

A Review Study of Numerical Simulation of Lid-Driven Cavity Flow with Nanofluids

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Perhaps the most deliberated fluid problem in the field of Computational Fluid Dynamics is the lid driven cavity flow whose simple geometry is used to study the thermal behavior of many engineering applications such as cooling of electronic equipment, solar collectors, thermal storage systems, food processing, solar ponds, crystal growth, lubrication technologies and cooling of electrical and mechanical components. Researchers have been devoting much of their time in order to discover innovative methods to enhance the thermal conductivity of conventional fluids. With the development of nanotechnology, the concept of nanofluids has gained ground considerably as a new kind of heat transfer fluid. Nanofluid is a new kind of fluid with high thermal conductivity is a mixture of solid nanoparticles and a liquid. This review recapitulates the recent progress of the various numerical methods that are used in predicting the influence of several parameters such as type of nanoparticle and host liquid, particle volume concentration, particle size and shape, Brownian diffusion and thermophoresis effect on hydrodynamic and thermal characteristics of convective heat transfer using nanofluids in a lid driven cavity.

KEYWORDS: Lid Driven, Cavity Flows, Nanofluids, Hybrid Nanofluids, Homogeneous Flow.

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1. INTRODUCTION

Perhaps the most deliberated fluid problem in the field of Computational Fluid Dynamics is the lid driven cavity flow whose simple geometry not only exhibits the complex hydrodynamics flow structure and phenomena such as vortex merging,^{1–3} flow separation^{4,5} corner singularities^{6,7}

boundary layers,^{8,9} chaotic mixing^{10,11} but also retain the flow physics with rotating vortices appearing at the corners of the cavity which are often used as a numerical benchmark platform to study particular physical effect and to validate new CFD codes and results.¹² Shanker and Deshpande⁶ and Kuhlmann and Romanò¹³ had given a broad assessment on lid driven cavity flow. Cavities or containers or enclosures as they are called, are always assumed to be completely filled with fluids and that gravity playing no role⁶ are some of the equipment that are frequently used in engineering applications as well as in different industries. Many applications of heat sources within a cavity can also be found in natural phenomena such as climate control, better understanding of meteorological and geophysical phenomena, and in industrial applications like cooling of electronics equipment, heat exchanger, nuclear power, lubrication, refrigeration, medicine, thermal storage and renewable energy, automotive, microfluidics, combustion etc., are just some of the few applications.^{14,15}

It was Prandtl's¹⁶ pioneer work on fluid flow with very low viscosity that has influenced researchers like Batchelor,¹⁷ Kawaguti,¹⁸ Burggraf¹⁹ and others to work in closed domains whereas, with the help of steady laminar flow within closed streamlines Batchelor¹⁷ studied separated eddies in the limit of infinite Reynolds number while Burggraf¹⁹ was influenced by the work of Kawaguti¹⁸ to study the viscous structure of separated eddies at finite

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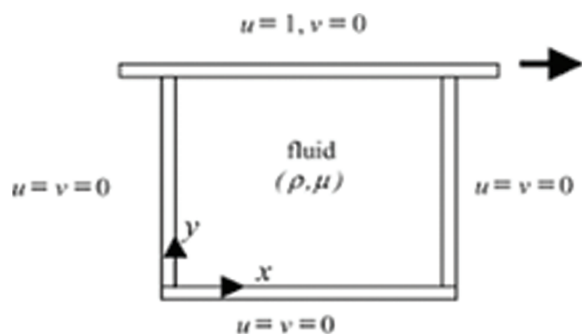


Fig. 1. Classical problem of lid driven square cavity flow. u and v represents the x and y components of velocity whereas μ and ρ represents the fluid viscosity and density respectively.²²

Reynolds number.²⁰ Since then the simple geometry of lid driven cavity, within which almost all physical phenomena can be generated has become a motivating area of study in Computational Fluid Dynamics.

1.1. Cavity Flow

Cavity flow (Fig. 1) which deals with the internal recirculating flow generated by the movement of one or more of the containing walls⁶ is a great motivation for fluid dynamists in understanding the flow within the cavity²¹ as these flows are not only technically significant but also of great scientific value since they display almost all fluid mechanical phenomena like corner eddies, longitudinal vortices, non-uniqueness, transition, and turbulence all taking place naturally within the cavity (Fig. 2).

Hussien et al.¹²⁸ considered an analysis of research papers on the applications of the effects of flow and heat transfer within cavities. For this review the researchers considered the impact of thermal properties of the fluid, the shape of the cavity, with initial and boundary conditions. Moreover, they also considered the influence of dimensionless numbers like Reynolds number, Richardson number, Grashof number, Rayleigh number, Darcy number, Hartmann number and Prandtl number.

1.1.1. Various Geometries

There has been a remarkable growth in the development of numerical algorithms with the advancement of computational power particularly in the field of Fluid Dynamics, thereby assisting fluid dynamists in solving their governing equations. In order to benchmark their observations based on these algorithms, fluid dynamists more often than not rely on flows of benchmark geometries like lid driven cavity.²⁴ Consequently, substantial progress in understanding the flow pattern within enclosures of different geometries like square, rectangular, triangular, trapezoidal,²⁵ L-shape,^{26,27} T-shape,^{28,29} C-shape,³⁰ semi-circular,^{31,32} cross shape, etc., can be found in the literature as flow inside a cavity has many practical applications in industries as well as in academic researches. A numerical

study was undertaken by Cakmak et al.,³³ to investigate mixed convection heat transfer. The geometrical setup for the study was a Al_2O_3 -water filled square lid driven cavity which had a moving cooled top wall while the other three walls were fixed. Using finite volume method they concluded that due to the presence of nanoparticles there was an increase in the heat transfer rate.

Ardalan, et al.³⁴ considered a square lid driven cavity filled with copper-water nanofluids. The top wall was in motion with constant velocity or sinusoidal function while other walls were fixed. The bottom wall was kept at a higher temperature than the top wall while the vertical walls as adiabatic. Using Lattice Boltzmann method, they observed that the mechanical power needed to keep the lid moving with a constant velocity increases in the case of nanofluid but the power decreases if the lid moves with a sinusoidal velocity. They also observed that heat transfer rate was directly related to volume fraction of nanoparticles and inversely related to Richardsons number. Shulepova et al.,³⁵ analyzed mixed convective heat transfer and fluid flow in a square enclosure filled with alumina-water nanofluid and with a mounted adiabatic fin and internal solid block on the bottom wall. They considered the side walls of the enclosure to be isothermal i.e., left was heated and right was cooled while the top wall was in motion. Solving the governing equations by finite difference method, they observed that the intensity of heat transfer was a function of position of the internal block and volume fraction of nanoparticles.

Muhammad et al.,³⁶ studied mixed convection in a square cavity filled with a combination of ethylene-glycol as base fluid and silver as nanoparticles. The square cavity had isothermally cooled moveable side walls. An isothermal heat source was placed at the lower horizontal wall. They inferred using finite volume method that heat transfer rate with this nanofluid was better when the heater was at center of gravity. Jamesahar et al.,³⁷ investigated the effect of mixed convection on heat transfer and fluid flow due to two oscillating fins inside a square cavity filled with nanofluid. The cavity was set up in such a way that horizontal walls were insulated while left wall was at a higher constant temperature than the right wall which was also at a constant temperature. Both the fins had same frequencies and amplitudes and were attached on the left wall. Finite element method was the mathematical tool used to solve the governing equations whereupon they concluded that increase of heat transfer was due to oscillations of fins. But lower heat transfer rate was due to increase in both thermal conductivity and viscosity.

