Special Topics & Reviews in Porous Media - An International Journal 14(3):63–72 (2023)

SCALING BEHAVIOR OF STOCHASTIC FLUID FLOW IN POROUS MEDIA: LANGEVIN DYNAMICS

Yassine Hariti,[1,*] Ahmed Hader,[1,2] Layla Amallah,[1] Ali J. Chamkha,[3] Ahmed El hana,[1] Mohammed Tanasehte,[1] & Yahia Boughaleb[1,4]

[1]*LBGIM, Ecole Normale Supérieure, University Hassan II, Casablanca, Morocco*
[2]*Regional Center for Education and Training, Casablanca - Settat, Morocco*
[3]*Faculty of Engineering, Kuwait College of Science and Technology, Doha District, Kuwait*
[4]*LPMC, Faculty of Sciences, University Chouaib Doukkali, El Jadida, Morocco*

*Address all correspondence to: Yassine Hariti, LBGIM, Ecole Normale Supérieure, University Hassan II, Casablanca, Morocco; Tel.: +21268048251, E-mail: yassinehariti@gmail.com

Original Manuscript Submitted: 1/10/2022; Final Draft Received: 5/18/2022

Transport phenomena of fluids in porous media occur in a variety of mediums with different properties. These phenomena are governed by a behavior of scaling law as a function of the different universal components. Hence, we study numerically the fluid transport phenomena in a porous medium under the effect of a static pressing force. Our numerical investigation is developed using the Langevin dynamics based on the competition between the stochastic and the dissipation processes. We study both average flow distance and average flow velocity. The results show that the time evolution of these two magnitudes exhibits exponential profiles with two different regimes, and they evince a decreasing behavior versus fluid viscosity, but an increasing behavior with both static pressure and medium porosity. Scaling law of the mean flow velocity is checked for different control parameters: static pressure, friction coefficient, and medium porosity. We have concluded that the exponent values $\beta \approx 0.5 \pm 0.01$ and $\alpha \approx 1 \pm 0.01$ are independent of these control parameters, which proves their universal character and their consistency with other experimental outcomes.

KEY WORDS: *porous medium, viscosity, porosity, static pressure, scaling law, characteristic time, Langevin dynamics*

1. INTRODUCTION

The flow of fluids in porous media exists in a wide variety of fields, for example geothermal energy (McCartney et al., 1992), hydraulics (Escobar et al., 2011; Pia et al., 2014), and chemical engineering used to filtrate and purify in chemical process, agricultural engineering to investigate the water flow in underground resources, and petroleum applications to study the motion of oil and natural gas (Cai et al., 2012; Xu et al., 2017). Transport phenomena include three primordial axes: fluid dynamics (Xiong et al., 2017) and heat and mass transfer flow in porous media (Mahmoud et al., 2010; Ghalambaz et al., 2020). These phenomena have been investigated with different methods in order to obtain a complete description of the transport procession, to understand all the laws governing them, and to determinate transport parameters such as permeability, porosity (Sheikholeslami et al., 2020; Xu et al., 2016), thermal conductivity (Huai et al., 2007; Ghalambaz et al., 2019), and mechanical proprieties (Gou and Schwartz, 2013).

Other studies have been used to discuss the influence of radiation conduction-adsorption and porosity of porous media on heat and mass transfer of nanofluids through a saturated porous medium (VeeraKrishna et al., 2021). In order to improve the properities of thermal conductivity, the rate of heat and mass transfer must be augmented by employing the Newtonian fluids in porours media (Verma et al., 2021). Many researchers have investigated heat and mass transfer on the unsteady magnetohydrodynamic (MHD) flow (VeeraKrishna and Chamkha., 2018), and on an MHD free convective flow in porous media with vertical, flat, and infinite plate under the influence of heat and chemical reaction

NOMENCLATURE

MHD	magnetohydrodynamic	v	velocity of fluid flow
τ	cross-over	v_{sat}	saturated mean flow velocity
ς	strain rate	T	temperature
μ	fluid viscosity	t	dimensionless time
γ	coefficient of friction	N	number of pores
S	porosity	L	system size
P	static pressure	δ	Diarc function
m_i	fluid mass	$g(t)$	scaling function
i	horizontal index	d	distance
j	vertical index	α	roughness exponents
R_i	random force	β	growth exponents
K_B	Boltzmann's constant		

(VeeraKrishna et al., 2018c). In addition, they discussed the unsteady MHD convective rotating flow of nanofluids and the MHD convective flow of elastico-viscous flow fluid through porous media under the effects of Hall and ion slip, as proved by VeeraKrishna and Chamkha (2020a,b). Various analytical and experimental investigations and computer simulations have often been used to study these physical parameters on transport phenomena in porous media. The analytical approaches are a complicated way to inquiry these phenomena. Hence, different models are used to determine the permeability and porosity in transport phenomena of porous media (Cao et al., 2016; Hariti et al., 2020) and to confront numerical results with experimental data (Hen et al., 2000; Yang et al., 2015). The capillary beam model has been used to predict the permeability of porous media based on the experimental parameters averting the difficult numerical calculations (Gou et al., 2013; Galindo et al., 2012), to predict the evolution of saturation profiles, and to predict the hydraulic conductivity of unsaturated porous media (Lebeau and Konrad, 2010).

The capillary bundle model using a developed interaction has been used to analyzed the immiscible displacement processes in porous media (Dong et al., 2005). Also, we found the complex model proposed by Tan et al. (2017), in which the porous media are considered as a set of tortuous capillaries with random diameters distributed in a solid frame randomly (Hariti et al., 2021). By using this model, they investigated the relationship between permeability and porosity versus the tortuosity of the capillaries, and they determined the deviation of their distribution throughout the solid frame, which gives an idea about the mean diameter of the capillaries (Tan et al., 2017). Then, they checked the scaling behavior of permeability and porosity with different parameters. The scaling laws for permeability-porosity relation present a power law with two universal exponents validated to previous results by other investigations (Hader et al., 2021).

