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Finite difference analysis on radiative flow on a perpendicular plate using the influence of thermal conductivity

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A R T I C L E I N F O

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A B S T R A C T

This study aims to investigate the influence of radiation, thermal conductivity and variable viscosity on natural convective flow on a semi-infinite perpendicular plate. Variable viscosity, thermal conductivity and thermal radiation are considered for the given study. The dimensional governing equations are framed with the use of the mentioned parameters and then these equations were converted into dimensionless equations by applying non dimensional quantities. The main aim of this study is to find the Nusselt number and skin friction for both air and water for considered parameters. Using the finite difference method through Fortran software, numerical solutions to the governing heat equations and dimensionless momentum equations were computed. The results for the parameters thermal conductivity, variable viscosity, radiation, and Prandtl number for both air and water are displayed via various graphs. The skin friction coefficients, Nusselt parameter, and local Nusselt numbers were discussed for both the air and water. The key conclusions of this study are that the succeeding velocity declines as the radiation's increases. By increasing the radiation value and the fluctuation time, the temperature distribution increases. Notably, the temperature profile increases significantly when the variable viscosity parameter decreases.

1. Introduction

Heat transport by thermal radiation does not necessitate the presence of a material medium. In the case of radiation emanating from a solid surface, the medium through which the radiation flows could be a vacuum, liquid or gas. Atoms with molecules inside the medium can absorb, reflect, or transfer radiation energy. If the medium is a vacuum, with atoms or no molecules, the thermal radiation energy is not attenuated and is consequently transferred in its entirety. In a vacuum, thermal radiation is more efficient. In the case of an energy and gas (e.g., air) can be absorbed or imitated somewhat by air molecules, and the balance is communicated. However, the radiation outcome on natural convection flow with heat transfer issues has assumed a greater industrial significance. The radiation effect can be impartially strong at high operating temperatures. Numerous engineering progressions occur at high temperatures, making familiarity with radiation and heat transmission crucial for designing relevant apparatus. Such engineering fields include gas turbines and many impulsion technologies for missiles, nuclear power plants, satellites, aircraft, and spacecraft.

The petroleum industry, crystal growth reactor technology, reactor cooling, boundary layer control in aeronautics, and plasma research are among the technological and engineering sectors that address this issue. Examining heat generation repercussions in affecting fluids is crucial to understanding a number of the significant problems, and fluids undergo an exothermic. Payable to the development of electronical knowledge, the effective freezing of electronic devices has been validated. These devices range from the individual transistor to the primary computer and from thermal energy benefactors to telephone panels.

Numerous engineering applications include laminar boundary layer flows over free convection. The application of practical examples consists of the aerodynamic bump of the cooling in a metallic plate in a steam bath, plastic sheet, the boundary layer flow, and a polymer sheet continuously from a dye, liquid film in condensation processes. At peak performance, the influence of heat and radiation may be significant. Extreme heat is a factor in various manufacturing processes, making knowledge of radiation's role in heat transfer crucial for developing major equipment.

Because of its applicability to boundary layer control, geophysics, aerodynamics, meteorology, and other research disciplines, applying the

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Nomenclature

$\overline{T}^*$	Fluid temperature
$\overline{u}^*, \overline{v}^*$	Dimensional velocity of the fluid
q_r	Radiative heat flux
$\overline{T_w}^*$	Temperature of the plate
k	Thermal conductivity
t, t^*	Dimensional and dimensionless Time
Pr	Prandtl number
ρc_p	Specific heat capacity
$\overline{T_\infty}^*$	Temperature of the fluid far away from the plate
C_f	Skin-friction coefficient
x, y	Dimensional Coordinate axis
Gr	Thermal Grashof parameter
Nu_x	Local Nusselt number
R	Radiation
U, V	Dimensionless Fluid velocity
$\overline{Nu}$	Average Nusselt number
X, Y	Dimensionless coordinates

Greek symbols

λ	Variable viscosity
ρ	fluid density
γ	Thermal conductivity
μ	Fluid viscosity
θ	non-dimensional temperature
μ_∞	free stream viscosity
υ	Kinematic viscosity
β	Volume expansion coefficient
g	gravitation

integral convection method across a semi-infinite perpendicular plate was the topic in Siegel [1] research paper. Soundalgekar and Ganesan [2] examined the effects of mass transport on a plate with heat flux. Loganathan et al. [3] discoursed heat conductivity influences unsteady MHD flow. The convective flow was shown to exist on a vertical plate with the inspiration of thermal conductivity with the aid of variable viscosity with MHD by Iranian et al. [4]. The above investigators used the implicit finite-difference approach to solve the problems without radiation parameters with unsteady flow. But Elbashbeshy and Ibrahim [5] explored steady incompressible flow across a heated plate.