Adibi et al.,³⁸ studied the effect of nanofluid on heat transfer in a closed domain with different aspect ratio. The enclosure was filled with water-Titanium oxide nanofluid with the upper wall having a temperature different from that of the lower wall and moving with a constant velocity. The result was obtained using finite volume method

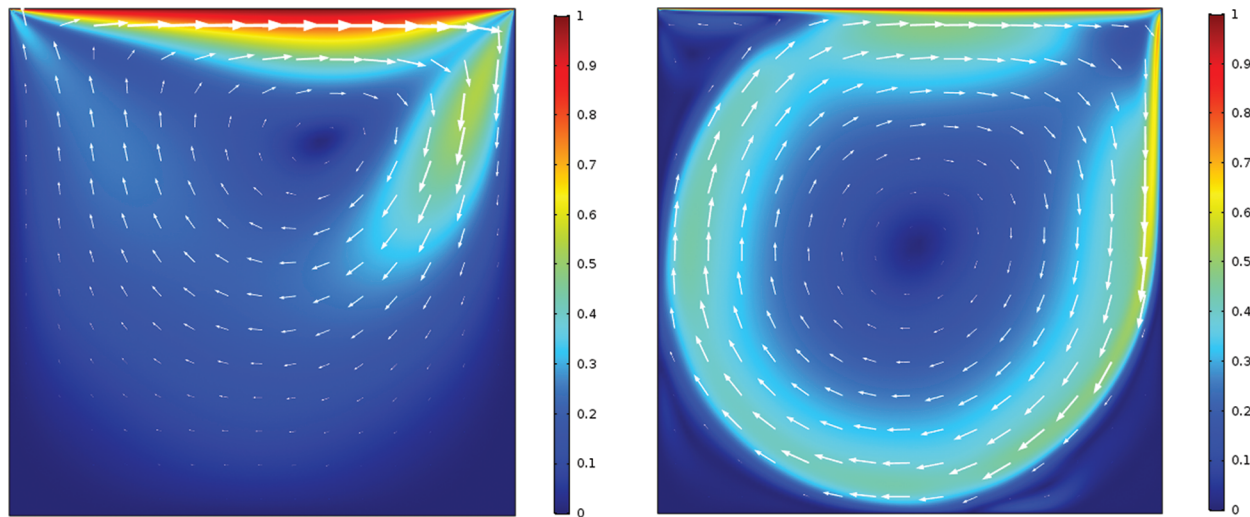


Fig. 2. The magnitude of the velocity and the direction of the flow in a cavity for Reynolds numbers of 100 (left) and 10,000 (right).²³

whereby they concluded that the average Nusselt number could be enhanced by 23% when using nanofluid than with simple base fluid water. Ghasemi and Siavashi,³⁹ investigated the effect of mixed convection magneto-hydrodynamic inside a three-dimensional enclosure with lid driven wall which was filled with Cu-water nanofluid. The cavity had fixed hot and cold vertical walls and also fixed adiabatic bottom wall. The top adiabatic wall moved in three different directions and with different velocities so as to form a 3-D mixed convection. They considered the multi-relaxation time lattice Boltzmann method (MRT-LBM) as their numerical method to solve the governing equation from where they concluded that improvement in the heat transfer could be achieved by continuously increasing the volume fraction of nanofluids but higher Nusselt number could be achieved with higher Reynolds number and lower Richardson number. Abu-Hamdeh et al.,⁴⁰ investigated mixed convection in a square lid driven cavity with one side open. The cavity was equipped with a flush mounted heater on the bottom wall. Solving the governing equations by finite volume technique, they concluded that heat transfer and flow characteristics were complex due to moving lid, open side wall and heater. However, they observed that heat transfer was a function of Grashof number and length of the heater.

In order to investigate mixed convection and heat lines of nanofluid inside a wavy bottom wall square cavity Azizul, et al.,⁴¹ placed solid blocks and a mixture of alumina-water inside this cavity. The wavy bottom was heated to a constant temperature while both the vertical walls were kept at cold temperature. Top horizontal wall was adiabatic and moved with constant velocity. Solving the mathematical equations by finite element method they concluded that better fluid flow and a reasonable rate of heat transfer could be obtained with high Reynolds number. Alsabery et al.⁴² investigated the

magneto-hydrodynamics mixed convection of nanofluid (water- Al_2O_3) in a lid driven square cavity with heated triangular wall. The governing equations were solved using the finite element method. The influence of the dimensionless numbers like Reynolds number, Richardson number, Hartmann number, along with the volume fraction of nanofluids and the thickness of the triangular wall were considered. They concluded that heat transfer depended on Reynolds and Richardson numbers and the maximum average Nusselt number was obtained for when Reynolds number was 500 and volume fraction of nanofluid was 0.02. Gorla et al.,⁴³ examined the influence of nanofluids on the magneto-hydrodynamic mixed convection flow in a square cavity inside which there was heat generation system, copper-water nanofluids and four square thermally insulated obstacles. The top and bottom walls were adiabatic and moving with constant speed. Using SIMPLER algorithm, they concluded that the pattern of isotherms and its strength significantly changed along with the position of heater.

In almost a similar setup, Shirani, et al.,⁴⁴ numerically investigated mixed convection in a square lid-driven cavity filled with Cu-water nanofluid and four insulated rotating cylinder with harmonic motion. The numerical tool for this setup with top and bottom walls of the cavity at different temperature was finite volume method whereupon they observed that heat transfer was a function of angular velocity of the cylinder, type of rotation and volume fraction of nanoparticles. In another study of Shirani and Toghraie⁴⁵ investigated the heat transfer in a lid driven cavity filled with Cu-water nanofluid and four rotating cylinders having harmonic motion. They considered seven different possibilities in order to find the best position and diameter ratio of cylinders. The conclusions made after their observations was that the behaviour of heat transfer was unpredictable because of the combinations of harmonic motion and the decrease in the space between the

wall and the cylinders. Ahmed et al.,⁴⁶ investigate mixed convection and heat transfer rate in a two-sided lid driven square cavity filled with copper-water nanofluid. The geometrical setup of the problem was a square cavity with top and bottom walls moving in horizontal direction whereas the side walls were fixed. The solution was obtained using finite volume method whereby they found that increasing the volume of nanoparticles indicated an increase in the heat transfer rate for higher values of Hartmann number.

Shulepova, et al.,⁴⁷ studied mixed convection-radiation in a square cavity which was lid driven and filled with a combination of alumina-water nanofluid. The top vertical wall was moving with a constant speed and constant low temperature while the other walls were adiabatic. Using finite difference method, they concluded the heater temperature as well as convective flow rate could be decreased by adding nanoparticles. Nazari, et al.,⁴⁸ undertook an investigation of mixed convection of non-Newtonian fluid. For this investigation they considered a geometrical setup which consists of a two lid driven square cavity filled with water-alumina nanofluid along with a porous media. The vertical walls kept at different constant temperatures were in up and down motion while the other walls were obviously insulated. They concluded using finite volume method that temperature gradient was a function of volume fraction of nanoparticles, Darcy as well as Richardson number. Colak et al.,⁴⁹ considered mixed convection in a chamfered lid driven cavity filled with nanofluid and heated from bottom. This cavity had top wall moving in both positive as well as negative directions. While the side walls were adiabatic the top wall was cooler than the bottom wall. Finite volume method was used to discretize the field. Their finding was that the negative direction of the lid increased the average Nusselt number by 57.2% in comparison to the positive direction.

Chamkha, et al.,⁵⁰ studied the effect of a rotating cone on mixed convection in a double lid driven porous trapezoidal cavity filled with CNT-water nanofluids with the help of finite element method. The adiabatic parallel sides were moving in opposite directions whereas the two side walls were fixed with different temperatures. Their findings say that because of the aspect ratio of the cone, better heat transfer rate was achieved along with 95% increase in the average Nusselt number. One study undertaken by Al-Rashed et al.,⁵¹ to investigate flow field and heat transfer of copper-water nanofluid was within a trapezoidal cavity. They considered the unparallel sides to be insulated while the parallel sides i.e., the bottom and the top were treated as hot and cold respectively. With the help of finite volume method, they concluded that the average Nusselt number was a function of Reynolds number, Darcy number, aspect ratio and concentration of nanoparticles. Ahmed, et al.,⁵² investigated the simulation of (MHD) Ferro-convective flow in an inclined L-shaped double lid driven cavity with heated corners. The enclosure which

was filled with Ferro fluid had the top and right boundaries of the L-walls moving. The Galerkin finite element method was used to obtain the result. The length of the heated corner had a strong impact on the isothermal distributions. With Hartmann number ranging from 0 to 50 and volume fraction of nanoparticles at 5%, there was a minimization of the heat transfer rate by 30.66%. Jabbar et al.,⁵³ investigated convective heat transfer of copper-water nanofluid in an inclined porous cavity. The setup that they used for this study was a square inclined cavity with the bottom wall moving and kept at a higher temperature while side walls were insulated. The top wall was sealed with a rotating cylinder. The governing equations were solved with the help of Galerkin finite element method. The observation made by the researchers was that the rotation of the cylinder played a big role in enhancing the average Nusselt number. Nguyen et al.,⁵⁴ investigated heat transfer of ethylene glycol-Fe₃O₄ nanofluid inside a curved porous cavity with two lids in motion and also under the impact of an electric field. Using CVFEM they inferred that convective flow becomes stronger because of high voltage.

