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REVIEWING THE IMPACT OF MAGNETIC PRANDTL NUMBER AND MAGNETIC FORCE PARAMETER ON CONVECTIVE HEAT TRANSFER IN BOUNDARY LAYERS

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This review paper provides a comprehensive inspection how the magnetic Prandtl number (Prandtl number influenced by a magnetic field) and the magnetic force parameter affect convective heat transfer in boundary layers. The investigation delves into the intricate interplay between these parameters and their implications for heat-transfer phenomena. Understanding the influence of magnetic fields on convective heat transfer holds significant importance for diverse engineering applications such as advanced technologies, heat exchangers, and cooling systems. Through a thorough analysis of existing literature, this review offers valuable insights into the complex relationship between the magnetic Prandtl number, the magnetic force parameter, and convective heat transfer in boundary layers. The main novelty of the current review is based on the perception that this review is very beneficial for the readers to establish future work in the field of magneto-material and magneto heat transfer. This review will provide the clear knowledge about the physical significances of the magnetic Prandtl number and magnetic force parameter on chief physical quantities like velocity profile, temperature distribution, skin friction, and heat-transfer rate.

KEY WORDS: *Prandtl number, boundary layer, heat transfer, magnetohydrodynamics*

1. INTRODUCTION

On a semi-infinite flat plate with a continuous stream of highly conductive gas and an aligning magnetization at great distances from the plate, Ingham (1967) researched the flow in the boundary layer. The boundary layer is discovered to thicken with either an increment in magnetization or a reduction in drag coefficient for a certain magnetic force. It is revealed that boundary-layer dissociation arises when the generated field's strength exceeds a particular critical value in Glauert (1962). Glauert (1961) evaluated the mechanism on a moderately flat plate in a smooth stream of conductive fluid, with a magnetic field positioned in the stream direction such that the Alfven rate is slower than

NOMENCLATURE

X	distance along *x*-direction	y	distance along *y* direction
U	*x* component of velocity	v	*y* component of velocity
$H_!$	*x* component of magnetic field	H_2	*y* component of velocity
η	magnetic diffusion	β	magnetic force parameter
ϵ	magnetic Prandtl number	V_o	mass flux at the surface
H_∞	magnetic field far from the surface	U_∞	free-stream velocity

the movement of the unaltered fluid. The flow disruption induced by a small point source of fluid in a stream with a vortex sheet is determined by Glauert's method of images (Glauert, 1960). To demonstrate the method's variety of relevance, calculations are made for a variety of these shear flows. In reference to two simulations of the movement of a conducting fluid via a body in an aligning magnetization, Glauert (1964) addressed the flow separation deriving from finite viscosity and magneto-diffusivity. The expansion of the flow separation may have massive implications in each scenario, such as casting doubt on the accuracy of the supposed fundamental flow patterns. A uniformly aligned semi-infinite flat plate is presented by Greenspan and Carrier (1959) which distorts the uniform/steady flow of the incompressible, viscous, highly conductive fluid. Specifically, no uniform steady flow exists at great distances from the surface when the electromagnetic force is greater than 1.

In a steady flow under the effect of an external magneto-dynamic pressure gradient related to a certain power of separation along the edge, Gribben (1965a) reviewed the study of hydro-magnetic flow separation. The method of evaluation where series approximations for the magnetic and velocity fields are appropriate in the frictional and electromagnetic flow conditions was examined by Gribben (1965b). The exact result is utilized to deduce the extension of the series that match in the two phases that correspond to this flow. Ramamoorthy (1965) demonstrated in the plate thermometer scenario that the adiabatic thermal resistance falls as the field strength rises. This is due to the heat exchange between the plate and the liquid occurs at lower adiabatic wall heats as the viscous heating reduces with higher magnetic field intensity. Davies (1963a) investigated the electromagnetic fluid mechanism by assuming the magnetodynamic differential pressure across the un-magnetized plate to extend the Falkner and Skan electromagnetic problem. Again, Davies (1963b) developed the electromagnetic flow model through a fixed un-magnetized but conducting plate by assuming the uniform parallel magnetic flux and liquid flow to the plate. To develop the magnetic and momentum flow separation, the large values of magnetic Reynolds and Reynolds number are assumed for given model. When the gravitational flux fluctuates in amplitude about a constant non-zero mean, the momentum boundary surface on a polarized plate has been probed by Chawla (1967).