In this work, we study the transport phenomena in porous media by adopting new dynamics based on the Langevin dynamics, as a simple reformulation of Newton's second law, that takes into account the stochastic behavior of the studied system. It appears in many theories of physics, such as fluctuation-dissipation (Cui et al., 2018), the diffusion process (Li et al., 2020; Kwok, 2020), quantum theorems (Hollowood et al., 2017; Breuer and Petruccione, 2003). The Langevin dynamics have been used to study diverse scientific phenomena, for example the active matter of biological systems (Bakir et al., 2017; Amallah et al., 2020a) and dynamics of Brownian motion, which considers a particle immersed in a fluid (Petrosyan and Zaccone, 2021). In order to understand the random motion of molecules in liquids, on which they undergo collisions and interactions with other surrounding molecules (Hansen and McDonald, 2006) developed a modern, molecular theory of the structural, thermodynamic interfacial and dynamical properties of the liquid phase of materials constituted of atoms, small molecules or ions. Einstein and Smoluchowski are the first researchers to give an analytical explication of Brownian motion in 1905 and 1906. In their models, they did not take into account the inertia of the Brownian motion of molecules. Then, Langevin, developed and described the Brownian motion by taking into account the effects of inertia (Czirok et al., 1999). However, our numerical approach allows us to investigate the motion of fluids in this medium, and it gives readily available solutions, thus avoiding the need to solve difficult differential equations with the analytical approach. The fluid flow is initiated by a static pressure force and is controlled by several parameters, chiefly fluid viscosity, medium porosity, and static pressure. The first part focuses on the mean flow velocity, checks its scaling law, and verifies its universality by calculating the growth and roughness exponent values corresponding to its temporal evolution. The second part studies the mean flow distance.

This paper includes four sections. In Section 2, we discuss the used model based on the Langevin dynamics, where we discuss the mathematical formalism corresponding to the temporal evolution of the average velocity of the fluid flow. Section 3 represents and discusses our numerical results. In the end, we conclude and summarize our investigations.

2. THE USED MODEL

Recently, scientists and engineers have widely studied transport phenomena of fluids in porous media experimentally and analytically. In our investigation, we use a numerical method based on the Langevin dynamics, which combines Brownian motion (Mazroui and Boughaleb, 1996) to the Newtonian dynamics of a particle inserted in a base fluid. This particle is under the action of a complicated and erratic movement. However, the Langevin dynamics is a simple reformulation of Newton's second law, which takes into consideration the inertia of the particles and a random force, which represents the stochastic process of the flow of fluids in a porous medium. The flow is considered laminar, and the fluid is considered Newtonian intruded in a set of capillaries of infinite length with random diameters and distributed on a square lattice. The probability of the fluid taking a certain pore is based on cyclic boundary conditions. The fluid exhibits a direct proportionality between stress and strain rate in laminar flow, and it is given by $\tau = \mu.\varsigma$. In order to study the fluid flow in porous media, we consider N pores of random diameters confined on a square-shaped surface of size $L \times L$, with periodic boundary conditions, as shown in Fig. 1. We take three parameters to dissect the reality of fluid flows in porous media, which are the friction coefficient, the medium porosity, and the static pressure. The intrusion of the fluid in the porous medium is done under the effect of a static pressure P, as described by the Langevin dynamics (Hariti et al., 2019):

$$m_i \frac{d\overrightarrow{v_i(t)}}{dt} = -m_i \gamma \overrightarrow{v_i(t)} + \overrightarrow{R_i(t)} + P\overrightarrow{S_i}. \tag{1}$$

The random force $R_i(t)$ is called force of Langevin, while $PS_i\vec{n}$ is a pressing force resulting from the static pressure P. The stochastic force and the friction force are given by the second fluctuation-dissipation theorem, which is obtained by calculating the variance of the speed in thermal equilibrium as the following equation:

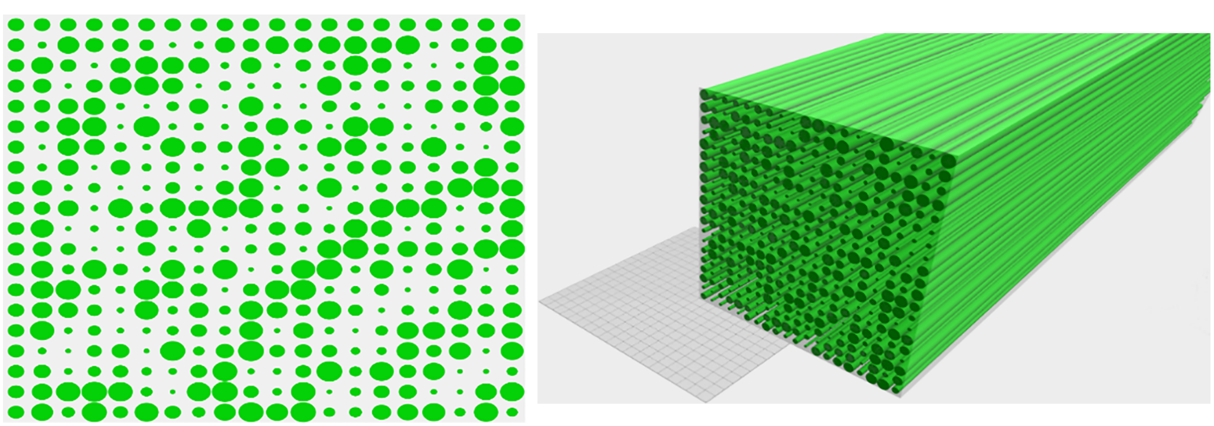

FIG. 1: System image on the right and a cross-section of it on the left (the black cylinders represent the random diameter pores)