1.1.2. Types of Flow

Fluid flow can generally be categorized as (i) Free or Natural and (ii) Forced. In free flow, the motion is due to the natural means like buoyancy effect i.e., the rise of hot or warmer (and thus lighter) fluid and the fall of cooler (thus denser) fluid whereas a flow due to an applied external force is called a force flow.⁵⁵ Shafee et al.,⁵⁶ used a magnetic force to numerically investigating forced convection within a permeable square lid driven cavity filled with nanofluid (Copper oxide-water). Lattice Boltzmann method was engaged to obtain the numerical solution. The result shows that with the increase in the Darcy and Reynolds numbers, the isotherm gathered next to the upper surface. It was established by them that the role of convective heat transfer increased with the increase of Reynolds number whereas it decreases with increase of Lorentz powers. A study was undertaken by Sivanandam et al.,⁵⁷ to understand the effects of the directions of the moving wall on mixed convection in a lid driven cavity filled with nanofluid. The top wall of the cavity moved in either directions and control volume technique was used to solve the governing equations. They concluded that direction of the moving wall had great impact on the rate of energy transfer. They observed that a higher rate of heat transfer could be achieved when the lid moved from left to right rather than when the lid moved from right to left. Hadize and Poshtiri⁵⁸ considered a square lid driven cavity with the top moving wall and the bottom wall as insulated and hot respectively while side walls were cold and filled with micropolar nanofluid. This set up was considered in order to study rate of heat transfer due to mixed convection. Applying SIMPLE algorithm based finite volume method for the solution of this laminar, steady and incompressible flow, they inferred that enhancing the Grashof number

enhanced the Nusselt number and heat transfer rate. Moreover, they observed that with the addition of nanoparticles the rate of heat transfer was more in the case of Newtonian fluid than in the micropolar nanofluid.

Muthukumar et al.,⁵⁹ investigated mixed MHD convection and thermal radiation of Cu-water nanofluid in a lid driven cavity. The side walls were heated sinusoidal while the other walls were thermally insulated. SIMPLE algorithm based on finite volume method was used to solve the governing equations for this 2D, Newtonian, laminar steady and incompressible flow. The final conclusion was that with the increase of volume fraction of nanofluid there was an increase of rate of heat transfer which was more effective in the presence of thermal radiation than in its absence. Hussain et al.,⁶⁰ undertook a study of heat transfer and fluid flow due to mixed convection in a lid driven cavity which was heated from the bottom while cooled from the top and filled with alumina-water nanofluids and an adiabatic bar. Applying Galerkin finite element method to the governing equations of this 2D, steady, incompressible, Newtonian and laminar flow, they observed that the position of the bar was a good parameter for heat and fluid flow with the vertical position being more effective than the horizontal position. Furthermore, the role of the bar becomes ineffective at low Richardson numbers. Rodrigues et al.,⁶¹ investigated geometrical optimization in a lid driven cavity with two rectangular intrusions. For this they considered the flow to be 2D, unsteady, incompressible, laminar and mixed convection. Finite volume method was used to solve the equations of mass, momentum and energy. They found that the increase in the heat transfer rate was the result of increase of heat exchange area along with a drop in the Nusselt number and increase in the ratios of height to length of the fins. Asiaei et al.,⁶² considered an unsteady, incompressible, Newtonian and laminar 2D flow inside a two sided lid driven cavity which was filled with a combined Cu-water fluid and a multi-layer porous material with an internal heater. Using finite volume method they concluded that there was an increase in the heat transfer rate by about 17% which was particularly due to the multi-layer porous media. Selimefendigil and Oztop⁶³ considered an inclined lid driven L-shaped cavity to study MHD mixed convection of nanofluid like CuO-water under the effect of internal heat generation. The fluid for this study was considered to be 2D, incompressible and Newtonian. Numerical solution were obtained by using finite element method from wherein they concluded that there was an increase in the average Nusselt number along with increase in the volume of nanoparticles.

1.2. Nanofluids

During the last few decades, researches were conducted in order to increase the thermophysical properties of conventional fluids. In order to enhance the thermal properties of water, Maxwell in the nineteenth century, introduced

the concept of suspending millimetre size metallic particles in the base fluid.⁶⁴ However, with the development of technology, Choi and Eastman⁶⁵ for the first time introduced particles with diameter less than 100 nm and named them as nanoparticles. They developed a combination of distilled water and nanoparticles and called this combination as nanofluid.⁶⁵ Today, nanofluids have attracted lot of attentions among the thermal scientists, engineers and the experimental researchers because of its thermo-physical properties and extensive use as a coolant in many industries like heat exchangers, automobiles, renewable energy, nuclear reactors, electronic cooling, etc.,^{64,66-69}

1.2.1. Homogeneous and Non-Homogeneous or Two-Phase Flow Model

There are generally two models to study the numerical simulation of nanofluids namely homogeneous and non-homogeneous models. The homogeneous model is mostly used to study the fluid flow and temperature distribution of the nanofluid with the assumptions that the nanoparticles and the base fluid move with the same velocity and are in thermal equilibrium or in other words it could be said that in homogeneous model the nanoparticles are assumed to be uniformly distributed within the base fluid thereby effectively change the properties of the fluid and hence are treated as single phase. In the non-homogeneous model the distribution of the nanoparticles and the base fluid are no longer considered to be uniform and there exists a relative velocity between them due to Brownian motion and thermophoresis.⁷⁰ Hadavand et al.,⁷¹ studied heat transfer enhancement of nanofluid in a semi-circular lid driven cavity. The combination of silver nanoparticles and base fluid water was studied as laminar, single phase and Newtonian and the governing equations were solved through the finite volume method. They observed that temperature of the heated fluid could be reduced by moving the heated fluid in the channel. Sheikholeslami,⁷² considered a lid driven porous cavity to investigate the effects of forced convection in homogeneous nanofluid flow. Numerical solutions were obtained with the help of CVFEM. The observations were that increasing the Darcy and Reynolds numbers did improve the heat transfer rate but the reverse happened for Hartmann numbers. A square cavity containing two rotating cylinders and filled with nanofluid and also subjected to a magnetic field was used by Barnoon et al.,⁷³ to numerically study mixed convection and entropy generation. The moving upper wall was hot while the lower horizontal wall was under the influence of a uniform and constant magnetic field. Solving the governing equations for two phase flow by Finite Volume method, they inferred that the rate of heat transfer could be enhanced by reducing the Hartmann number, Richardson number and by increasing the volume fraction of nanoparticles.

1.2.2. Hybrid Nanofluid

In order to have additional improvement in the properties of heat flow, researchers have been studying heat transfer with hybrid nanoparticles which is a mixture or composite of two or more different nanoparticles. Selimefendigil and Chamkha,⁷⁴ undertook a study of magnetohydrodynamics mixed convection in a triangular shaped partitioned lid driven square cavity filled with a hybrid nanofluid of Ag–MgO/water involving a porous layer. The vertical walls were heated with two different temperatures while the top horizontal wall was moving with a constant speed. Using finite element method, they observed that by changing the position of the porous medium significant changes could be seen in the average Nusselt number. Moreover, they also observed that the triangular shaped porous compound was an excellent tool for convective heat transfer. Cimpean et al.,⁷⁵ numerically investigated mixed convection of hybrid nanofluid in a porous trapezoidal lid driven cavity. The bottom wall was heated while the top moving wall was cooled with the other walls as adiabatic. Finite difference method was used as the numerical method to obtain the result. They concluded that an increase in the Darcy number gave a better rate of heat transfer.