The phase difference and intensity of the skin-friction fluctuations grow initially in the low-frequency region and gradually decline to their corresponding skin-wave quantities. The steady flow separation flow caused by a continuous moving fluid across a semi-infinite magnetic flat plate has been discussed by Mohanty (1972) in relation to the consequences of harmonic fluctuations of frequency in the magnitude of the magnetization. It has been discovered that the surface current distribution is unaffected by the strong magnetic dispersion. Ishigaki (1971a) developed the analytical results for transient skin friction phenomenon across the plate which is placed in the fluctuating stream at zero incidences. Consequently, the developing transient friction force is inversely proportional to square of frequency. Again, Ishigaki (1971b) analyzed the scientific research on the time assuming thermal performance of the incompressible flow separation on a plate under the action of fluctuation. The lower frequency and high amplitude was examined for the case of finite-velocity amplitude.

Both theoretically and practically, the boundary-layer flow over a circular cylinder at a significant Reynolds number has been addressed by Dwyer and Mccroskey (1973). The objective of the research was to emphasize on the flow's self-induced fluctuations. When thermal conductance and viscosity are both considered to be thermal, Ashraf et al. (2016) proposed a computational response to the issue of multiple, constant stream separation buoyant flow on a vertical magnetic surface. Ashraf et al. (2012a,b) have looked at how radiation affects the flow of a viscous, incompressible, semiconducting fluid across a magnetic vertical plate during a hydro-magnetic convection flow. Ashraf et al. (2010) looked at how radiation affected the flow of a fluid that conducts electricity through a magnetic porous plate with constant permeability in a continuous magnetohydrodynamic (MHD) flow separation. Ilyasa et al. (2022) talked about the investigation into the periodic characteristic of heat transfer by convection along electrically insulating cone implanted in a permeable material. Ilyas et al. (2021) studied the thermal buoyancy-assisted irregular hydro-magnetic boundary-layer flow along the internal magnetic cone immersed in porous material. Ashraf et al. (2022a) evaluated the transmission of heat in Newtonian fluid across thermal and electrical-insulating shape implanted in porous material when it came into contact with the varied surface temperature.

By considering differential density, the periodic heat transmission movement of maximum conductive fluid along a non-conducting horizontal cylindrical cylinder has been examined by Ashraf and Ullah (2020). Ullah et al. (2022a) looked into the effects of lower gravitation and hydro-magnetic on fluctuating free/forced-convective magnetohydrodynamic flow pattern over an un-magnetized cylinder. Ullah et al. (2022b) used a computer algorithm to statistically estimate the role of thermal density on the hydro-magnetic forced convection processes of electrical-conductive fluid beyond a vertically magnetic and warmed surface situated in stratifying material. In the presence of a thermally stratified material, Ullahet et al. (2020a) looked into the importance of periodic convection stratification and heat transport properties at different stations of non-conducting cylinders. Ullah et al. (2022c) investigated how magnetohydrodynamics and slip velocity affected alternating mixed-convective electrical conductive flow pattern above the non-conducting cylinder. The cylinder was implanted into a porous material and had a thermal conductivity of zero. The non-conducting circular cylinder immersed in a porous medium features an oscillating mixed-convection flow field process that has been calculated by Ashraf et al. (2021).

With the inclusion of entropy generation influences in the mathematical model, Ashraf et al. (2022b) evaluated the initial assessment of thermal transfer attributes of fluid along electric charged cone implanted in malleable materials. Ullah et al. (2022d) investigated the periodical mixed-convective fluid flow's amplitude and phase angle along a circular, non-conducting object. Ullah et al. (2022d) used mathematical solution to examine the consequences of radiant heat and thermal slip across vertically symmetrical heating geometry submerged in malleable material. Ullah et al. (2020b) studied the oscillating made by mixing flow of a fluid that conducts electricity around a fixed circular horizontal cylinder. Ullah et al. (2020b) examined the effects of drag coefficient on the mixed-convection flow of a fluid that conducts electricity with fluctuating free-stream rate and surface heat over a non-conducting cylinder. Reasonably accurate computational outcomes are obtained by Ashraf et al. (2022c) for persistent mixed-convective flow in the forms of phase angle and amplitude angle. By incorporating algebraic decline in stream velocity, Ashraf et al. (2017) studied computer model of continuous convective flow under the conditions of a low magnetic Prandtl number and a strong magnetic force. Abbas et al. (2021) studied the impacts of changing density, heat conduction, and magnetization impacts on the models of heat transmission movement over an angled accelerating surface (Sakiadis flow).