$$\prec R_i(t) \succ = 0 \text{ and } \prec R_i(t)R_i(t') \succ = 2m_i\gamma k_B T\delta(t-t') \tag{2}$$

where $\prec R_i(t) \succ = 0$ is the mean value of $R_i(t)$ and $2m_i\gamma k_B T$ is the variance of noise determined by $\prec v^2 \succ = \frac{k_B T}{m}$. In our numerical calculations, we distribute N pores of random diameters and infinite lengths on a square lattice of $L \times L$, and we focus on the mean flow velocity and the mean flow distance for the different parameters: static pressure, friction coefficient, and porosity—changing one parameter at a time. The probability that the fluid takes a certain pore at a certain moment is based on its traveled distance in the previous moment in that pore and its four contiguous pores, under boundary conditions, as shown:

$$p_{i,j}(t) = \frac{d_{i,j}(t-1)}{d_{i-1,j}(t-1) + d_{i+1,j}(t-1) + d_{i,j}(t-1) + d_{i,j-1}(t-1) + d_{i,j+1}(t-1)} \tag{3}$$

where i and j are the horizontal and the vertical indexes of the pore on the lattice, respectively. The contiguous pores for the ones on the boundaries are obtained following the periodic boundary conditions, such that the pores at indexes –1 and L are considered to be the same, be it on the horizontal or the vertical axes. The list of symbols used in our numerical calculations is below in Table 1.

3. RESULTS AND DISCUSSION

In our calculation, we consider N pores with random diameter sections S, distributed randomly on a square lattice of size $L \times L$, where the fluid is injected with constant pressure P. The temporal evolution of the mean flow velocity is modeled by the above formalism based on the Langevin dynamics. Therefore, the kinetic and dynamical properties of the injected fluid transport phenomena in the considered porous lattice are characterized by average flow distance and average flow velocity versus pressure P, medium porosity S, friction parameter γ, and size of the system L. We notice that the mean flow velocity of the system evolves exponentially over time in two regimes—transient and permanent—separated by a crossover τ. The results obtained for the different control parameters are represented in Fig. 2. The corresponding profile is fitted by the growth of the exponential function, given by the following equation:

$$v(t) = v_{sat}(1 - e^{-t/\tau}) \tag{4}$$

TABLE 1: Table of numerical simulation parameters

N	Number of pores
τ	Stress
ς	Strain rate
μ	Fluid viscosity
γ	Coefficient of friction
P	Static pressure
m_i	Fluid mass
R_i	Random force
K_B	Boltzmann's constant
S	Medium porosity
δ	Diarc function

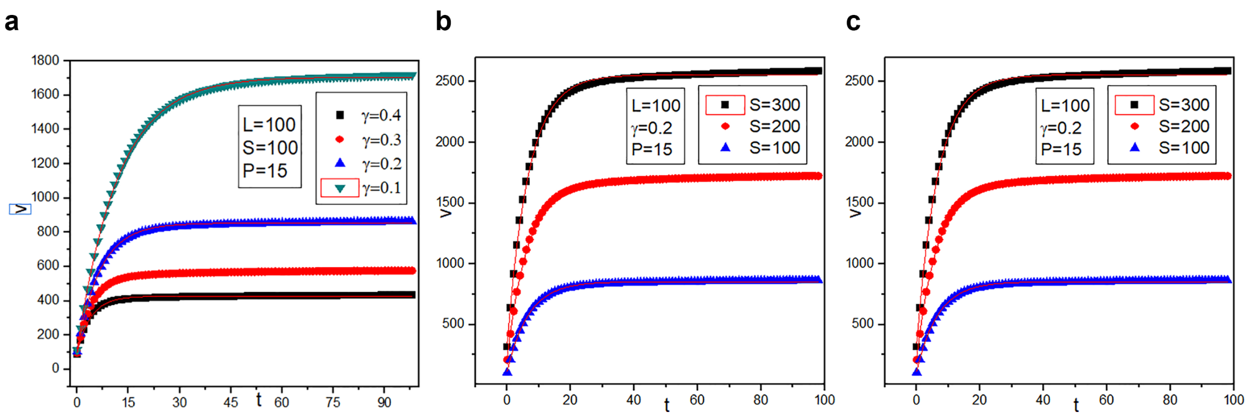

FIG. 2: The time evolution of the mean flow velocity for the different parameters: (a) friction coefficient, (b) porosity, and (c) the static pressure

In addition, the fluid mean flow velocity evolves exponentially with two regimes (transient and permanent) separated by a crossover τ. In the transient regime ($t << \tau$) the fluid mean flow velocity increases with time as power law t^β. In the permanent regime ($t >> \tau$), the saturated mean flow velocity v_{sat} is proportional to the parameter χ as $v_{sat} \approx \chi^\alpha$. Where χ is one of the controlled parameters, such as pressure *P*, medium porosity *S*, inverse of friction parameter $1/\gamma$, and size of the system *L*. These results are more consistent with our previous work (Hariti et al., 2019). Hence, the fluid mean flow velocity scales with time *t* and noise value χ, according to this equation (Amallah et al., 2021):

$$v(\chi,t) = \chi^\alpha . g\left(t \big/ \chi^{\alpha/\beta} \right), \tag{5}$$

where $g(t)$ is a scaling function defined by:

$$g(t) = \begin{cases} t^\beta \; if \; t \ll \tau \\ \chi^\alpha \; if \; t \gg \tau \end{cases} \tag{6}$$