Many authors solved thermal and mass transfer on plate problems using radiation parameters by different methods. For instance, Soundelgekar and Takhar [6] talked about the impact of thermal radiation on free convection. Chamkha et al. [7] looked at radiation's effects on a horizontal plate for their research. Muthucumaraswamy and Ganesan [8] proved the effects of radiation by propelling a vertical plate at an accelerated speed while varying the plate's temperature. Takhar et al. [9] looked into how radiation affected the semi-infinite vertical plate problem and found some interesting results. The possessions of thermal radiation on natural convection flow across a perpendicular plate using a non-constant temperature surface are considered by Abd El et al. [10], who provided a finite difference solution for the problem but not with variable viscosity. Hossain et al. [11] demonstrated the radiation parameter possessions on a permeable vertical plate with changing thickness. The flow of Walter's Liquid-B in a radiative MHD setting through a semi-infinite plate was presented by Damala et al. [12]. Hydrodynamic convective fluid flow when thermal radiation is present was also studied by Dahake and Dubewar [13]. Natural convective MHD flow with viscoelastic fluid across a permeable plate with a heat source in unsteady form has been presented by Kehinde et al. [14]. Abdul Gaffar et al. [15] also looked into the effects of Jeffrey fluids past a moving plate. Manna et al. [16] have looked at the impact of radiation on an oscillating vertical porous plate immersed in an oscillating heat-flux porous medium. Amanulla et al. [17] discussed Williamson fluid on a semi-infinite perpendicular plate with convection flow and the inspiration of thermal radiation impacts. Impact of thermal conductivity and the influence of variable viscosity on the dynamics of nanofluids via a perpendicular plate was done by Gladys and Ramana Reddy [18]. Damala et al. [19] considered radiative MHD with the occurrence of viscous dissipation and the effect of energy source and flows of Walter's liquid-B on a semi-infinite perpendicular plate. Onwubuoya et al. [20] investigated the impacts of viscous dissipation, Soret, and the outcome of thermal radiation on the MHD natural convection flow of Williamson liquid with thermal conductivity and variable viscosity. MHD nanofluid and mixed convective flow of varying thermal conductivity was examined by Darvesh et al. [21]. Saikia et al. [22] examined a mathematical study on MHD flow on a perpendicular extending sheet with thermal conductivity and the consequence of variable viscosity. Sudarmozhi et al. [23,24] investigated on vertical plate by adding Maxwell fluid in their study.

According to the findings of earlier research, it is necessary to employ computational methods to evaluate the outcome of radiation with the impact of variable viscosity and thermal conductivity's effect on natural convection on a semi-infinite plate. To examine the behaviour of the fluid velocity, x-component of the induced velocity outline, and energy outlines, curves are generated for a range of flow-description values.

2. Problem Statement

Consider incompressible, viscous, radiating fluid travelling across a semi-infinite distance in the two-dimension perpendicular plate. The fluid's viscosity with thermal conductivity is taken into justification. Consider the plate being vertical, whereas the y-axis is upright to the plate and $\overrightarrow{u*}$, $\overrightarrow{v*}$ are called the fluid velocity in the directions of the x & y axis correspondingly, the fluid and plate are also expected to have the same initial temperature. For time $t^* > 0$, the plate's temperature constantly increases. The schematic diagram of the problem is revealed in Fig. 1. with these assumptions and including the Boussinesq's approximation within the boundary layer, the governing equations of

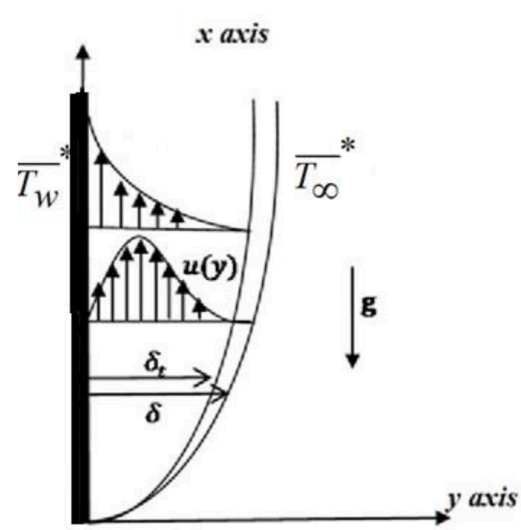

Fig. 1. Physical mode of the problem.

continuity, velocity and heat respectively are assumed by interchanging the heat generation to radiation parameter from [Loganathan et al. 25].