The analysis was done by Mahmood et al. (2009) on the flow of a thick, electrically charged fluid through a wedge with a permeable surface. In the case of fluid blowing, they have seen the beginning of reverse flow in the downstream side. Regarding the convective heat transport of MHD Jeffrey fluid above a shrinking sheet, Ahmad et al. (2015) concentrated on the analytical solutions. The confluent hyper-geometric function is designed to represent the precise solutions of the reduced conventional differential equations. Using the finite-element analysis, a computational model is utilized by Kumar et al. (2021) to simulate the flow of nanofluid, the transmission of heat from an infinite vertical plate in the presence of a magnetization, and the dissipation of viscous fluid. As the Grashof number rises, they noticed an improvement in the velocity distribution. Aina and Malgwi (2019) used the velocity slipping and heat-jump characteristics at the micro-porous flow path to achieve the analytical solutions for the velocity distribution and heat flux. The results demonstrate that the amount of the wall ambient thermal gradient influences how the inclination angle alters fluid motion. The computational research was done by Hussain et al. (2020) on the contribution of suspension on MHD slipping thermal convection of heat transmission across vertically malleable plate. They realized that drag force had a noticeable impact on heat transmission.

Homotopy analysis approach (HAA), a semi-analytical algorithm proposed by Rashidi et al. (2014), was used to evaluate the hydromagnetic mix convective heat exchange for an incompressible, laminar, and electrically insulating viscoelastic flow pattern over a transparent wedge. HAA is implemented to expand the existing research to mix convective heat-transport computations and is exhibited to demonstrate great convergence and consistency. Ferdows et al. (2022) created a computer simulation for the assessment of MHD flow across a curvy stretched surface using a curvilinear reference frame. They gathered the results of the comparison with earlier literature in tabulated form and discovered reasonable agreement. The fluid's movements are governed by the plate's stochastic movement action, thermal gradients, and transport phenomena using Laplace approximating technique in Riaz et al. (2021). They determined that thermal conduction rises in buoyant forces to accelerate fluid flow. In Aziz-Ur-Rehman et al. (2021), a thorough examination of the mass and energy transfer of a second MHD turbulent fluid in the context of ramping conditions close to a porous surface was conducted. The management of momentum thickening and expansion of thermal conductivity may be indicated in the existence of a Prandtl number.

Nanofluid innovation, which is being pursued in nanotechnology, has a wide range of uses in engineering and industry, involving heating cooling towers, renewable power collection, and energy production by Othman et al. (2021). The shrinking case is used by the researchers to demonstrate that dual solutions exist inside a particular variety of values. Alshehri et al. (2021) researched the joint influence of vacuum, momentum slip, and heating element across a porosity adiabatic stretching/shrinking surface in a radiating turbulent flow of a viscous nanofluid around an aligning magnetization. They discovered that, in contrast to increasing rates of stretching, the rate of heat transmission and frictional drag continuously increases. To create a flow model that represents moment momentum, energy, and concentration equations, a flow-separation assumption is designed by Afikuzzaman et al. (2018). They used the graphical package tecplot-9 to plot and discuss the stability converging test. Irfan et al. (2019) combined constant and variable fluid characteristics with the MHD flowing-stream nanofluid to transfer heat over an exponential radiating stretchable sheet. On the transient flow of MHD flow over a stretching surface in the presence of a magnetic field, the impact of Navier slip and convective surface conditions have been explored by Haritha et al. (2017). They discovered that as the heat source parameter's quantities increases, temperature rises. Reddy et al. (2022) looked into the impact of chemical process and magnetic Casson nanofluid dynamics that occur when radiant energy passes through a porous inclined stretchable material.

The upper layer jets created by the Lorentz force effect in hydro-magnetic flows have the ability to significantly alter the heat-exchange characteristics in Kaneesamkandi et al. (2012). The flow and heat transmission characteristics of a dusty fluid above a smooth stretchable sheet in the context of viscous dissipation have been studied by Gireesha et al. (2012). They noticed that the temperature of the surrounding fluid is higher than that of the dust component. Swarnalathamma (2018) talked about the mass and heat transport of a homogeneous sphere-based magnetohydrodynamic movement of an Eyring-Powel nanofluid with temperature slip effects. Ishak et al. (2008) used the Keller-box methodology for the simulation results of MHD flow and transfer of heat beyond a stretching cylinder. In contrast to Prandtl number, the skin friction coefficient's intensity grows with the magnetic parameter and Reynolds number. In the presence of a volume fraction of ferrous nanoparticles, a mathematical model is developed by Kakaç et al. (2009) and Naramgari et al. (2018) for analyzing the thermal source/sink behavior on hydro-magnetic flow through a cone and vertically plate. Additionally, it has been discovered that the flow above a plate transfers heat more effectively than the flow above a cone. Williamson nanofluids' radiation heat influence on MHD heat transmission and flow modeling has been quantitatively examined in Kho et al. (2017). The quantitative evaluation of the Casson nanofluid boundary-layer mass and heat transport above a stretching sheet with a uniform wall temperature under the influences of the magnetization has been discussed by Kho et al. (2018). Mahmoud et al. (2009) investigated how the heat and flow transmission of an electrically conducting non-Newtonian power-law fluid inside a thin layer of liquid across an unstable stretchy sheet were affected by the viscosity and thermal expansion. Ashraf et al. (2012a,b), Ilyas et al. (2020), Tan et al. (1968), Mohanty (1972), Rollins et al. (1997), Krishna and Chamkha (2019, 2020); Krishna et al. (2019a,b, 2020), and Shao et al. (2023) discuss the different cases of magnetohydrodynamics nanofluid along flat and porous medium. Dogonchi et al. (2021, 2023), Nayak et al. (2023), Shao et al. (2022), and Pasha et al. (2023) studied the different aspects of entropy analysis in the presence of aligned and applied magnetic field. Ashraf et al. (2022c) studied the periodic magnetohydrodynamic mixed-convection flow along a cone embedded in a porous medium with variable surface temperature.