The two exponents α and β are similar to the roughness and growth exponents used in the study of the collective behavior of self-propelled complex systems (Amallah et al., 2021). Hence, the first exponent α is the roughness exponent, which describes the variation of the mean velocity with the system size. The second exponent β is the growth exponent, which means the temporal evolution of the mean flow velocity in the transition phase (Hariti et al., 2019; Amallah et al., 2021). To study the effect of the static pressure, friction coefficient, and medium porosity on the fluid intrusion process in porous media, and to characterize the scaling law, we have determined the different values of roughness and growth exponents. The scaling law is defined in Eq. (5), and the calculated exponent values are regrouped in Table 2. The obtained results are plotted in Figs. 3 and 4. Our results are consistent with the scaling law defined in Eq. (6).

The viscosity has a profound effect on the mean flow velocity, i.e., the bigger the viscosity, the lower the mean flow velocity. Thus, the viscosity hinders the flow of the fluid in the medium. Based on these results, it is clear that the effect of porosity and static pressure have the same effect on the system's mean flow velocity in the two regimes (the same exponent values). These results are qualitatively similar to the results found by Seiwert et al. (2017). They measured the horizontal and vertical velocity field in a vertical foam film supported on a rectangular solid frame in a draining film, thus providing the first quantitative experimental evidence of this flow pattern (Seiwert et al., 2017). Scaling law is determined for all characteristic velocities coupling gravitational effects and capillary

TABLE 2: The exponent values corresponding to the fluid mean flow velocity

	The exponent β	**The exponent α**
By varying the friction coefficient γ	0.64 ± 0.01	0.99 ± 0.01
By varying the porosity parameter *S*	0.54 ± 0.01	0.99 ± 0.01
By varying the pressure *P*	0.54 ± 0.01	0.99 ± 0.01

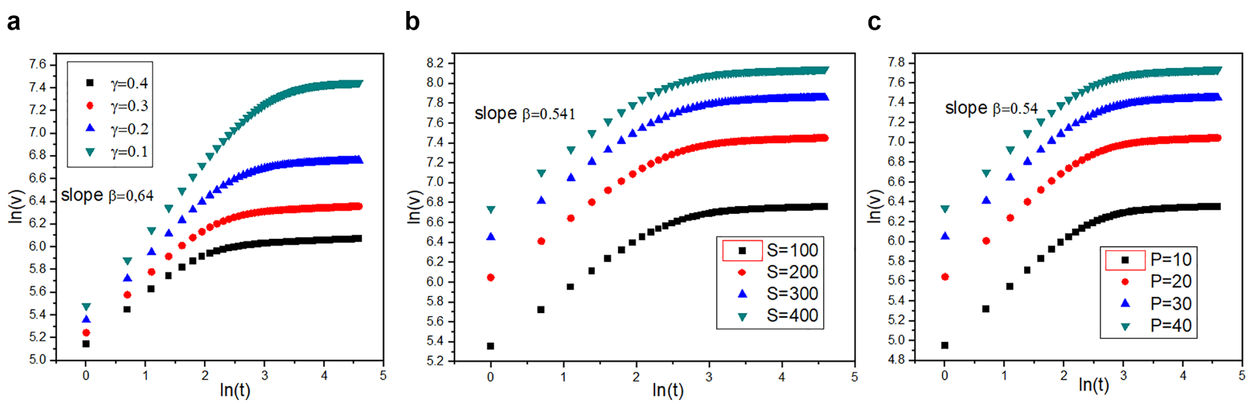

FIG. 3: The log-log plot of the fluid mean flow velocity versus time for the different parameters: (a) friction coefficient, (b) porosity, and (c) the static pressure

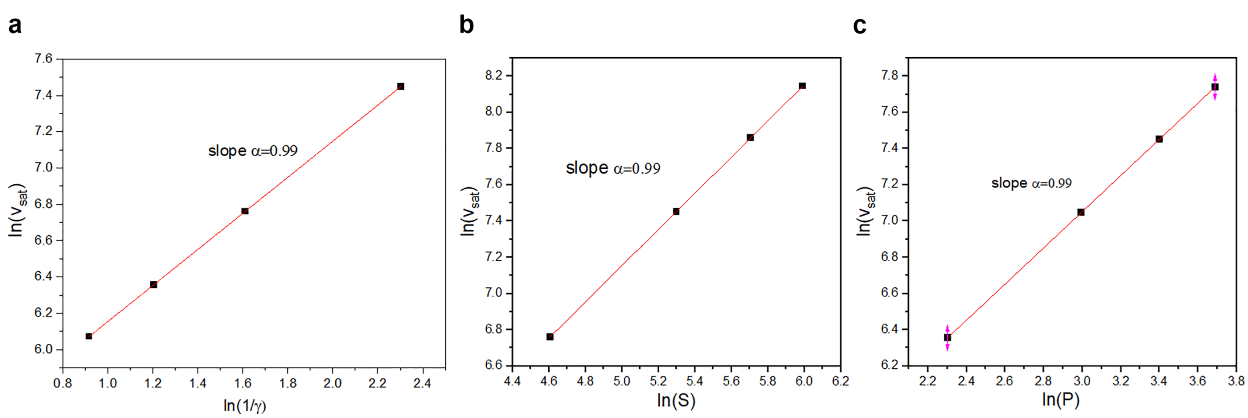

FIG. 4: The log-log plot of the saturated value of the fluid mean flow velocity versus time for the different parameters: (a) friction coefficient, (b) porosity, and (c) the static pressure

sections. We have obtained that the exponent values $\beta \approx 0.5$ and $\alpha \approx 1$ are independent from the control parameters such as the static pressure, viscosity, and medium porosity, which proves their universal character and their consistency with other experimental results (Horváth et al., 1991a). We have calculated the ratio v/v_{sat}, which represents the fluid mean flow velocity normalized by its saturated value versus the normalized time t/τ. The corresponding results for the different controlled parameters are plotted in Fig. 5. Hence, the normalized fluid mean flow velocity profiles collapse on a unique curve, which proves the scaling law, which is similar to the scaling behaviors of interface growth (Family and Vicsek, 1985; Horváth et al., 1991b) and of the collective motion in biomaterials systems (Amallah et al., 2020b). In addition, we have investigated the effect of the different control parameters on