Constructed these traditions, the modelling of the problem that demonstrates about the physical situations is detailed by

$$\frac{\partial \overline{u}^*}{\partial x}+\frac{\partial \overline{v}^*}{\partial y}=0 \tag{1}$$

$$\frac{\partial \overline{u}^*}{\partial t^*}+\overline{u}^*\frac{\partial \overline{u}^*}{\partial x}-g\beta\left(\overline{T}^*-\overline{T}^*_\infty\right)+\overline{v}^*\frac{\partial \overline{u}^*}{\partial y}=\nu\frac{\partial^2 \overline{u}^*}{\partial y^2} \tag{2}$$

$$\rho c_p\left(\frac{\partial \overline{T}^*}{\partial t^*}+\overline{u}^*\frac{\partial \overline{T}^*}{\partial x}+\overline{v}^*\frac{\partial \overline{T}^*}{\partial y}\right)=k\frac{\partial^2 \overline{T}^*}{\partial y^2}-\frac{\partial q_r}{\partial y} \tag{3}$$

The momentum equation's first two terms of RHS show the variable viscosity and thermal Grashof values. The first two terms are in the RHS of the energy transport, which resembles thermal conductivity, the energy of the fluid with radiation.

The boundary conditions connected to Eqs. (1)—(3) are considered as

$$\begin{aligned}
&t^*\le 0: \overline{u}^*=0,\ \overline{T}^*=\overline{T}^*_\infty,\ \overline{v}^*=0,\ \textit{for all } x \textit{ and } y\\
&t^*>0: \overline{u}^*=0,\ \overline{T}^*=\overline{T}^*_w,\ \overline{v}^*=0,\quad at\quad y=0\\
&\qquad \overline{u}^*=0,\ \overline{v}^*=0,\ \overline{T}^*=\overline{T}^*_\infty\quad at\quad x=0\\
&\qquad \overline{u}^*\to 0,\quad \overline{T}^*\to\overline{T}^*_\infty\quad as\quad y\to\infty
\end{aligned} \tag{4}$$

Assume Rosseland calculation which results in $k_e$the mean absorption coefficient, a relative heat flux using σ_s as Stefan-Boltzman constant is written as

$$q_r=\frac{-4\sigma_s}{3k_e}\frac{\partial t^{*4}}{\partial y} \tag{5}$$

It is worth noting that the Rosseland approximation limits our study to optically thick fluids. If the thermal differences inside the fluid flow are minor enough, Eq. (5) can be formed by increasing t^{*4}into$\overline{T}^*_\infty$, by excluding complex order terms, has the following form:$t^{*4}\cong 4\overline{T}^{*3}_\infty\overline{T}^*-3\overline{T}^{*4}_\infty$

$$\begin{aligned}
&\frac{U^{n+1}_{i,j}-U^n_{i,j}}{\Delta t^*}+U^n_{i,j}\left[\frac{U^{n+1}_{i,j}-U^{n+1}_{i-1,j}+U^n_{i,j}-U^n_{i-1,j}}{2\Delta X}\right]+V^n_{i,j}\left[\frac{U^{n+1}_{i,j+1}-U^{n+1}_{i,j-1}+U^n_{i,j+1}-U^n_{i,j-1}}{4\Delta Y}\right]\\
&\quad=\exp(-\lambda\theta)\left[\frac{U^{n+1}_{i,j-1}-2U^{n+1}_{i,j}+U^{n+1}_{i,j+1}+U^n_{i,j-1}-2U^n_{i,j}+U^n_{i,j+1}}{2\Delta Y}\right]-\lambda\exp(-\lambda\theta)\\
&\quad\left[\frac{U^{n+1}_{i,j+1}-U^{n+1}_{i,j-1}+U^n_{i,j+1}-U^n_{i,j-1}}{4\Delta Y}\right]\left[\frac{\theta^{n+1}_{i,j+1}-\theta^{n+1}_{i,j-1}+\theta^n_{i,j+1}-\theta^n_{i,j-1}}{4\Delta Y}\right]+\frac{\theta^{n+1}_{i,j}+\theta^n_{i,j}}{2}
\end{aligned} \tag{14}$$

$$\begin{aligned}
&\frac{\theta^{n+1}_{i,j}-\theta^n_{i,j}}{\Delta t^*}+U^n_{i,j}\left[\frac{\theta^{n+1}_{i,j}-\theta^{n+1}_{i-1,j}+\theta^n_{i,j}-\theta^n_{i-1,j}}{2\Delta X}\right]+V^n_{i,j}\left[\frac{\theta^{n+1}_{i,j+1}-\theta^{n+1}_{i,j-1}+\theta^n_{i,j+1}-\theta^n_{i,j-1}}{4\Delta Y}\right]\\
&\quad=\frac{1}{\Pr}\left[\left(1+\gamma\left(\frac{\theta^{n+1}_{i,j}+\theta^n_{i,j}}{2}\right)+R\right)\left(\frac{\theta^{n+1}_{i,j-1}-2\theta^{n+1}_{i,j}+\theta^{n+1}_{i,j+1}+\theta^n_{i,j-1}-2\theta^n_{i,j}+\theta^n_{i,j+1}}{2(\Delta Y)^2}\right)\right]\\
&\qquad+\frac{\gamma}{\Pr}\left(\frac{\theta^{n+1}_{i,j+1}-\theta^{n+1}_{i,j-1}}{2\Delta Y}\right)\left(\frac{\theta^n_{i,j+1}-\theta^n_{i,j-1}}{2\Delta Y}\right)
\end{aligned} \tag{15}$$