2. CLASSICAL AND RECENT STUDIES ON MAGNETOHYDRODYNAMICS CONVECTIVE HEAT TRANSFER

The characteristics of magnetohydrodynamics fluid to change its physical properties in heat and fluid flow mechanism takes it to the group of smart materials. Due to its effects on thermal properties, researchers found that magnetohydrodynamics heat and fluid flow has potential applications in heat-transfer device design. Table 1 shows the detailed research on the application of magnetohydrodynamics convective heat transfer (Figs. 1–6).

3. DISCUSSION ON THE EFFECTS OF MAGNETOHYDRODYNAMICS ON HEAT AND FLUID FLOW

When a consistent magnetization is supplied in the streaming direction, the basic principles of the movement of a conducting fluid with low viscosity through a solid plate have been examined in the research by Glauert (1964). He discussed the effects of small and large values of the electrically conducting parameter and found solution by using extended series method. In this study he showed that if the strength of the applied magnetic field exceeds s certain value, boundary-layer separation occurs. The equations that govern the magnetohydrodynamic flow by following Glauert (1964) are given in the following form:

$$f''' + ff'' - \beta gg'' = 0 \tag{1}$$

$$g' + \in \left(fg' - f'g \right) = 0 \tag{2}$$

Subject to the following boundary conditions:

$$f(0) = 0,\ f'(0) = 2,\ g(\infty) = 0,\ g'(\infty) = 2 \tag{3}$$

In the vicinity of a semi-infinite surface, Davies (1963) researched the flow separation of a viscous electrically conductive fluid with a magneto-dynamic differential pressure obstructing the flow. The electromagnetic field is considered to be parallel to the surface and far from the surface, assuming that the plate is unchanging. The adverse magneto-dynamic pressure gradient being taken by Cx^n, an investigation is made of viscous stress at the plate and also the critical values of the parameter C and n when this viscous dissipation stress vanishes. The orientation of the plate was adjusted at $y = 0,\ 0 < x < \infty$, and $(u, v), (H_1, H_2)$ are designated as velocity and magnetic field components in the x and y directions. The exact equations governing the steady motion of the incompressible liquid are given as below,

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} - \frac{\mu}{4\pi\rho}H_2\left(\frac{\partial H_2}{\partial x} - \frac{\partial H_1}{\partial y}\right) + \nu\nabla^2 u \tag{4}$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} - \frac{\mu}{4\pi\rho}H_1\left(\frac{\partial H_2}{\partial x} - \frac{\partial H_1}{\partial y}\right) + \nu\nabla^2 v \tag{5}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{6}$$

TABLE 1: Research on the application of magnetohydrodynamics convective heat transfer

Author	Year	Title of investigation	Geometry of the problem	Non-dimensional quantities/outcomes	Solution procedure
Rollins et al. (1997)	1958	The magnetohydrodynamic flow past a flat plate	Flat plate with symmetrical orientation and corresponding external electromagnetic and accelerating fields	Effects of magnetic force on velocity field, and for strong magnetic field, no constant, homogeneous circulation that extends far from the surface	Asymptotic method
Glauert (1964)	1961	The boundary layer on a magnetized plate	Flat plate and magnetic field is applied in the stream direction	It is demonstrated that the flow separation forms surpass a particular critical value with strong magnetic impacts	Extended series solution
Davies (1963)	1963	The magnetohydrodynamic boundary layer in the two-dimensional steady flow past a semi-infinite plate: The influence of an adverse magneto dynamic pressure gradient	Flat sheet with a semi-limit	A gradient in magneto-dynamic potential resists the flow	The method of iteration and first approximation solution
Ramamoorthy (1965)	1964	Heat transfer in hydromagnetics	Electrically insulated flat plate in the presence of a uniform magnetic field parallel to the plate	It is found that the temperature distribution in the boundary layer is considerably reduced because of the slowing down of the flow by the magnetic field. The dissipation of currents due to Joule heating is small. It is also found that the presence of magnetic field increases both the viscous and temperature boundary layer	Approximation method
Gribben (1965a)	1965	The magnetohydrodynamic boundary layer in the presence of a pressure gradient	An external hydro-magnetic differential pressure corresponding to a certain magnitude of the distance along the boundary alters the flow along a semi-infinite plate	The tangential component, electromagnetic intensity at the wall, and skin friction are the properly functioning characteristics of importance. The skin friction and field intensity at the wall are reduced with the increase of magnetic field	Extended series solution