Batool and Nawaz⁷⁶ analyzed thermal performance of fluid with microstructure with the help of hybrid nanoparticles in a square lid driven cavity. The bottom of the cavity was heated whereas the other walls were cooled with the top wall moving with a constant speed. Results were obtained with the help of finite volume method. They predicted that with the decrease of Reynolds number there was an increase in the rate of heat transfer. Yan et al.,⁷⁷ studied the average Nusselt number in a tall and narrow rectangular lid driven cavity using hybrid nanofluid (water, copper and Titanium oxide). The geometrical setup for this study consists of a rectangular cavity with a moving top wall and rest of the walls being fixed. The left side wall was heated while the right side wall was cool. The governing equations were solved using finite volume method. They observed that the average Nusselt number increased with the increase in the volume fraction of the nanofluid. A study on mixed convection was undertaken by Alsabery et al.,⁷⁸ inside a wavy lid driven cavity having localized solid blocks and filled with hybrid nanofluid Cu–Al₂O₃. Both the horizontal walls were insulated with the top one as the lid driven. Using Finite element method, they concluded that the heat transfer characteristics could be controlled by the position of the solid blocks, the wavy nature of the surface as well as by the concentration distribution of the nanoparticles.

Hussien et al.¹²⁹ undertook a survey on the preparation methods of hybrid nanofluids as well as on the rate of heat transfer of hybrid nanofluids. From this review, the researchers observed that because of hybrid nanofluids there had been an enhancement in the rate of heat transfer as compared to base fluids along with a reasonable

increase in the pressure drop. Hussien et al.¹³⁰ considered MWCNTs/GNPs hybrid nanofluids in microtubes in order to study their heat transfer and entropy generation capacities. This study revealed that due to these hybrid nanofluids, there was an improvement in the rate of heat transfer and pressure drop along with a decrease in the generation of total entropy.

1.3. Objectives of the Review and the Focus of the Paper

A review paper gives immense information to researchers on a particular subject that they are looking for in a very comprehensive manner. It makes them aware with previous results, precludes duplication and helps in exploring new areas of research. This paper focused on the various Numerical Methods that are been extensively used in solving the equations governing the different heat transfer and fluid flow characteristics arising in a lid driven cavity filled with different nanofluids.

2. NUMERICAL SIMULATION OF CAVITY FLOW WITH NANOFLUIDS

In spite of the advantages and disadvantages of numerical methods in solving equations of heat transfer and fluid flow characteristics, it has been widely accepted as an important method for the simulation of lid driven cavity flow with nanofluids.⁷⁹ As it is not always possible to measure the flow details, numerical techniques are used not only as compliment to experimental studies but also provides supplementary information regarding the flow.⁸⁰ However, there is no mechanism to decide which method is suitable to apply as a given problem is not always straightforward and every method has its own advantages and disadvantages.⁸¹

2.1. CFD: FDM, FVM, CVFEM Approaches

The three mostly commonly used numerical methods to solve partial differential equations are finite difference (FDM), finite element (FEM) and finite control volume methods (CVFEM).⁸² FDM are a group of numerical techniques for solving differential equations by approximating the derivatives present in the differential equations with finite differences into a system of linear equations which are then solved with the help of matrix algebra. FEM is another numerical technique which is frequently used to solve partial differential equations arising in engineering, like problems related to structural analysis, heat transfer, fluid flow, etc., This method subdivides a large system into simple and smaller parts called finite elements, which finally reduces into a system of algebraic equations.⁸³ Finite volume method which is also known as control volume finite element methods or finite volume element methods or mixed co-volume methods⁸² is a

hybrid numerical method⁸⁴ and lies in between finite difference and finite element methods. They have grid flexibility similar to finite element and can be used similar to finite difference methods.⁸²

Astanina et al.,⁸⁵ investigated mixed convection in a lid driven cavity using alumina-water nanofluid. The moving upper wall was kept at a constant cold temperature while the bottom wall was at hot temperature. The mathematical solutions were obtained by solving the governing equations with the help of finite difference method. They observed that with the addition of nanofluid, there was enhancement of heat transfer for free convection while it was the opposite for mixed as well as forced convection. Gibanov et al.,⁸⁶ analyzed mixed convection along with entropy generation of alumina-water nanofluid inside a lid driven cavity. They considered the cavity geometry to be moving top isothermal wall while the bottom wall was thick heat conducting solid wall. Finite difference method was used as the mathematical tool to arrive at the results. Their results indicated that with the increase of nanoparticles volume there was an increase in the rate of heat transfer but with a reduction of average Bejan number. Oztop et al.,⁸⁷ undertook a computational study to investigate the effects of mixed convection in a partially heated wavy walled lid driven cavity filled with nanofluid. The top wall was in motion with the side walls being adiabatic. The governing equations of mixed convection was numerically analysed by finite difference technique. They concluded that optimization of the rate of heat transfer was a function of volume fraction, Hartmann and Richardson numbers. Ismael et al.,⁸⁸ studied the heat transfer and fluid flow characteristics of CuO-water nanofluid inside a lid driven cavity. The left vertical wall moved upward and was isothermally cooled while the remaining walls were adiabatic with the exception that a heater was placed at the corner between the right vertical and horizontal walls. The governing equations were solved with finite difference method. The conclusion was that because of nanoparticles Nusselt number increased at high values of Richardson number.

Khan et al.,⁸⁹ investigated thermal conductivity and heat transfer enhancement of hybrid nanofluid inside a split lid driven cavity. They used alumina + copper + water combination as the hybrid nanofluid. The side walls were insulated, bottom wall was kept cold while the moveable upper lid was split in the middle with the left part moving towards the right and right part moving towards the left. Galerkin finite element method was used to solve the governing equations. They observed that due to the movement of the split lids there was an increase in the local Nusselt number and reached its maximum value when the two lids meet. Alsabery et al.,⁹⁰ used aluminum oxide water combination to study mixed convection and thermal transmission inside a lid driven wavy cavity containing a solid cylinder. The moving vertical walls were

kept at different but fixed temperature whereas horizontal walls were adiabatic. The governing equations were solved by finite element technique. Their study concludes that with low Reynolds number the overall energy transport could be increased by increasing the volume fraction of nanoparticles. Mixed convection was studied by Hussain et al.⁹¹ using a square two-sided lid driven cavity filled with alumina—water nanofluid. Both the top and bottom lids were adiabatic and moved in the directions of right and left respectively with the same velocity. Left vertical wall was hot while the right vertical wall was cold. Galerkin finite element method was used to solve the governing equations. They observed that by increasing the volume of the nanoparticles, they could achieve an increase in the rate of heat transfer while this rate decreased when the Hartmann number increased. Hussain et al.⁹² analyzed the influence of fins and inclined magnetic field on Cu-water nanofluid within a single lid as well as a double lid driven square cavity. Solving the governing equations with the help of Galerkin finite element method they concluded that presence of fins and inclination of the magnetic field did had an impact on the fluid flow and heat transfer characteristics as compared to their absence.

Ali et al.⁹³ studied mixed convection in a double lid driven rectangular cavity. They undertook this study in order to understand the effect of hybrid nanofluids (Cu–Al₂O₃–water) on heat transfer rate. They considered both the horizontal top and bottom to be moveable but kept them at difference temperature. Using finite volume method, they concluded the due to the hybrid nanofluids there was a significant increase in the rate of heat transfer. They also observed that the local Nusselt number decreased with respect to the cavity length. Jakeer et al.,⁹⁴ studied the impact of heated obstacle on the magneto-hybrid nanofluids (Cu–Al₂O₃ + water) inside a square lid driven cavity. The geometry for this study was a square cavity with the right vertical wall treated as a heat sink while the other three walls were adiabatic. Using finite volume method, they concluded that the hybrid nanofluids provided a higher heat transfer rate than nanofluids. Along with this they also concluded that average Nusselt number could be lowered by increasing the Hartmann number. Mansour et al.,⁹⁵ undertook a study in which they studied free, forced as well as Marangoni convective flow in a square cavity filled with magnetic micropolar nanofluids. The side walls were moving while fixed bottom wall was heated. Finite volume method was used as the numerical method to solve the governing equations. Their observation was that the average Nusselt number increased alongside the Marangoni number.