Resulting in dimensionless quantities being introduced

$$X=\frac{x}{L},\quad Y=\frac{yGr^{1/4}}{L},\quad U=\frac{\overline{u}^*LGr^{-1/2}}{\nu},\quad V=\frac{\overline{v}^*LGr^{-1/4}}{\nu},\quad R=\frac{16\sigma_s\overline{T}^{*3}_\infty}{3k_\infty k_e}$$
$$\theta=\frac{\overline{T}^*-\overline{T}^*_\infty}{\overline{T}^*_w-\overline{T}^*_\infty},\quad t^*=\frac{t\overline{v}^*Gr^{1/2}}{L^2},\quad \Pr=\frac{\overline{v}^*\rho c_p}{k_\infty},\quad Gr=\frac{g\beta L^3\left(\overline{T}^*_w-\overline{T}^*_\infty\right)}{\nu^{*2}} \tag{6}$$

Many studies [4,5,10,11] presumed that the thickness of the fluid with the effect of thermal conductivity was proportional to the temperature of the fluid. Therefore, the viscosity can be written as from (Ockendon and Ockendon [26]):

$$\frac{\mu(\theta)}{\mu_\infty}=\exp(-\lambda\theta) \tag{7}$$

In addition, thermal conductivity can be introduced as

$$\frac{k(\theta)}{k_\infty}=(1+\gamma\theta) \tag{8}$$

By applying the above dimensionless quantities (6) and (7-8), the non-dimensional equations are

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \tag{9}$$

$$\frac{\partial U}{\partial t^*}+U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=\theta+\exp(-\lambda\theta)\left(\frac{\partial^2 U}{\partial Y^2}-\lambda\frac{\partial U}{\partial Y}\frac{\partial\theta}{\partial Y}\right) \tag{10}$$

$$\frac{\partial\theta}{\partial t^*}+U\frac{\partial\theta}{\partial X}+V\frac{\partial\theta}{\partial Y}=\frac{1}{\Pr}\left((1+\gamma\theta+R)\frac{\partial^2\theta}{\partial Y^2}+\gamma\left(\frac{\partial\theta}{\partial Y}\right)^2\right) \tag{11}$$

In the dimensionless form, the equivalent initial & boundary conditions;

$$\begin{aligned}
&t^*\le 0:\ U=0,\quad V=0,\quad \theta=0 \text{ for all } X \text{ and } Y\\
&t^*>0:\ U=0,\quad V=0,\quad \theta=1\quad \text{at}\quad Y=0\\
&\qquad U=0,\quad V=0,\quad \theta=0\quad \text{at}\ X=0\\
&\qquad U\to 0,\quad \theta\to 0\quad \text{as}\quad Y\to\infty
\end{aligned} \tag{12}$$

3. Numerical Solution

To obtain solutions for the above equations with the suitable boundary conditions is impossible to answer in closed form. By the implicit finite difference approach of the Crank-Nicholson method, formed governing equations were answered, the finite difference technique of the governing equations is as below

$$\frac{U_{i,j-1}^{n+1} - U_{i,j}^{n+1} + U_{i,j}^{n} - U_{i-1,j}^{n}}{2\Delta X} + \frac{V_{i,j-1}^{n+1} - V_{i,j}^{n+1} + V_{i,j}^{n} - V_{i-1,j}^{n}}{2\Delta Y} = 0 \tag{13}$$

4. Outcomes of results

This article attains thermal radiation influences on laminar flow, unsteady thermal conductivity, and variable viscosity through a semi-infinite perpendicular plate. The current study is validated to previously published solutions to determine the exactness of the numerical outcomes. Comparisons are made between the velocity and temperature outlines with the available resolutions. It is seen that the current results correspond well with the solution of [Loganathan et al. 25].

Fig. 2 displays the velocity profile calculated dimensionless for $Pr = 0.73$ (air). It shows the velocity outline for various R values and fixed values of $\lambda = -0.6$ and $\gamma = 1.0$. The time to attain steady state reductions steadily as R is increased, the fluid velocity grows with increasing time until it reaches a temporal maximum ($U = 0.53922$), following which it decreases until reaching a steady state. Therefore, velocity decreases with R and t near the plate. When radiation impinges on the surface of the semi-infinite perpendicular plate, it can be absorbed by the plate material. The absorbed radiation leads to a rise in the plate's temperature. The absorbed radiation raises the temperature of the plate, creating a non-uniform temperature distribution along its surface. The plate becomes hotter close to the region where the radiation is absorbed and gradually cools down as we move away from that region. The non-uniform temperature distribution on the plate surface induces a temperature gradient. Hotter regions near the absorbed radiation region create a thermal gradient in the air adjacent to the plate. The buoyancy effects cause the air to move, generating a free convection flow along the vertical plate. The air near the hotter regions rises due to buoyancy, while the cooler air descends to replace it. This flow establishes a velocity profile adjacent to the plate. The velocity profile near the semi-infinite vertical plate is typically characterized by upward motion of air near the heated region and downward motion in the cooler regions. The exact shape of the velocity contour depends on factors such as the temperature distribution, the intensity of radiation absorption, and the plate material properties.