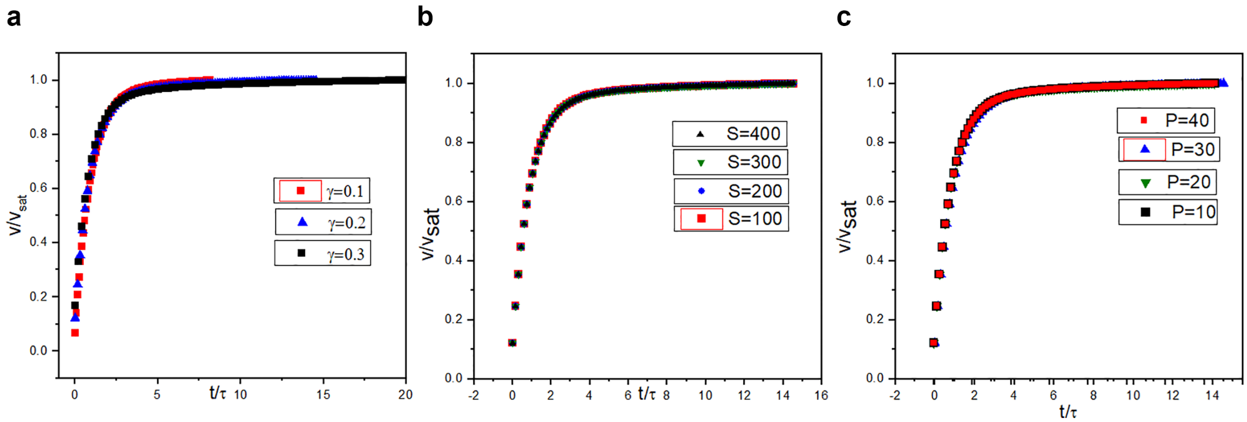

FIG. 5: The time evolution of the fluid mean flow velocity normalized by its saturated value for the different controlled parameters: (a) friction coefficient, (b) porosity, and (c) the static pressure

cross-over τ of the mean flow velocity (see Fig. 6), where the cross over τ decreases exponentially with the fluid viscosity, but it increases exponentially with both porosity S and static pressure P. These results are consistent with experimental findings (Seiwert et al., 2017) and show that the flow of a fluid in a porous medium evolves from a disordered state to an ordered one.

The fluid flow in the capillaries is constrained by its viscosity and friction with the capillaries walls. At any given moment in time, the traversed distance depends on the flow velocity at that moment and the flow distance in the previous one. Hence, the mean flow distance is given by:

$$\prec d(i,j,t) \succ = \prec d(i,j,t-1) \succ + \prec v(i,j,t) \succ .\Delta t. \tag{7}$$

Figure 7a–7c show, respectively, the time evolution of the mean flow distance for the different controlled parameters: viscosity, porosity, and static pressure. The mean flow distance decreases exponentially with the fluid viscosity γ, but it increases exponentially with both porosity S and static pressure P. The same behavior was also observed for the fluid mean flow velocity v.

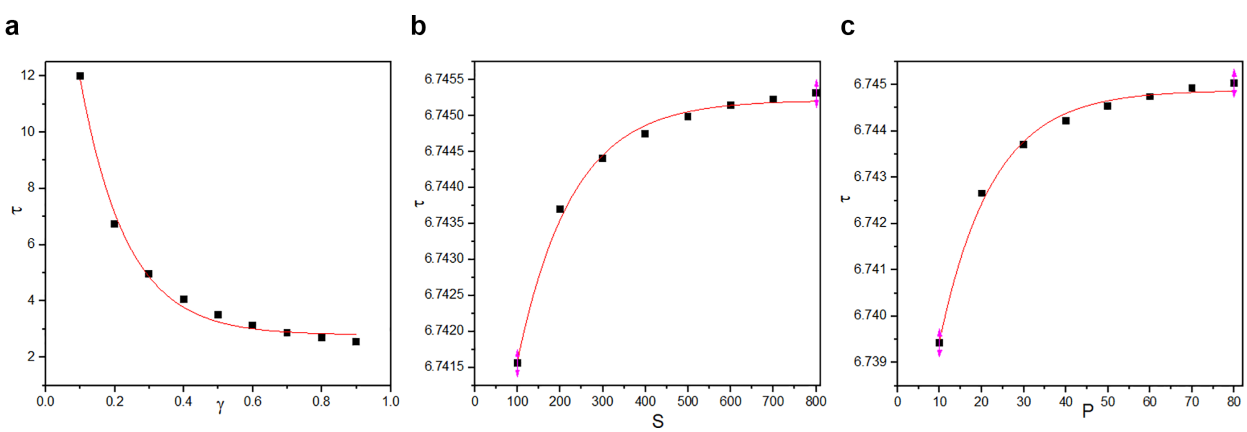

FIG. 6: The evolution of the characteristic time τ corresponding to the fluid means flow velocity versus different parameters: (a) friction coefficient, (b) porosity, and (c) the static pressure