2.2. Fully HOC Solution (Higher Order Scheme) F D:

Central finite difference schemes are very popular numerical methods when solving partial differential equations for their easy and straight-forwardness. These methods

Table I. Depicting the various parameters of heat transfer.

S. No	Authors	Geometry	Flowregime	Workingfluid	Numerical methods	Dimensionless numbers	Observations
1.	Ardalan, et al. ³⁴	Square	Unsteady, Laminar, Single phase	Water Copper	LBM	Ri = 0.1–10 Gr = 10 ⁴ Pr = 6.2	The power needed to move the lid increased with increase of viscosity
2.	Shulepova et al., ³⁵	Square	Newtonian	Water Alumina	FDM	Ra = 10 ⁴ –10 ⁶ Re = 50–200	Rate of Heat Transfer depended on the position of the internal block
3.	Muhammad et al., ³⁶	Square	Viscous, Laminar, Incompressible	Ethylene-glycol Silver	FVM	Ri = 1–100 Re = 100	Change between Fourier's law and Cattaneo-Christov heat flux due to the effect of gravity.
4.	Jamesahar et al., ³⁷	Square	Newtonian, Incompressible, Laminar	Nanoparticles	FEM ALE	Ra = 10 ⁴ –10 ⁶	Better heat transfer in the middle part due to the oscillation of fins.
5.	Adibi et al., ³⁸	Closed Domain	Incompressible	Water Titanium oxide	FVM	Gr = 100–104 Re = 100–1000	Nusselt number is 23% more in case of nanofluid
6.	Ghasemi and Siavashi, ³⁹	Cube	Newtonian, Laminar, Incompressible	Water Copper	MRT-LBM	Ri = 0.5–5 Ra = 10 ³ –10 ⁵ Ha = 20–400	Magnetic field had an adverse effect on heat transfer.
7.	Azizul, et al., ⁴¹	Square	Steady Laminar, Incompressible	Water Alumina	FEM	Re = 5–500 Gr = 10 ³ –10 ⁶	High Grashof number contributed highest heat transfer rate.
8.	Alsabery et al. ⁴²	Square	Laminar, Incompressible, Two phase	Water Alumina	FEM	Re = 1–500 Ri = 0.01–100 Ha = 0–500	Heat transfer was directly related to Reynolds number as well as to Richardson number.
9.	Gorla et al., ⁴³	Square	Newtonian Steady Laminar, Incompressible	Water Copper	FVM SIMPLER	Ri = 0.01–100 Ha = 0–30	There was an inverse relation between average Nusselt number and the length of the heat source.
10.	Shirani, et al., ⁴⁴	Square	Incompressible, Laminar	Water Copper	FVM	Ri = 0.1–10	Addition of nanoparticles had a negative impact on the performance evaluation criteria (PEC)
11.	Shirani et al., ⁴⁵	Square	Incompressible, Laminar	Water Copper	NUMERICAL STUDY	Ri = 0.1–10 Re = 86.2–862	Rate of heat transfer was influenced by the size of the cylinders
12.	Ahmed et al., ⁴⁶	Square	Viscous, Laminar, Incompressible Single Phase	Water Copper	FVM	Ha = 0–10 Bi = 0.1–10	Decrease in the velocity components, Nusselt number could be observed with an increase in the Hatrmann number
13.	Shulepova, et al., ⁴⁷	Square	Newtonian	Water Alumina	FDM	Ra = 10 ³ –10 ⁶ Re = 50–500	For passive cooling system, alumina nanoparticles and the moving cold wall were good controlling parameter
14.	Nazari, et al., ⁴⁸	Square	Non-Newtonian	Water Alumina	FVM	Ri = 0.01–100 Da = 10 ⁻⁴ –10 ⁻²	Factor for reducing the temperature gradients could be surface penetrability
15.	Chamkha, et al., ⁵⁰	Porous trapezoidal	Newtonian Laminar, Incompressible	Water CNT	FEM	Ri = 0.5–50 Ha = 0–50 Da = 10 ⁻⁴ –10 ⁻²	Increase of heat transfer depended on the aspect ratio of the cone
16.	Al-Rashed et al., ⁵¹	Trapezoidal	Steady Laminar, Incompressible	Water Copper	FVM SIMPLER	Re = 10–1000 Da = 10 ⁻⁴ – 10 ⁻²	The average Nusselt number enhanced by increasing the reynolds number
17.	Ahmed, et al., ⁵²	Inclined L-shape	Steady Laminar, Incompressible	Water Iron oxide	FEM	Gr = 100 Ri = 0.01–10 Ha = 0–50	The average Nusselt number could be enhanced by increasing the volume fraction
18.	Jabbar et al., ⁵³	Square inclined	–	Water Copper	FEM	Ra = 10 ⁵ Ri = 0.01–100	Nusselt number could be increased by the rotation of the cylinder
19.	Nguyen et al., ⁵⁴	Curved porous cavity	Impact of electric field	Ethylene-glycol Iron oxide	CVFEM	Da = 10 ² –10 ⁵ Re = 3000	Radiative factor and average Nusselt number were directly associated
20.	Shafee et al., ⁵⁶	Square	Magnetic field	Water Copper	LBM	Da = 0.001–100 Ha = 0–60 Re = 50, 200	The convective flow was directly related to reynolds number whereas inversely related to Lorentz number

Table I. Continued.

S. No	Authors	Geometry	Flowregime	Workingfluid	Numerical methods	Dimensionless numbers	Observations
21.	Sivanandam et al., ⁵⁷	Square	Unsteady Laminar, Incompressible	Water Alumina	CVFEM SIMPLE	Ri = 0.01–100 Re = 10–1000 Gr = 10000	The rate of energy transfer was highly affected by the direction of the moving wall
22.	Hadize and Poshiri ⁵⁸	Square	Steady Laminar, Incompressible	Water Alumina	FVM SIMPLER	Gr = 10 ² –10 ⁵	The rate of heat transfer was lower in case of micropolar nanofluid than Newtonian fluid
23.	Muthukumar et al., ⁵⁹	Square	Newtonian Steady Laminar, Incompressible	Water Copper	FVM SIMPLE	Ha = 0–70 Da = 10 ⁻⁴ –10 ⁻¹ Ri = 0.01–100	At Ha = 50 the magnetic field dominated the impact of Darcy number
24.	Hussain et al., ⁶⁰	Square	Newtonian Steady Laminar, Incompressible	Water Alumina	FEM	Re = 1–200 Ha = 0–100 Ri = 0–10	By enhancing the Hartmann number, there was a decrease in the heat transfer rate as well as kinetic energy
25.	Asiaei et al., ⁶²	Square	Newtonian Unsteady Incompressible	Water Copper	FVM	Ri = 10 ⁻⁴ –10 ³ Gr = 10 ⁴	The rate of heat transfer could be improved by using multi-layered porous medium
26.	Selimefendigil and Oztop ⁶³	Inclined L-shape	Newtonian Incompressible	Water Copper oxide	FEM ALE	Ri = 0.03–30 Ha = 0–50 Ra = 10 ⁴ –10 ⁶ Ri = 1–10	Elastic wall had influenced the flow characteristics
27.	Hadavand et al., ⁷¹	Semi-circular	Newtonian Single phase Steady	Water Silver	FVM SIMPLEC	Ri = 1–10	Entropy generation could be decreased by increasing the nanoparticles
28.	Sheikholeslami, ⁷²	Irregular	Porous Media	Water Copper oxide	CVFEM	Re = 100–500 Da = 0.01–100 Ha = 0–40	Nusselt number and Hartmann were negatively related
29.	Barnoon et al., ⁷³	Square	Newtonian Laminar Two phase	Water Alumina	FVM	Ri = 1–100 Ha = 0–30	Heat transfer increased because of the existence of the cylinder and its angular velocity
30.	Selimefendigil and Chamkha, ⁷⁴	Square	Newtonian Steady Incompressible	Water Magnesium oxide, Silver	FEM	Ri = 0.01–100 Ha = 0–50 Da = 10 ⁻⁴ –5 × 10 ⁻²	The position of the porous medium had a significant impact on the average Nusselt number
31.	Cimpean et al., ⁷⁵	Trapezoidal	–	Water Copper, Alumina	FDM	Ra = 10 ⁶ Re = 50–500 Da = 10 ⁻⁴ –10 ⁻²	Rate of heat transfer was a function of Darcy number
32.	Batool and Nawaz ⁷⁶	Square	Non-newtonian incompressible	Copper oxide Alumina	FVM	Gr = 0.1 Ha = 20 Re = 200 Ri = 0.01–100	reynolds number and heat transfer rate were inverse related
33.	Yan et al., ⁷⁷	Rectangular	Steady, Laminar, Incompressible	Water Copper Titanium oxide	FVM SIMPLER	Ri = 0.01–100	Narrow cavity with small Richardson's number and tall cavity with high Richardson's number had highest heat transfer rate.
34.	Alsabery et al., ⁷⁸	Wavy	Two phase	Water Copper Alumina	FEM	Ri = 0.01–10 Re = 100	The optimization of heat transfer was achieved by placing a conducting block
35.	Astanina et al., ⁸⁵	Square	Porous media	Water Alumina	FDM	Ri = 0.01–10 Da = 10 ⁻⁷ –10 ⁻³	Volume of nanoparticles positively related to mixed and forced convections
36.	Gibanov et al., ⁸⁶	Square	–	Water Alumina	FDM	Ri = 0.01–10 Re = 100	Thickness of the bottom wall played a significant role
37.	Oztop et al., ⁸⁷	Wavy	Steady, Laminar, Incompressible	Water Copper oxide	FDM	–	Richardson number and hartmann were directly related to rate of heat transfer
38.	Ismael et al., ⁸⁸	Square	Steady, Laminar	Water Copper oxide	FDM	Ri = 0.01–100 Re = 100–300	Role of nanoparticles on the rate of heat transfer were insignificant at low and midway values of Richardson numbers