Fig. 3 displays the velocity curve for water; this figure represents the velocity field for diverse values of R. While R increases, velocity decreases. At ($U = 0.82015$), the velocity field attains a temporal maximum and then declines monotonically to a scope steady state. The air velocity gradient is continuously more significant than the fluid velocity of the water gradient. Substantially, this is accurate since the rise in Prandtl value is due to a growth in the fluid's viscosity, which means the fluid to become thicker and consequently reduces its velocity. Unlike air, which has relatively low thermal conductivity and is strongly affected by radiation, water has a much higher thermal conductivity. As a result, radiation's direct impact on the velocity profile of water near a vertical plate is typically limited. Radiation can still influence the velocity profile indirectly through its effect on the temperature distribution of the vertical plate. When radiation is absorbed by the plate, it can cause localized heating and lead to a non-uniform temperature distribution along the plate's surface. In the occurrence of a thermal boundary layer, natural convection can be induced due to buoyancy effects. The temperature gradients within the boundary layer cause variations in water density, leading to upward or downward motion of fluid. The velocity profile near the semi-infinite vertical plate in water will depend on the interplay between natural convection and other factors, such as the plate temperature distribution, thermal conductivity of water, and boundary conditions. The specific shape of the velocity profile can vary, but it generally exhibits upward or downward motion near the plate surface.

During the steady-state phase, velocity influences on many parameters are examined. Fig. 4 depicts the velocity outlines with fixed Pr = 0.71 for air Pr = -0.3 and radiation parameter R = 5.0. As the value increases, velocity drops. Before returning to its steady state value, the fluid velocity influences its temporal maximum. In addition, the width of the border layer decreases as it increases. The outcome of variable viscosity on the velocity outline for air is primarily governed by the concept of boundary layers and the relationship between shear stress and velocity gradients. When a fluid flows over a surface, such as air flowing over a solid object, a thin layer of fluid near the surface is known as the boundary layer is formed. The behavior of the boundary layer is influenced by the viscosity of the fluid. In the case of air, which is considered a low-viscosity fluid, the boundary layer near the surface is typically thin. This thin boundary layer experiences minimal viscosity effects, and the velocity profile within this region remains relatively uniform, with the fluid particles moving at a similar velocity. However, as the distance from the surface rises and the boundary layer expands, the effects of viscosity become more significant. Viscosity causes adjacent layers of air to exert shear forces on each other, resulting in velocity gradients within

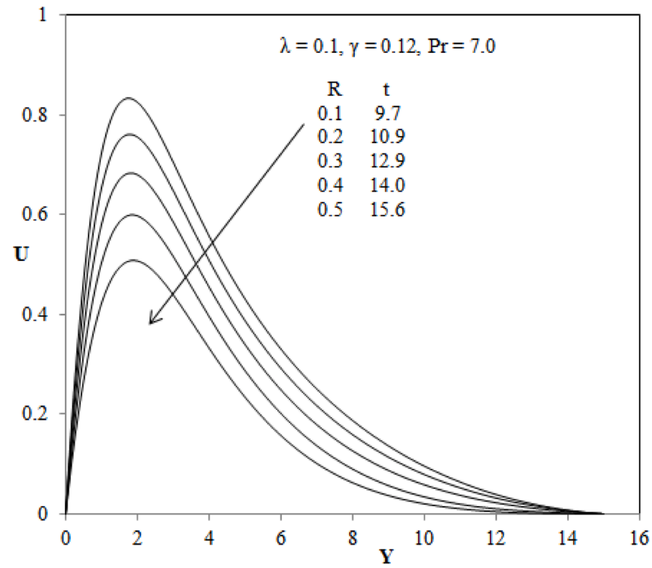

Fig 3. Impression of R on velocity outline for water.