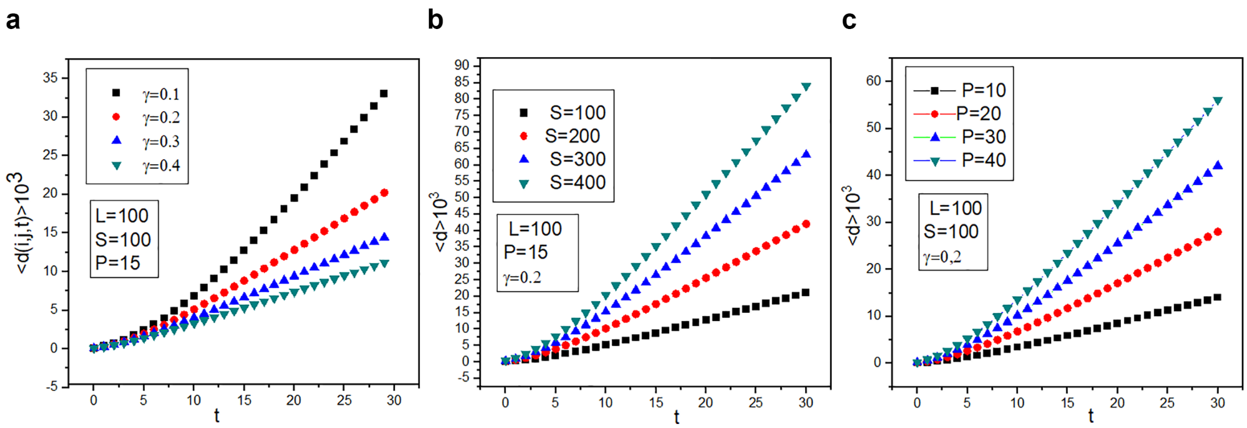

FIG. 7: The time evolution of the mean flow distance for the different parameters: (a) friction coefficient for fixed values of (S = 100, L = 100, P = 15), (b) porosity for fixed values of (γ = 0.2, L = 100, P = 15), and (c) the static pressure for fixed values of (S = 100, L = 100, γ = 0.2)

The obtained interface behaves with the above parameters similar to the results obtained by Stokes, Kushnick, and Robbins of the fluid flow interface (Stokes et al., 1988). They found that the interface is pinned for pressures below a threshold pressure value and moves with a finite velocity.

4. SUMMARY AND CONCLUSION

In this work, we have numerically studied the fluid intrusion process in a porous medium under the effect of a static pressing force by using the Langevin dynamics framework based on stochastic and dissipation processes. The results show that the system exhibits a kinetic phase transition from a non-equilibrium state to an equilibrium state. The mean flow velocity has two different regimes, transient and permanent, with a crossover time decreasing exponentially with the fluid viscosity, and increasing exponentially with static pressure and porosity. We were particularly interested in the scaling behavior of the mean flow velocity and mean flow distance. The mean flow velocity varies with viscosity, porosity, and static pressure as a power law with universal exponents $\beta \approx 0.5 \pm 0.01$ and $\alpha \approx 1 \pm 0.01$. The porosity and static pressure have the same effect on the characteristic time of mean flow velocity. The mean flow distance shows the same profile with respect to viscosity, porosity, and static pressure.

REFERENCES

Amallah, L., Hader, A., Bakir, R., Achik, I., Sbiaai, H., Tarras, I., and Boughaleb, Y., Correlations and Diffusions Process in Self-Propelled Particles Dynamic: Langevin Equation Treatment, *Sensor Lett.*, pp. 688–692, 2018.

Amallah, L., Hader, A., Bakir, R., Tarras, I., and Boughaleb, Y., Langevin Dynamic Simulation of Self-Propelled Particles in Two-Dimensional Systems, *IOP Conf. Ser.: Mater. Sci. Eng.*, vol. **948**, 2020.

Amallah, L., Hader, A., Tanasehte, M., Hariti, Y., and Boughaleb, Y., Dynamical Properties and Scaling Behavior of Self-Propelled Particles: Langevin Dynamics, *Multidisc. Model. Mater. Struct.*, vol. **17**, no. 2, pp. 418–425, 2021.

Amallah, L., Hader, A., Tanasehte, M., Hariti, Y., Sbiaai, H., and Boughaleb, Y., Kinetic and Scaling Behavior of Collective Motion in Biomaterials, *Mater. Today: Proc.*, vol. **30**, pp. 918–922, 2020.

Bakir, R., Tarras, I., Amallah, L., Sbiaai, H., Hader, A., Mazroui, M., and Boughaleb, Y., Effects of Agent's Orientation in 2D Flocking Models, *Chin. J. Phys.*, vol. **55**, pp. 692–697, 2017.

Breuer, H.P. and Petruccione, F., *The Theory of Open Quantum Systems*, Oxford, UK: Oxford University Press, 2003.

Cai, J., Hu, X., Standnes, D.C., and You, L., An Analytical Model for Spontaneous Imbibition in Fractal Porous Media Including Gravity, *Colloids Surf. A*, vol. **414**, pp. 228–233, 2012.

Cao, L., Li, X., Luo, C., Yuan, L., Zhang, J., and Tan, X., Horizontal Well Transient Rate Decline Analysis in Low Permeability Gas Reservoirs Employing an Orthogonal Transformation Method, *J. Nat. Gas Sci. Eng.*, vol. **33**, pp. 703–716, 2016.

Cui, B. and Zaccone, A., Generalized Langevin Equation and Fluctuation-Dissipation Theorem for Particle-Bath Systems in External Oscillating Fields, *Phys. Rev. E*, vol. **97**, p. 060102, 2018.