Table I. Continued.

S. No	Authors	Geometry	Flowregime	Workingfluid	Numerical methods	Dimensionless numbers	Observations
39.	Khan et al., ⁸⁹	Square	Single phase Steady	Water Copper	FEM	–	The rate of heat transfer could be increased by placing a Y-shaped obstacles
40.	Alsabery et al., ⁹⁰	Wavy	Incompressible Two phase Newtonian	Water Alumina	FEM	Ri = 0.01–100 Re = 1–100 Ha = 0–50	For natural convection at low reynolds number the presence of nanoparticles enhanced energy transport
41.	Hussain et al. ⁹¹	Square	Laminar, Incompressible Newtonian, Unsteady	Water Alumina	FEM	Ri = 0.01–10 Ha = 0–100	Rate of heat transfer was a function of hartmann number
42.	Hussain et al., ⁹²	Square	Incompressible Steady Laminar, Incompressible	Water Copper	FEM	Ri = 0.01–1 Ha = 0–100	The heat transfer diminished with the increase of Hartmann and Richardson numbers
43.	Ali et al. ⁹³	Rectangular	Steady	Water Copper Alumina	FVM	Re = 10–500 Ri = 0.01–10	The inclusion of a big solid body can increase the rate of heat transfer
44.	Jakeer et al., ⁹⁴	Square	Newtonian, Laminar, Incompressible	Water Copper Alumina	FVM SIMPLER	Ri = 0.1–100 Ha = 0–100 Da = 10 ⁻⁶ –10 ⁻²	Hybrid nanofluids gave better heat transfer
45.	Mansour et al. ⁹⁵	Square	Two phase Non-newtonian Incompressible	MicroPolar nanofluid	FVM	Ha = 0–50 Da = 10 ⁻⁵ –10 ⁻²	The cup-mixing temperature could be increased by increasing the field number
46.	Goswami et al., ⁹⁷	Square	Newtonian, Steady Laminar, Incompressible	Water Copper oxide	CFDM	Ri = 0.1–20 Re = 50–750 Ha = 0–60 Gr = 5 × 10 ⁴	Brownian motion gave better heat transfer rate
47.	Taghikhani, ⁹⁸	Wavy	Newtonian, Single phase Incompressible	Water Copper	5PCC- SOC	Re = 0–100 Ha = 0–100 Gr = 10 ⁵	reynolds number with a constant volume of nanoparticles had insignificant role on Nusselt number
48.	Li et al. ⁹⁹	Inclined square	Steady Laminar, Incompressible	Water Alumina	HOC FDM	Ri = 0.01–100 Gr = 100 Pr = 6.2	The increase in the rate of heat transfer was more pronounced due to the inclination of the cavity
49.	Kefayati, ¹⁰⁴	Square	Non-newtonian Steady Laminar, Incompressible	Water Copper	FDLBM	Ri = 0.001–1 Gr = 100 Ha = 0–30	The increase of hartmann numbers improves the impact of nanoparticles on rate of heat transfer
50.	Kefayati, ¹⁰⁵	Square	Non-newtonian Steady Laminar, Incompressible	Water Alumina	FDLBM	Ri = 0.001–1 Gr = 100 Ha = 0–60	The increase of hartmann numbers improves the impact of nanoparticles on rate of heat transfer
51.	Kefayati, ¹⁰⁶	Square	Non-newtonian Steady Laminar, Incompressible	Water Alumina	FDLBM	Ri = 0.001–1 Gr = 100	The impact of nanoparticles on the rate of heat transfer was seen for power-law index was $n = 0.2$
52.	Kefayati, ¹⁰⁷	Square	Non-newtonian Steady Incompressible	Water Alumina	FDLBM	Ri = 0.001–1 Gr = 100	Impact of nanoparticles heat transfer was negatively related to power-law index
53.	Karimipour et al. ¹⁰⁸	Inclined	Laminar	Water Copper	LBM	Re = 10, 100	In comparison to Re = 10, Re = 100 gives better average Nusselt number
54.	Sheikholeslami, ¹⁰⁹	Square	Incompressible	Water Alumina	LBM	Re = 30, 60, 180 Ha = 0–40 Da = 0.001–100	The observation was that the decrease in the convection heat transfer was due to Lorentz force
55.	Sheikholeslami, et al., ¹¹⁰	CUBIC	–	Water Alumina	LBM	Re = 30–180 Ha = 0–60 Da = 0.001–100	The observation was that the decrease in the convection heat transfer was due to Lorentz force
56.	Dahani et al. ¹¹¹	Square	Incompressible	Water Alumina	LBM	Ri = 0.01–100 Re = 1–100 Gr = 100	The mid-point of the heater was most suitable position for heat transfer
57.	Rahmati et al., ¹¹²	Square	Steady Laminar	Water Copper	LBM	Ri = 0.01–100 Gr = 100	The flow pattern was not disturbed by the temperature phase deviation with low Richardson number

Table I. Continued.