TABLE 1: (*continued*)

Chawla (1967)	1967	Fluctuating boundary layer on a magnetized plate	Semi-infinite plate, where the electromagnetic field's intensity pulsates around a fixed non-zero mean	The phasing lead and intensity of skin friction oscillates in the low-frequency region, increasing initially and subsequently decreasing to their individual "skin-wave" amounts	Karman-Pohlhausen method
Tan et al. (1968)	1968	Heat transfer in aligned field magnetohydrodynamic flow past a flat plate	The flow rate and magnetization components are identical in aligned flow past a semi-infinite flat plate	The effect of Eckert number along with other parameters involved in the flow model on viscous and thermal boundary layer is highlighted. It is found that for the increasing values of magnetic field, the viscous, magnetic, and thermal boundary-layers thickness are increased. The rate of heat transfer, however, is decreased with increasing magnetic field for Eckert number greater than or equal to zero ($Ec \geq 0$)	Numerical method with the help of IBM 7094
Mohanty (1972)	1972	The effects of magnetic field oscillations on the boundary-layer flow past a magnetized plate	Uniform free stream through a magnet surface with the magnetization circulating in the free-stream direction while fluctuating round the constant mean	Viscous drag is increased as magnetic force parameter is increased	Extended series solution
Mahmood et al. (2009)	2009	Hydromagnetic flow of viscous incompressible fluid past a wedge with permeable surface	The movement of a fluid that conducts electricity via a permeable-surfaced wedge	It has been established that as fluid is extracted, both the momentum and electromagnetic boundary layer drop out as the magnetic force component rises	Extended series solution, local non-similarity method, Keller box technique

TABLE 1: (*continued*)

Author	Year	Title of investigation	Geometry of the problem	Non-dimensional quantities/outcomes	Solution procedure
Ashraf et al. (2012)	2012	Computational study of the combined effects of conduction-radiation and hydromagnetics on natural convection flow past a magnetized permeable plate. The obtained results for both parameters are given in Figs. 1 and 2	A magnetically permeable sheet is passed by the free convection of a viscous, incompressible, highly conductive fluid	On the factors of skin friction, temperature transfer rate, and electrical field, the consequences of changing the Prandtl number, magnetic Prandtl number, induced magnetic component, and radiative heat component are shown. The increase in the magnetic force parameter increases the coefficient of skin friction actively but rate of heat transfer and current density are increased slowly	Finite-difference method and extended series solution
Ilyas et al. (2020)	2020	Periodical analysis of convective heat transfer along electrical conducting cone embedded in porous medium. The obtained graphical results are given in Figs. 3 and 4	Electrically conducting cone embedded in porous medium and the magnetic field is considered at the surface along the stream direction	The material parameters of interest are porous medium parameter, magnetic force parameter, Prandtl number, mixed-convection parameter, and magnetic Prandtl number. Numerical solutions reflect that velocity profile and magnetic intensity at the surface of plate are reduced but temperature field is increased with the increase of magnetic Prandtl number	Implicit finite-difference method
Ullah et al. (2022)	2022	The analysis of the amplitude and phase angle of periodic mixed-convection fluid flow across a non-conducting horizontal circular cylinder. The obtained graphical results are given in Figs. 5 and 6	Transient flow field in heat transmission along the circular, non-conducting surface	Calculations have been made on the amplitude and phase angle of the time-dependent friction factor, the temperature transfer rate, and the electrical field near the ideal places of the non-conducting rotating shape	Implicit finite-difference method in conjunction with primitive variable formulation