Czirok, A., Barabasi, A.B., and Vicsek, T., Collective Motion of Self-Propelled Particles: Kinetic Phase Transition in One Dimension, *Phys. Rev. Lett.*, vol. **82**, pp. 209–212, 1999.

Dong, M., Dullien, F.A.L., Dai, L., and Li, D., Immiscible Displacement in the Interacting Capillary Bundle Model Part I. Development of Interacting Capillary Bundle Model, *Transp. Porous Media*, vol. **59**, pp. 1–18, 2005.

Escobar, F.H., Zambrano, A.P., Giraldo, D.-V., and Cantillo-Silva, J.H., Pressure and Pressure Derivative Analysis for Non-Newtonian Pseudoplastic Fluids in Double-Porosity Formations, *Ciencia, Tecnología y Futuro*, vol. **4**, no. 3, pp. 47–59, 2011.

Family, F. and Vicsek, T., Scaling of the Active Zone in the Eden Process on Percolation Networks and the Ballistic Deposition Model, *J. Phys. A: Math. General*, vol. **18**, no. 2, p. L75, 1985.

Galindo, T.S., Scheuermann, A., and Li, L., Numerical Study on the Permeability in a Tensorial Form for Laminar Flow in Anisotropic Porous Media, *Phys. Rev. E*, vol. **86**, p. 046306, 2012.

Ghalambaz, M. and Zhang, J., Conjugate Solid-Liquid Phase Change Heat Transfer in Heatsink Filled with Phase Change Material-Metal Foam, *Int. J. Heat Mass Transf.*, vol. **146**, p. 118832, 2020.

Ghalambaz, M., Mehryan, S., Hajjar, A., and Veisimoradi, A., Unsteady Natural Convection Flow of a Suspension Comprising Nano-Encapsulated Phase Change Materials (NEPCMs) in a Porous Medium, *Adv. Powder Technol.*, vol. **31**, pp. 954–966, 2019.

Gou, X. and Schwartz, J., Fractal Analysis of the Role of the Rough Interface between $Bi_2Sr_2CaCu_2O_x$ Filaments and the Ag Matrix in the Mechanical Behavior of Composite Round Wires, *Supercond. Sci. Technol.*, vol. **26**, p. 055016, 2013.

Hader, A., Sbiaai, H., Tanasehte, M., Amallah, L., and Boughaleb, Y., Scaling Law in Avalanche Breaking of Composite Materials, *Multidisc. Model. Mater. Struct.*, vol. **17** no. 2, pp. 391–400, 2021.

Hansen, J.P. and McDonald, I., *Theory of Simple Liquids*, Amsterdam: Elsevier Science, 2006.

Hariti, Y., Hader, A., Amallah, L., Achik, I., and Boughaleb, Y., Langevin Dynamics Study of the Mean Flow Rate-Energy Stochastic Fluid Intrusion Process in Porous Media, *Int. Rev. Model. Simul.*, vol. **12**, no. 6, 2019.

Hariti, Y., Hader, A., Faraji, H., and Boughaleb, Y., Scaling Law of Permeability and Porosity for Fluid Transport Phenomena in Porous PCM Media, *J. Appl. Comput. Mech.*, vol. **7**, pp. 84–92, 2021.

Hariti, Y., Hajji, Y., Hader, A., Faraji, H., Boughaleb, Y., Faraji, M., and Saifaoui, D., Modelling of Fluid Flow in Porous Media and Filtering Water Process: Langevin Dynamics and Darcy's Law Based Approach, *Mater. Today: Proc.*, vol. **30**, pp. 870–875, 2020.

Hen, Z., Cheng, P., and Zhao, T., An Experimental Study of Two Phase Flow and Boiling Heat Transfer in Bi-Dispersed Porous Channels, *Int. Commun. Heat Mass*, vol. **27**, pp. 293–302, 2000.

Hollowood, T.J. and McDonald, J.I., Decoherence, Discord, and the Quantum Master Equation for Cosmological Perturbations, *Phys. Rev. D*., vol. **95**, p. 103521, 2017.

Horváth, V.K., Family, F., and Vicsek, T., Anomalous Noise Distribution of the Interface in Two-Phase Fluid Flow, *Phys. Rev. Lett.*, vol. **67**, no. 23, pp. 3207–3210, 1991b.

Horváth, V.K., Family, F., and Vicsek, T., Dynamic Scaling of the Interface in Two-Phase Viscous Flows in Porous Media, *J. Phys. A: Math. General*, vol. **24**, pp. L25–L29, 1991a.

Huai, X., Wang, W., and Li, Z., Analysis of the Effective Thermal Conductivity of Fractal Porous Media, *Appl. Therm. Eng.*, vol. **27**, pp. 2815–2821, 2007.

Kwok, S.F., A Class of Nonlinear Langevin Equation with the Drift and Diffusion Coefficients Separable in Time and Space Driven by Different Noises, *Physica A: Stat. Mech. Appl.*, vol. **545**, p. 123334, 2020.

Lebeau, M. and Konrad, J.M., A New Capillary and Thin Film Flow Model for Predicting the Hydraulic Conductivity of Unsaturated Porous Media, *Water Resour. Res.*, vol. **46**, pp. 1–15, 2010.

Li, L., Chahl, H.S., and Gopalakrishnan, R., Comparison of the Predictions of Langevin Dynamics-Based Diffusion Charging Collision Kernel Models with Canonical Experiments, *J. Aerosol Sci.*, vol. **140**, p. 105481, 2020.

Liu, R., Li, B., and Jiang, Y., A Fractal Model Based on a New Governing Equation of Fluid Flow in Fractures for Characterizing Hydraulic Properties of Rock Fracture Networks, *Comput. Geotech.*, vol. **75**, pp. 57–68, 2016.