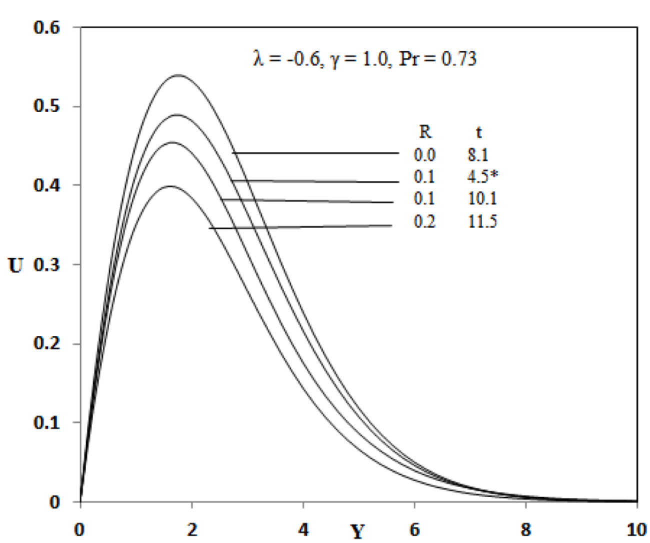

Fig. 2. Impression of R on velocity outline of air.

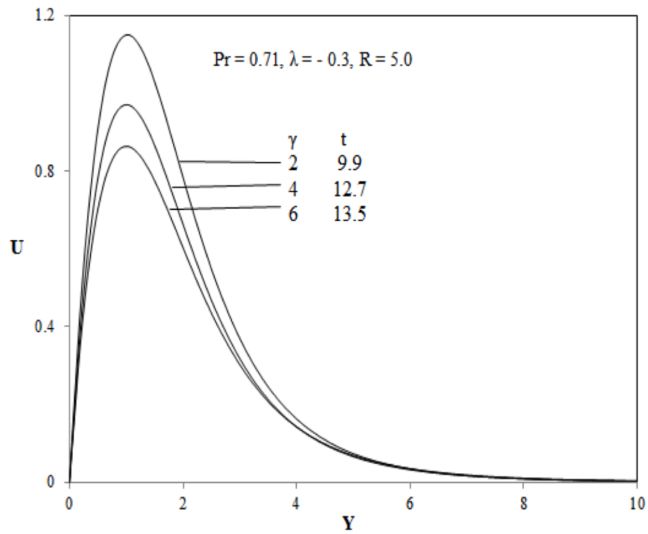

Fig 4. λ Impact on air.

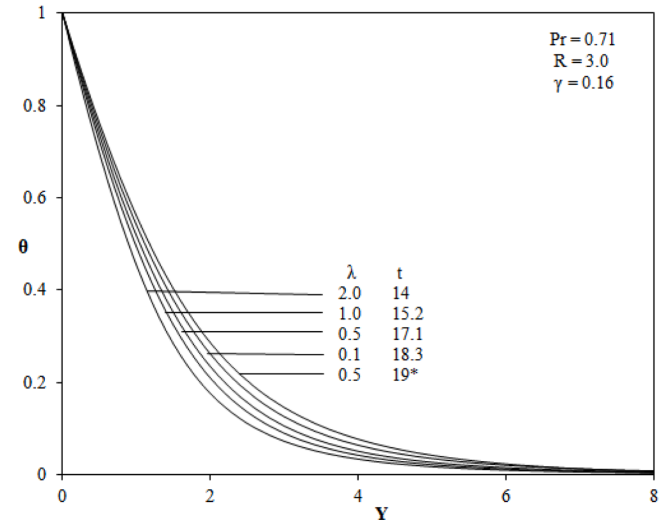

Fig. 6. Temperature profile for γ impression.

the boundary layer. That means that the fluid particles closer to the surface move slower, while those further away move faster. The velocity profile in the boundary layer can be classified into two regions: the laminar sublayer and the turbulent region. In the laminar sublayer, which is closer to the surface, the velocity gradient is relatively small, and the velocity profile follows a smooth, parabolic shape known as the Hagen–Poiseuille profile. In this region, viscosity dominates, and the velocity profile is directly influenced by changes in viscosity. As the boundary layer moves further away from the surface and enters the turbulent region, the velocity profile becomes more complex and irregular due to the interaction of eddies and vortices. In the turbulent region, viscosity still plays a role in influencing the velocity profile, but other factors like turbulence intensity become more significant.

Impacts of R on the temperature outlines were shown in Figs 5 **and** 6 for water and air for different values of λ, γ, temperature profiles grow as R increases with time. When R is large, it takes longer to attain a steady state temperature curve for several factors. At first glance, there is no variation in temperature profiles across time. When temperatures rise, the boundary layer thins and its temperature profiles drop. Radiation can also influence the boundary conditions at the surface of the plate. For example, if the surrounding environment is at a different temperature, the radiative energy transfer between the plate and the surroundings can modify the temperature gradient at the surface. It's important

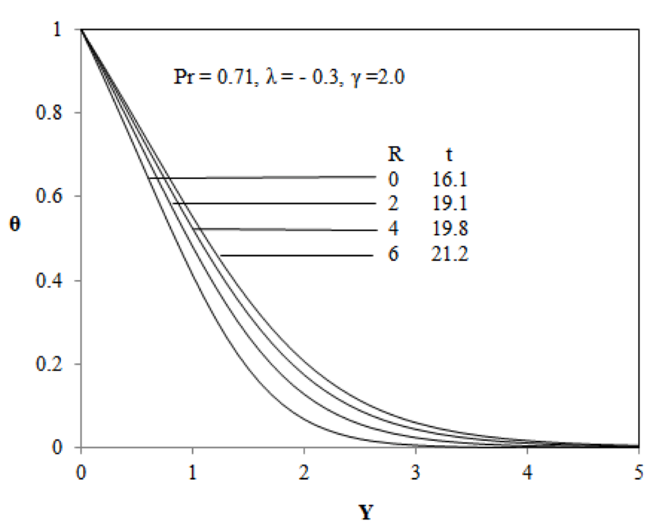

Fig 5. Temperature outline with Influence of R.