S. No	Authors	Geometry	Flowregime	Workingfluid	Numerical methods	Dimensionless numbers	Observations
58.	Nemati et al., ¹¹³	Square	Steady Laminar,	Water Copper Alumina Copper oxide	LBM	Re = 1–100 Ra = 10 ⁴	An important factor for better heat transfer was the choice of nanofluids
59.	Sheikholeslami, ¹¹⁴	POROUS CUBE	Single phase	Water Alumina	LBM	Re = 30–180 Ha = 0–40 Da = 0.001–100	A direct result was observed between temperature gradient and Darcy as well as reynolds numbers
60.	Gümğüm and Tezer-Sezgin, ¹¹⁵	Square	Unsteady	Water Alumina Copper	DRBEM	Re = 10–100	Negative relation between Nusselt number and Richardson number
61.	Nagehan Alsoy-Akgün ¹¹⁸	Square	Unsteady Incompressible	Water Copper	DRBEM	Ra = 10 ³ –10 ⁷	Numerical results were expressed in the form of graph and tables
62.	Serna et al., ¹¹⁹	Square	Newtonian Unsteady Incompressible	Water Copper oxide	NSM	Re = 50 Ri = 11.82	Averaged Nusselt number increases due to the sinusoidal velocity
63.	Faridzadeh, et al. ¹²¹	Inclined square	Laminar	Water Copper	FVM ANN	–	0.9999 was the coefficient of multiple determinants
64.	Aminossadati et al. ¹²³	Square	Laminar	Water Alumina	ANFIS	Ri = 0.01–100 Gr = 10 ⁴	Cavity aspect ratio played a significant role in the process of heat transfer
65.	Maghsoudi and Siavashi ¹²⁴	Square	Newtonian Steady Incompressible	Water Copper	Optimization	Ra = 10 ³ –10 ⁶ Ri = 0.01–100	Increase in heat transfer could be optimized through heterogeneous porous medium
66.	Selimefendigil et al. ¹²⁶	Square	Steady Laminar	Water Copper oxide	FEM ALE	Ri = 0.01–5 Ha = 0–50 Re = 100–1000	In comparison to rigid wall, flexible wall gave a better rate of heat transfer
67.	Selimefendigil and Öztop ¹²⁷	Square	Steady Laminar	Water Copper oxide	FEM ALE	Ra = 10 ³ –10 ⁶ Ri = 0.01–100 Ha = 0–50	As the impact of magnetic field increased, the local as well as average Nusselt number decreased
68.	Hussien et al. ¹³⁰	Microtubes	Laminar	Water MWCNTs/GNP	Experimental	Re = 200–500	Due to hybrid nanofluids, there was an improvement in the rate of heat transfer and pressure drop

Notes: Ra = Rayleigh number; Re = Reynolds number; Ri = Richardson number; Pr = Prandtl number; Gr = Grashof number; Ha = Hartmann number; Da = Darcy number.

work reasonable well but both mesh refinement and increased band-width results in additional arithmetic operations which is computationally not feasible. Therefore, Higher Order Compact (HOC) finite difference schemes are becoming more common because of their high accuracy and compact difference stencils. These schemes operate at grid points located only directly adjacent to the node about which the differences are taken. Moreover, these schemes are called higher order compact methods if their order of accuracy is greater than two. There are various ways through which these schemes can achieve higher-order compactness.⁹⁶ Goswami et al.,⁹⁷ undertook a study of mixed convection within a partially heated 2-dimension square lid driven cavity filled with copper oxide water combination as nanofluid with the top moving wall in adiabatic condition while the irregular bottom wall was placed at a cool temperature. Two heating surfaces were placed on the side walls. The governing equations were solved using an adopted compact finite difference scheme. They observed that heat transfer rate was a function of nanofluids, Hartmann number, Richardson number and waviness

of the bottom surface. Taghikhani,⁹⁸ analyzed MHD mixed convection of Cu-water nanofluid inside a lid driven cavity using two sinusoidal heat sources. The left side of cavity was heated by these sinusoidal heaters with other sides at constant temperature. The cavity's top wall was moving in the +x direction. The governing equations were solved by compact finite difference approximation (five-point constant coefficient second order compact (5PCC-SOC) scheme). The observation was that with increase in Hartmann's number, there was decrease in the heat transfer rate, thereby increasing the heat transfer by conduction resulting in the decrease of average Nusselt number. Li et al.,⁹⁹ used a fully higher order compact (HOC) finite difference scheme to study the effect of mixed convection flow inside an Inclined square lid driven cavity filled with Al₂O₃ + water combination. Considering two cases depending on the direction of the temperature gradient, they concluded that presence of nanoparticles did enhance the heat transfer rate. Wang et al.,¹⁰⁰ studied the effect of mixed convection of copper-water nanofluid inside a lid driven cavity on the heat transfer rate. For this they

also considered two mutually orthogonal heated thin plates inside the cavity. The governing equations were solved by fourth-order accurate compact finite difference scheme, thereby concluding that there was substantial increase in the heat transfer rate.

2.3. Lattice Boltzmann Methods

LBM originating from the lattice gas automata model is a comparatively new technique for solving complex fluid systems. It is an alternative method to the macroscopic based conventional fluid dynamics system and can simulate a variety of transport phenomena and fluid dynamics problems; depending on the kinetic approach at mesoscopic level that conserves the physical laws.^{101–103} Kefayati,¹⁰⁴ analyzed mixed convection of Non-newtonian nanofluid in a square lid driven cavity in the presence of magnetic field. The cavity was filled with the combination of water and copper. The left cavity wall was at a higher temperature than the right wall while both the moving top and stationary bottom walls were adiabatic as well as impermeable. The governing equations were solved by finite difference Lattice Boltzmann method. The conclusion was that heat transfer rate decreased with an increase in Richardson number but increased with addition of nanoparticles. Kefayati,¹⁰⁵ analyzed mixed convection of Non-newtonian nanofluid in a square two sided lid driven cavity in the presence of magnetic field. The cavity was filled with the combination of water and alumina. The left cavity wall was at a higher temperature than the right wall while both the moving horizontal walls were adiabatic as well as impermeable. The governing equations were solved by finite difference Lattice Boltzmann method. The conclusion was that heat transfer rate decreased with an increase in Richardson number but increased with addition of nanoparticles. Kefayati,¹⁰⁶ analyzed mixed convection of Non-newtonian nanofluid in a square two sided lid driven cavity. The cavity was filled with the combination of water and alumina. The left cavity wall was at a higher temperature than the right wall while the moving horizontal walls were adiabatic as well as impermeable. The governing equations were solved by finite difference Lattice Boltzmann method. The conclusion was that heat transfer rate decreased with an increase in Richardson number but increased with addition of nanoparticles. It was also observed that the greatest impact of nanoparticles was for power-law index $n = 0.2$.

Kefayati,¹⁰⁷ undertook a study of laminar mixed convection of Non-newtonian nanofluid in a 2-dimensional square lid driven cavity. The cavity was filled water and alumina nanoparticles. The cavity under consideration had its left wall at a higher temperature than the right wall which had as sinusoidal temperature. The top moving horizontal wall as well as the stationary bottom wall was adiabatic and impermeable. The governing equations were solved by finite difference Lattice Boltzmann method. The result

was that heat transfer rate increased along with the addition of nanoparticles but on the other side the influence of nanoparticles decreased with the increase in Richardson number and power-law index. Karimipour et al.¹⁰⁸ studied the effect of mixed convection inside an inclined lid driven cavity filled with copper-water nanofluid. The geometry of the cavity was that upper wall which was at a higher temperature than the bottom wall moved with a constant velocity while the side walls were adiabatic. The numerical results were obtained using Lattice Boltzmann method. They observed that at higher Richardson number and greater angle of inclination of the cavity along with volume fraction of nanofluids, there was an abrupt increase in the Nusselt number. A porous lid driven cavity filled with aluminum oxide was used by Sheikholeslami,¹⁰⁹ to examine the MHD forced convection. Considering the top cool surface to be stationary and the bottom hot surface to be moving, the researcher concluded by using Mesoscopic method that Lorentz force was the cause for the decreasing of heat transfer rate while the temperature gradient over the hot surface increased along with the increase of Reynolds number, Darcy number and the volume fraction of nanoparticles. For analyzing MHD forced convection around an elliptic obstacle Sheikholeslami, et al.,¹¹⁰ considered a permeable 3-d lid driven cavity filled with Al_2O_3 + water based nanofluid with the bottom wall moving. Mesoscopic simulation was used for this study. Using Lattice Boltzmann method, they observed that the average Nusselt number increases with the increase in Darcy as well as Reynolds number. Dahani et al.,¹¹¹ analyzed fluid flow and heat transfer characteristics for forced convection in a square lid driven cavity using Ag-water combination as nanofluid. A thin heater placed inside the cavity used for forced convection. The side walls were insulated with the vertical walls were at constant temperature but lower than the temperature of the heater. Using Lattice Boltzmann method, they observed that due to the presence of nanofluids there was an increase in the heat transfer rate while the average Nusselt number decreased with the increase of Richardson number.