Mahmoud, A.A. and Chamkha, A.J., Non-Similar Solutions for Heat and Mass Transfer from a Surface Embedded in a Porous Medium for Two Prescribed Thermal and Solutal Boundary Conditions, *Int. J. Chem. Reactor Eng.*, vol. **8**, pp. 1–24, 2010.

Mazroui, M. and Boughaleb, Y., Interacting Brownian Particles in a Two Dimensional Periodic Potential, *Physica A: Stat. Mech. Appl.*, vol. **227**, pp. 93–107, 1996.

McCartney, J.S., Sánchez, M., and Tomac, I., Energy Geotechnics: Advances in Subsurface Energy Recovery, Storage, Exchange, and Waste Management, *Comput. Geotech.*, vol. **75**, no. 2, pp. 244–259, 2016.

Petrosyan, A. and Zaccone, A., Relativistic Langevin Equation Derived from a Particle-Bath Lagrangian, *Condensed Matter Stat. Mech*., pp. 1–20, 2021.

Pia, G. and Sanna, U., An Intermingled Fractal Units Model and Method to Predict Permeability in Porous Rock, *Int. J. Eng. Sci.*, vol. **75**, pp. 31–39, 2014.

Pia, G., Sassoni, E., Franzoni, E., and Sanna, U., Predicting Capillary Absorption of Porous Stones by a Procedure Based on an Intermingled Fractal Units Model, *Int. J. Eng. Sci.*, vol. **82**, pp. 196–204, 2014.

Pollmann, N., Larsson, F., Runesson, K., and Jänicke, R., Diffuse Interface Modeling and Variationally Consistent Homogenization of Fluid Transport in Fractured Porous Media, *Eur. J. Mech. A/Solids*, vol. **84**, pp. 2–16, 2020.

Seiwert, J., Kervil, R., Nou, S., and Cantat, I., Velocity Field in a Vertical Foam Film, *Phys. Rev. Lett.*, vol. **118**, p. 048001(5), 2017.

Sheikholeslami, M., Jafaryar, M., Shafee, A., and Babazadeh, H., Acceleration of Discharge Process of Clean Energy Storage Unit with Insertion of Porous Foam Considering Nanoparticle Enhanced Paraffin, *J. Clean. Prod.*, vol. **261**, p. 121206, 2020.

Stokes, J.P., Kushnick, A.P., and Robbins, M.O., Interface Dynamics in Porous Media: A Random-Held Description, *Phys. Rev. Lett*., vol. **60**, pp. 1386–1389, 1988.

Tan, X.H., Jiang, L., Li, X.P., Li, Y.Y., and Zhang, K., A Complex Model for the Permeability and Porosity of Porous Media, *Chem. Eng. Sci.*, vol. **172**, pp. 230–238, 2017.

VeeraKrishna, M. and Chamkha, A.J., Hall and Ion Slip Effects on MHD Rotating Flow of Elastico-Viscous Fluid through Porous Medium, *Int. Commun. Heat Mass Transf*., vol. **113**, p. 104494, 2020b.

VeeraKrishna, M. and Chamkha, A.J., Hall and Ion Slip Effects on Unsteady MHD Convective Rotating Flow of Nanofluids-Application in Biomedical Engineering, *J. Egypt. Math. Soc*., vol. **28**, no. 1, pp. 1–14, 2020a.

VeeraKrishna, M. and Chamkha, A.J., Hall Effects on Unsteady MHD Flow of Second Grade Fluid through Porous Medium with Ramped Wall Temperature and Ramped Surface Concentration, *Phys. Fluids*, vol. **30**, p. 023106, 2018.

VeeraKrishna, M., Gangadhar Reddy, M., and Chamkha, A.J., Heat and Mass Transfer on MHD Free Convective Flow over an Infinite Nonconducting Vertical Flat Porous Plate, *Int. J. Fluid Mech. Res*., vol. **45**, no. 5, pp. 1–25, 2018.

VeeraKrishna, M., Gangadhar Reddy, M., and Chamkha, A.J., Heat and Mass Transfer on Unsteady MHD Flow through an Infinite Oscillating Vertical Porous Surface, *J. Porous Media*, vol. **24**, pp. 81–100, 2021.

Verma, A.K., Gautam, A.K., Bhattacharyya, K., Banerjee, A., and Chamkha, A.J., Boundary Layer Flow of Non-Newtonian Eyring–Powell Nanofluid over a Moving Flat Plate in Darcy Porous Medium with a Parallel Free-Stream: Multiple Solutions and Stability Analysis, *Pramana - J. Phys*., vol. **95**, p. 173, 2021.

Xiong, J., Liu, X., Liang, L., and Zeng, Q., Adsorption of Methane in Organic-Rich Shale Nanopores: an Experimental and Molecular Simulation Study, *Fuel*, vol. **200**, pp. 299–315, 2017.

Xu, C., Kang, Y., You, L., and You, Z., Lost-Circulation Control for Formation-Damage Prevention in Naturally Fractured Reservoir: Mathematical Model and Experimental Study, *SPE J*., vol. **22**, pp. 1654–1670, 2017.

Xu, P., Sasmito, A.P., Yu, B., and Mujumdar, A.S., Transport Phenomena and Properties in Treelike Networks, *Appl. Mech. Rev.*, vol. **68**, p. 040802, 2016.

Yang, S., Liang, M., Yu, B., and Zou, M., Permeability Model for Fractal Porous Media with Rough Surfaces, *Microfluid. Nanofluidics*, vol. **18**, pp. 1085–1093, 2015.