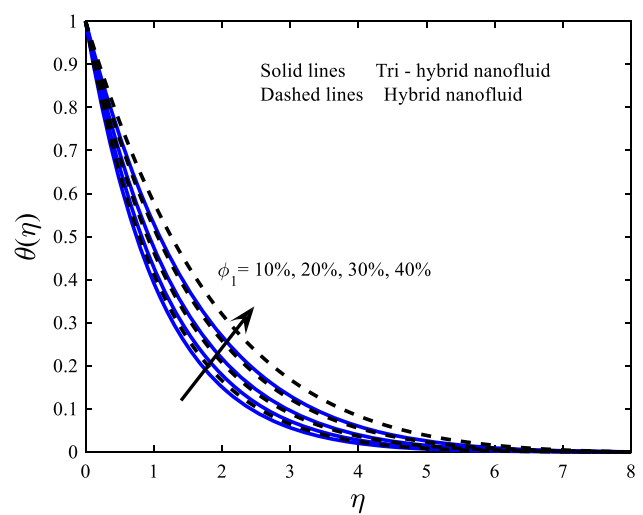
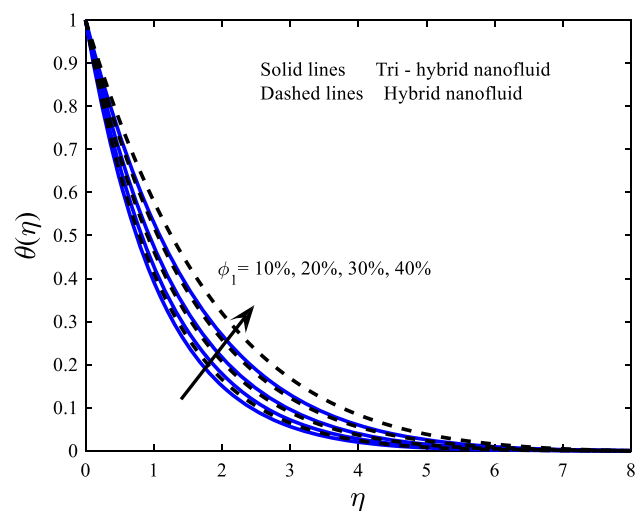


Fig. 12 Effect of nanoparticle volume friction ϕ_2 on temperature profile ($\theta(\eta)$)



Figures 11, 12 and 13 highlight the effect of the nanoparticles volume friction (ϕ_1 , ϕ_2 and ϕ_3) parameters on the temperature profile of the hybrid and tri hybrid nanofluid cases, respectively. When the nanoparticle releases its stored energy, it does so in the form of heat. Therefore, in both the hybrid and tri hybrid nanofluid examples, the mixing of extra nanoparticles may need more energy, increasing the breadth of the temperature and boundary layer. Additionally, fluid temperature may be better regulated in ternary hybrid nanomaterial situations than in hybrid nanomaterial cases. The impact that slip parameter (δ) had on the velocity

Fig. 13 Effect of nanoparticle volume friction ϕ_3 on temperature profile ($\theta(\eta)$)

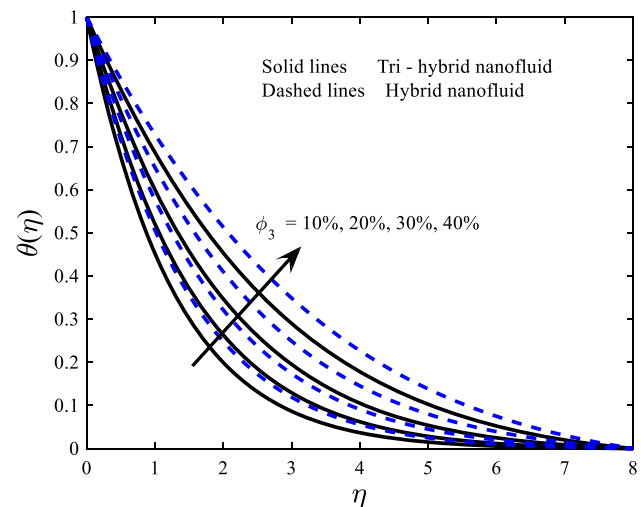
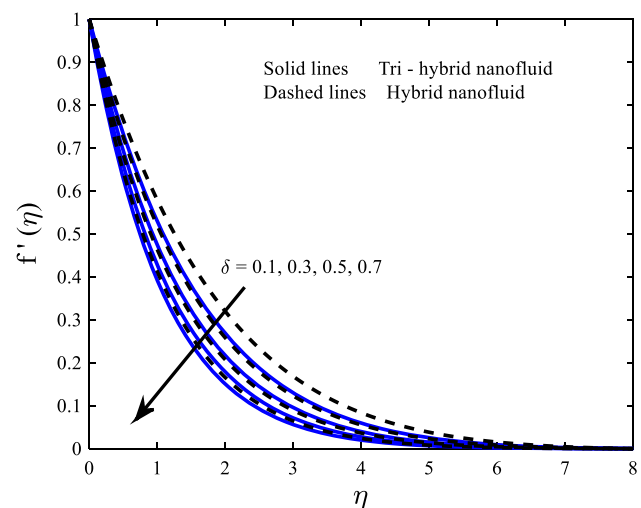


Fig. 14 Effect of slip parameter (δ) on velocity profile ($f'(\eta)$)



profile ($f'(\eta)$) for both hybrid and tri hybrid nanofluid cases is shown in Fig. 14. It is clear that the velocity field for both hybrid and tri hybrid nanofluid cases should be decreased in order to raise the velocity slip parameter.

3.1 Final remarks

In this paper, hybrid nanofluid, tri hybrid nanofluid model, and slip effects are demonstrated, taking into consideration the flow and radiative heat transfer across a stretched surface. Both numerical and graphical analyses are used to expound on the impact that the physical parameters have. The most important findings may be summarized as follows:

- The heat transfer rate of the ternary hybrid nanomaterial improves more than that of the hybrid nanomaterial fluid.
- The velocity profile decreases by increasing values of slip parameter. This is because the slip velocity parameter is increased, the slide speed will also be decreased accordingly.
- The effect of thermal radiation is to boost the transfer of heat since with increasing thermal radiation and thermal boundary layer for both dusty and ternary phases.
- The nanoparticle is so tiny which may be deduced from the amount of heat that it generates when it releases the energy that it has accumulated. Due to this, temperature profile enhances by increasing values of nanoparticle volume friction parameter.
- The Prandtl number increases, the liquid's thermal diffusivity reduces, and the temperature drops. Due to this, for higher values of Prandtl number decayed the temperature of the fluid for both cases.
- The larger values of nanoparticles volume friction are decayed the velocity profiles for both hybrid and tri hybrid cases.

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Availability of data and materials All data that support the findings of this study are included within the article (and any supplementary files).

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