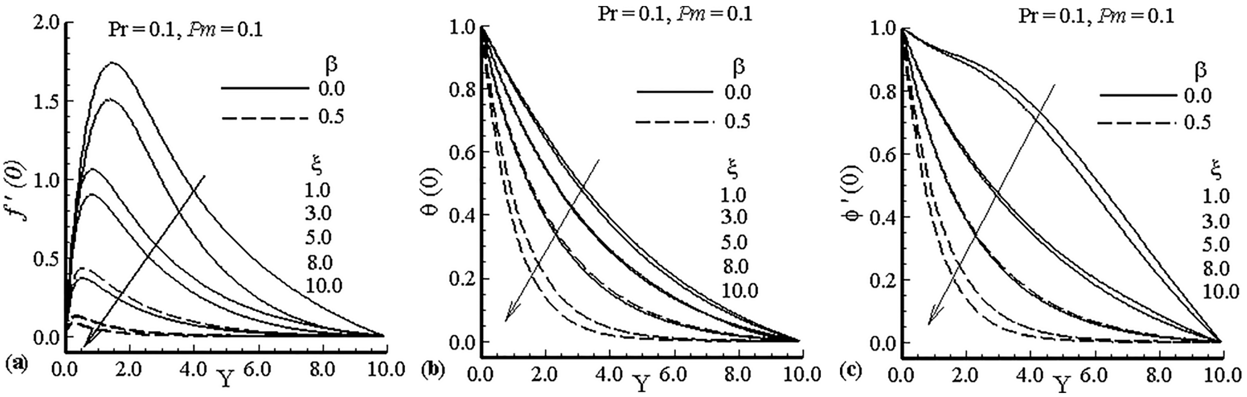

FIG. 1: The effects of magnetic force parameter on velocity, temperature, and magnetic flux along a permeable surface

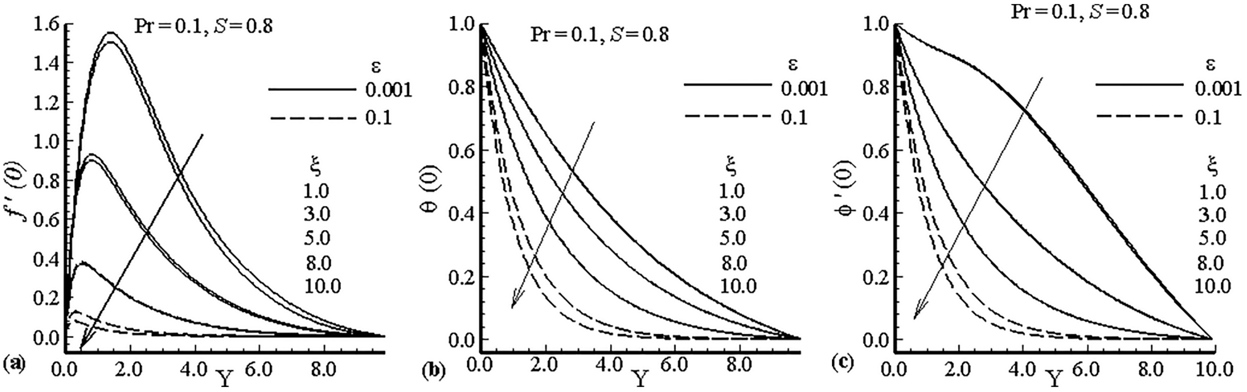

FIG. 2: The effects of magnetic Prandtl number on velocity, temperature, and magnetic flux along a permeable surface

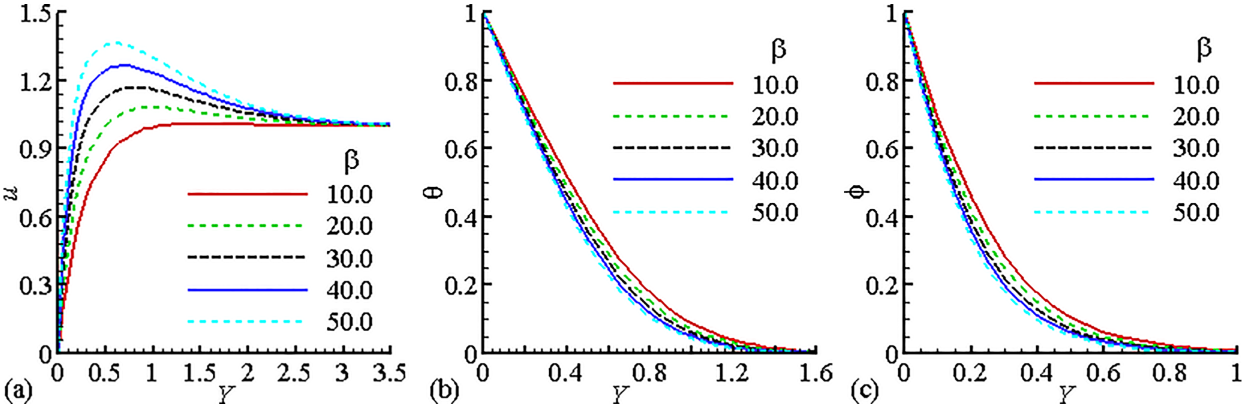

FIG. 3: Effects of the different values of magnetically force number on velocity, temperature, and electromagnetic field along the surface of a magnetized cone