Rahmati et al.,¹¹² undertook a numerical study of mixed convection in a double lid driven cavity filled with Cu-water nanofluid. They considered a geometrical setup in which the insulated horizontal walls were moving while the stationary side walls had sinusoidal temperature distribution. The governing equations were solved by Lattice Boltzmann method from whereby they concluded that the Nusselt number could be enhanced by lowering the Richardson number. Nemati et al.,¹¹³ undertook a numerical investigation of mixed convection using nanofluid. For this they considered a geometrical setup consisting of a square lid driven cavity filled with Cu, CuO and Al_2O_3 and with moving top and stationary bottom walls as adiabatic while the left side wall was hotter than the right side wall. Applying Lattice Boltzmann method, they observed that

the effect of nanoparticles on heat transfer was greater for Al_2O_3 than for CuO and Cu. They also observed that by increasing the Reynolds number there was a decrease in the effect of nanoparticles. Sheikholeslami,¹¹⁴ studied the impact of magnetic field on nanofluid inside a porous lid driven cavity. The cavity was filled with water and Al_2O_3 nanoparticles and a hot sphere was also placed inside the cavity. Using Lattice Boltzmann method, they concluded that (i) Nusselt number decreased with increase of Lorentz forces (ii) a direct relationship could be seen between temperature gradient, Darcy number and Reynolds number.

2.4. Dual Reciprocity Boundary Element Method (DRBEM)

Gümgüm and Tezer-Sezgin,¹¹⁵ analyzed mixed convection of nanofluids using different parameters in a square lid driven cavity. In order to understand the flow and heat transfer characteristics they considered two different setups. In the first setup they considered a square cavity with both vertical and top horizontal walls as fixed and cool while the bottom horizontal wall as moving and heated. This cavity was filled with Al_2O_3 -water combined nanofluid. However, the second setup was different from the first in the sense that it had Cu-water combined nanofluid and instead of the bottom wall, the top wall was moving. The side and the bottom walls were fixed. In addition, this the bottom wall had a heater covering a part of it. Using Dual Reciprocity Boundary Element Method (DRBEM) in solving the governing equations, they concluded that Nusselt number is directly proportional to volume fraction of nanoparticles and inversely proportional to Richardson number and length of the heat source. Gümgüm,¹¹⁶ studied the impact of slip boundary conditions on mixed convection inside a lid driven cavity using water-alumina based nanofluids. The moving bottom wall was heated while the other walls were stationary and cooled. The governing equations were solved using Dual Reciprocity Boundary Element Method (DRBEM). The result of the observation was that with the rise of Knudsen number there was a decrease of temperature gradient and average Nusselt number while Nusselt number increased with the rise of nanoparticles. Alsoy-Akgün¹¹⁷ numerically analyzed Cu-water based nanofluids under double diffusive mixed convection in a lid driven cavity. Numerical results were obtained by the method of Dual Reciprocity Boundary Element Method (DRBEM). The conclusions were (i) increase of nanoparticles had a positive impact on Nusselt number (ii) for higher Reynolds number, a fairly high heat transfer rate could be achieved by increasing the volume fraction of the nanoparticles. Using Dual Reciprocity Boundary Element Method to solve the governing equations Alsoy-Akgün¹¹⁸ numerically studied the performance of unsteady mixed convection of Cu-water combined nanofluid in a partly heated square lid driven cavity. A heat source was placed on the bottom wall while the

side and upper moving walls were at constant cold temperatures. The results of the problem were presented in the form of graphs and tables which were in agreement with previous results.

2.5. Network Simulation Method

Serna et al.,¹¹⁹ studied unsteady, viscous flow in a heated lid driven cavity filled with nanofluids. Using Network Simulation method, they inferred that the time averaged Nusselt number increases due to the sinusoidal velocity waves at the lid by 16%.

2.6. An Artificial Neural Network Analysis

Artificial Neural Network is a type of computational intelligence resulting from the biological nervous systems.¹²⁰ An Inclined square ventilated lid driven cavity filled with Cu-water based nanofluid was considered by Faridzadeh, et al.¹²¹ in order to study laminar mixed convection. Using Artificial Neural Network (ANN) to obtain a relation between the independent parameters, they concluded that the coefficient of multiple determinations between the real values and ANN was 0.9999, the maximum error was less than 0.5829 whereas the mean square error was 0.0000537. A parametric study of mixed convection in a lid driven right triangle filled aluminum oxide-water based nanofluid was undertaken by Triveni and Panua.¹²² For this study they considered the base of the triangle to be hot and caterpillar shaped while the two sides to be cold. Furthermore, in order to understand the effect of mixed fraction they considered different volume fraction of nanoparticles as well as the direction of the movement of the lid. Finite volume and SIMPLE algorithm was used to discretize the governing equations. Their result indicated that there was a positive relation between heat transfer rate, volume fraction, Richardson number and Grashof number. Artificial Neural Network was used to validate the obtained numerical results.

2.7. Adaptive Network Based Fuzzy Inference System Analysis

Aminossadati et al.,¹²³ studied mixed convection of nanofluid in a double lid driven cavity using Adaptive network-based fuzzy inference system. For this, they considered a setup consisting of a square cavity filled with water-alumina based nanofluids. Both the horizontal walls of which the top wall which was at higher temperature than the bottom wall were moving either in the same direction or in the opposite direction while the vertical walls were thermally insulated. They observed that Adaptive network-based fuzzy inference system method had comparatively reduced the computational time and effectively calculated the fluid velocity and its temperature. They further observed that an enhanced rate of heat transfer could be achieved with higher volume fraction of

nanoparticles and low Richardson number and was dependent on the aspect ratio and in the direction of the lid movement.

2.8. Optimization

Maghsoudi and Siavashi¹²⁴ undertook mixed convection study of nanofluid in a double sided lid driven cavity. The horizontal walls of the cavity were adiabatic as well as moving while the side walls were at different temperatures. The cavity was filled with heterogeneous porous media. Using Optimization technique, they observed that for optimized heterogeneous porous medium the rate of heat transfer increased in the case when the flow was dominated by convection while for natural convection flow the optimal porous medium was homogeneous.

2.9. FSI Analysis

We come across many physical phenomena where a fluid flow encounters a solid object thereby leading to the deformation of the object. Depending on the pressure and velocity of the fluid and material properties of the object, these deformations can be quite large or very small. If the deformation is quite large then the pressure and velocity of the fluid will change as a result of the interaction. These type of phenomena are multiphysics in nature and so in order to understand these phenomena we need to study both fluid dynamics as well as structural mechanics. The combination of these two subjects is called FSI or Fluid-Structure Interaction.¹²⁵ A square lid driven cavity with flexible side wall and filled with nanofluid was considered by Selimefendigil et al.¹²⁶ in order to study MHD flow. The bottom wall of the cavity was hotter than the top horizontal moving wall whereas the vertical walls were insulated. The governing equations were solved through the finite element method and fluid flow was defined with the help of The Arbitrary-Lagrangian-Euler method with the flexible wall in fluid-structure interaction model. They observed that averaged heat transfer rate decreases with increase of Hartmann number and decreasing Richardson number while it increases when Young's modulus of the flexible wall decreases. Selimefendigil and Öztöp¹²⁷ undertook a study of MHD mixed convection in a lid driven cavity filled with CuO nanofluid in which the fluid flow was described by The Arbitrary Lagrangian-Euler method with the elastic wall in fluid-structure interaction model. The cavity had the cold left vertical wall moving in the +y direction whereas the right vertical wall was at hot temperature and other walls of the cavity were insulated. The governing equation for this study was solved by finite element method. The result revealed that average Nusselt decreased with Young's modulus for Richardson number 0.01 but increases for 1 and 100. The influence of volume fraction of nanoparticles could be seen for higher values of Richardson number.

3. CONCLUSION

This paper presents a comprehensive review on the research progress made on understanding the heat transfer characteristics and fluid flow characteristics using a lid driven cavity filled with different nanofluids. The objective of this review work is to present the different Numerical Methods used by different researchers in understanding the flow phenomena within a lid driven cavity. Much research can be done by using methods like Meshfree, Fluid-Structure Interaction model.

References and Notes

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