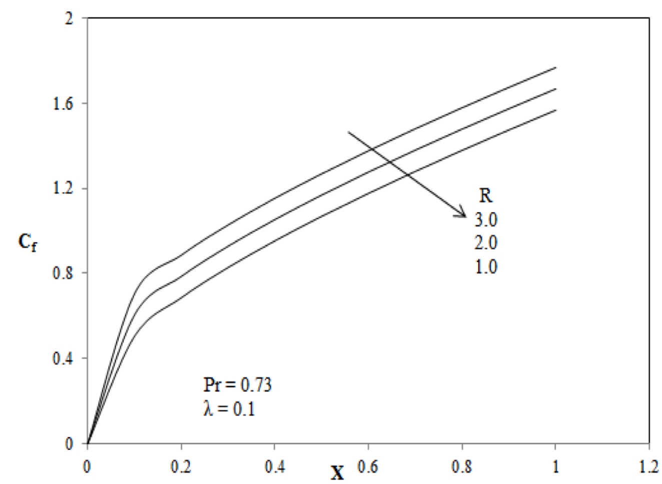

Fig. 7. Impression of C_f.

to note that the impact of radiation on the temperature profile depends on various factors such as the emissivity of the plate material, the temperature difference between the plate and its surroundings, the geometric configuration of the plate, and the presence of other heat transfer mechanisms (such as conduction or convection). The impact of thermal conductivity on the energy profile for an unsteady vertical plate is primarily governed by the conduction heat transfer mechanism. Thermal conductivity is a property that quantifies a material's ability to conduct heat, and it plays a crucial role in determining how energy is transferred through the plate. in the context of an unsteady vertical plate, the physical mechanism behind the influence of thermal conductivity on the temperature profile can be described as follows. Thermal diffusivity, which is the ratio of thermal conductivity to the product of material density and specific heat capacity, characterizes the rate at which temperature changes propagate through a material. Higher thermal conductivity generally corresponds to higher thermal diffusivity, meaning that temperature changes spread more quickly through the plate material. The thermal conductivity of the plate material also interacts with other heat transfer mechanisms, such as convection or radiation. In the presence of convection, for example, the temperature profile is influenced by the interplay between heat conduction and convective heat transfer. The thermal conductivity determines the conduction component of heat transfer, affecting the overall temperature distribution. To analyze the temperature profile for an unsteady

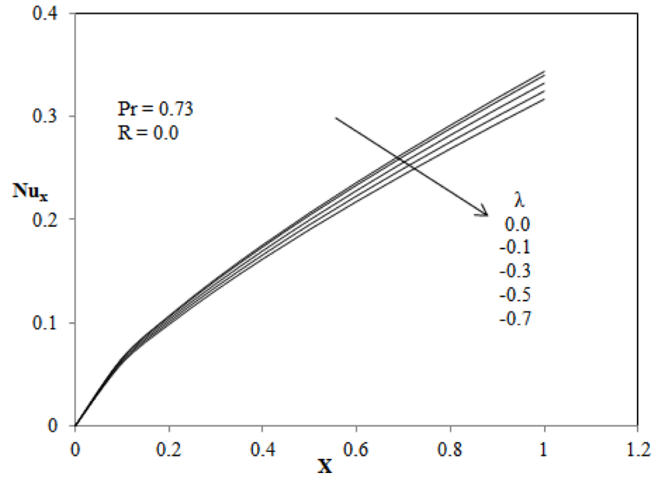

Fig. 8. Influence of Nu_x.

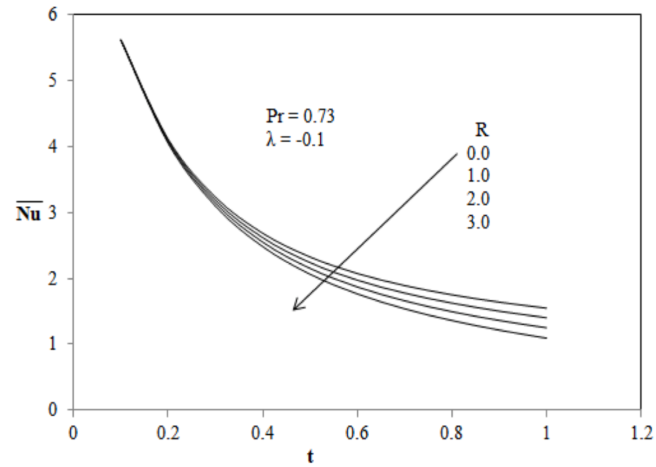

Fig. 9. Average Nusselt number.

vertical plate with thermal conductivity effects, it is necessary to consider the heat conduction equation, often known as the heat diffusion equation, along with appropriate boundary and initial conditions. This equation relates the temperature distribution to the material properties, including thermal conductivity, and describes how temperature evolves over time as a result of heat conduction.