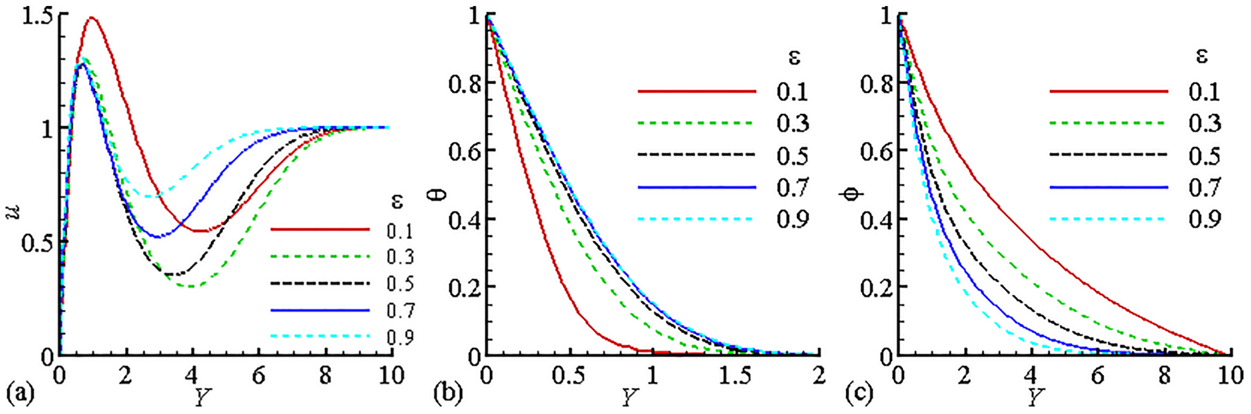

FIG. 4: Effects of the different values of magnetic Prandtl number on velocity, temperature, and magnetic field along the surface of a magnetized cone

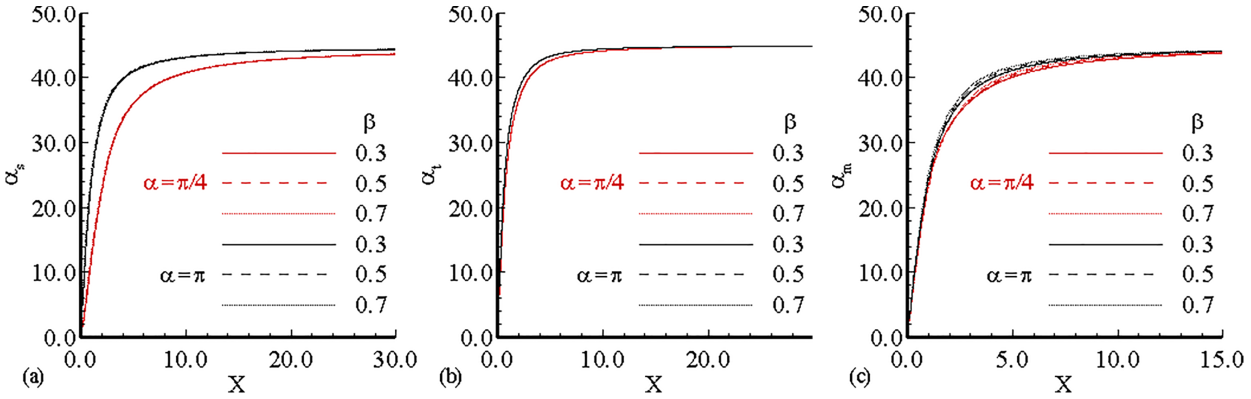

FIG. 5: Effects of distinct choices of magnetic force parameter on phase angle across two different positions of a circular cylinder