C_f decreases monotonically for the different values of R along the upward direction of the plate, as realized in Fig. 7. When the R is reduced, and the importance of $\lambda = 0.1$ for air is used, C_f decreases. Skin friction is mainly determined by the flow regime, viscosity of the fluid, and the nature of the boundary layer. The physical mechanisms that govern the skin friction profile for an unsteady vertical plate include viscous shear stresses. The flow of fluid near the surface of the plate experiences viscous effects, resulting in the generation of shear stresses. These shear stresses cause frictional drag on the surface, known as skin friction. The magnitude of skin friction is affected by the viscosity of the fluid and the velocity gradient near the surface. As the fluid flows over the plate, a boundary layer forms near the surface. The boundary layer thickness and its development depend on factors such as the velocity of the fluid, viscosity, and the characteristics of the flow (e.g., laminar or turbulent). In laminar flow, the boundary layer is relatively thin, while in turbulent flow, it is thicker. The transition from laminar to turbulent flow has a important impact on the skin friction outline. In the laminar boundary layer, the skin friction is relatively low, whereas in the turbulent boundary layer, the skin friction is higher due to increased momentum transport and turbulent fluctuations. Radiation, on the other hand, primarily affects the thermal aspects of the system. It influences the temperature delivery and energy transfer rate between the plate and the surrounding environment, but it does not directly influence the skin friction profile.

Fig. 8 shows the Nu_x for air (Pr =0.73) at various levels of λ. The Nu is seen to be unaffected by X for small values. However, as λ is decreased for higher values of X, the Nu_x reduces. When a fluid flows over a surface, a boundary layer is formed near the surface. Within the boundary layer, there exist velocity and temperature gradients. These gradients are influenced by the changes in fluid viscosity. Variable viscosity affects the shear stress distribution, which, in turn, affects the velocity gradient in the boundary layer. Additionally, the temperature gradient is influenced by the variable viscosity, as viscosity affects the convective heat transfer process. Convective heat transfer in the boundary layer is affected by the velocity and temperature gradients. In laminar flow, heat transfer primarily occurs through molecular conduction, and the Nusselt number is relatively low. In turbulent flow, on the other hand, heat transfer is enhanced by the presence of eddies and turbulent fluctuations, resulting in increased convective heat transfer and higher Nusselt numbers. The viscosity of a fluid typically decreases with increasing temperature. As the temperature of the fluid within the boundary layer changes, the viscosity also varies. This change in viscosity can alter the velocity and energy gradients, subsequently affecting the convective heat transfer rate and the Nusselt number profile.

Fig. 9 depicts the average Nusselt number ($Pr = 0.73$). It reduces as R increases and for the value of λ = - 0.1. The influence of radiation on the Nusselt number profile for an unsteady vertical plate is primarily related to the radiative heat transfer mechanism and its interaction with convective heat transfer. The physical mechanism behind this effect can be described as follows: Radiation is a mode of heat transfer that occurs through the emission, absorption, and re-emission of electromagnetic radiation. In the case of an unsteady perpendicular plate, radiation is emitted from the plate's surface and can be absorbed by the surrounding fluid or other nearby surfaces. The radiative heat transfer rate is influenced by factors such as the temperature and emissivity of the plate, the temperature of the surrounding environment, and the view factors between the plate and its surroundings.

5. Conclusion

A finite difference flow technique was conducted on a semi-infinite perpendicular plate with thermal radiation and heat transfer flow effect. An implicit Crank-Nicolson finite difference approach resolves the governing equations. In addition, a comparison is made with the existing numerical results. It is determined that validation between the two results is admirable. The influence of velocity and temperature fields on various parameters is investigated. Before reaching their respective steady-state values, temperature profiles and the transient velocity attain their maximum levels. The C_f, as measured by the Nu, are represented visually.

1. As R rises, the fluid velocity will continue to decrease when the velocity rises with an increase of γ.
2. When R increases, there is a corresponding rise in the temperature profile.
3. The temperature profile decreases when λ increases.
4. When R decreases, C_f decreases.
5. When Nu_x decreases, λ decreases.
6. Average Nusselt number reductions as R declines.

Declaration of Competing Interest

The authors declare that they have no competing interests.

Data availability

Data will be made available on request.

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