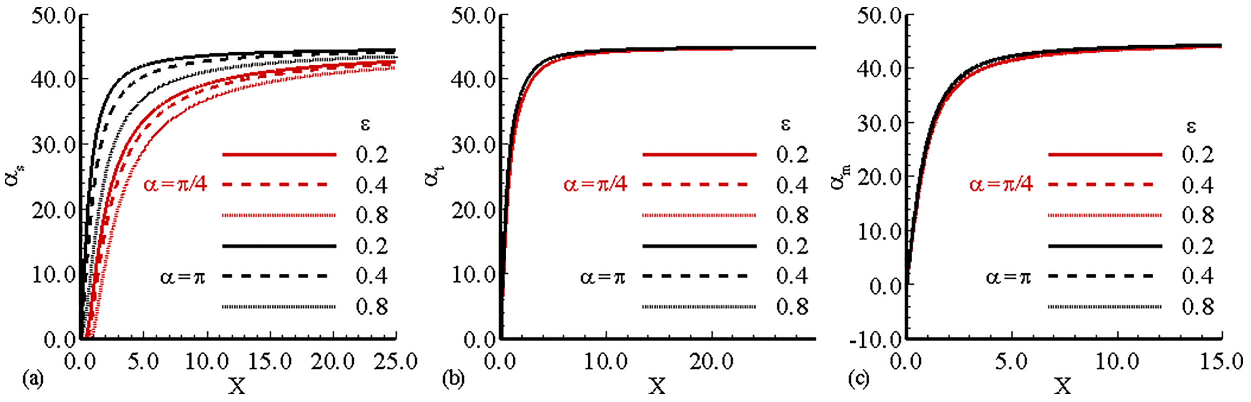

FIG. 6: Effects of different values of magnetic Prandtl number on phase angle around two different positions of a circular cylinder

$$u\frac{\partial H_1}{\partial x}+v\frac{\partial H_1}{\partial y}-H_1\frac{\partial u}{\partial x}-H_2\frac{\partial u}{\partial y}=\eta\nabla^2 H_1 \tag{7}$$

$$u\frac{\partial H_2}{\partial x}+v\frac{\partial H_2}{\partial y}-H_2\frac{\partial u}{\partial x}-H_1\frac{\partial u}{\partial y}=\eta\nabla^2 H_2 \tag{8}$$

$$\frac{\partial H_1}{\partial x}+\frac{\partial H_2}{\partial y}=0 \tag{9}$$

By using these equations, the results obtained by Glauert (1964) and Davies (1963) are given. In Table 2, they reported that for magnetic force parameter β = 0.0, and for increasing values of magnetic Prandtl number the coefficient of skin friction is increased. However, for magnetic force parameter β = 0.8 is increased slowly.

From Table 3, he concluded that for increasing values of magnetic Prandtl number the skin friction is increased; however, the magnetic intensity is decreased. Chawla (1967) studied magnetohydrodynamic boundary-layer flow along a magnetic surface. He came to the conclusion that for low-frequency range, the skin friction oscillation's phase angle lead and intensity expand initially before decreasing to their respective "skin wave" characteristics. Further, the

TABLE 2: The values of coefficient of skin friction at the leading edge for different values of magnetic Prandtl number when magnetic force parameter β = 0.0, 0.8

β = 0.0		**β = 0.8**	
ϵ	**Glauert (1964)**	**ϵ**	**Davies (1963)**
0.1	0.2669	0.1	0.3140
1.0	0.3016	0.2	0.3157
2.0	0.3078	0.3	0.3173
4.0	0.3128	0.5	0.3194
6.0	0.3152	0.7	0.3204
8.0	0.3167	0.9	0.3204
10.0	0.3178	—	—
50	0.3237	—	—
75	0.3247	—	—
100	0.3254	—	—

TABLE 3: Numerical results obtained from Glauert (1964) model for the effects of different values of magnetic force parameter β on skin friction and magnetic intensity when magnetic Prandtl number ϵ = 1.0, 10

	ϵ = 1.0		**ϵ = 10.0**	
β	**$f''(0)$**	**$g(0)$**	**$f''(0)$**	**$g(0)$**
0.0	0.3321	2.1797	0.3321	1.0095
0.1	0.3025	2.2448	0.3182	1.0238
0.2	0.2729	2.3099	0.3044	1.0380
0.4	0.2138	2.4402	0.2767	1.0665
0.6	0.1547	2.5704	0.2491	1.0950
0.8	0.0955	2.7006	0.2214	1.1235

phase angle of the surface current is decreased from 90° to 45° and its amplitude is increased with frequency. For this purpose he considered the two-dimensional flow past a semi-infinite plate when both Reynolds number and magnetic number are large. In addition, he took into account the fact that while the distant fluid has no magnetic field, the boundary layer has one that was created by outside forces inside the plate with the stream direction. The effect of magnetic Prandtl number on skin friction reported by Chawla (1967) is given.

From Table 4, they reported that for increasing values of magnetic Prandtl number the skin friction is slightly increased. In a flow that is parallel to the electromagnetic field vectors far from the surface and is aligned with a moderately surface, Tan et al. (1968) evaluated the stable-state heat transmission features. For this purpose he used the model as proposed in Eqs. (4)–(9) by considering the boundary conditions,

$$u_x = 0, v_y = 0, H_y = 0, T = T_w \text{ at } y = 0$$

$$v_x = V_\infty, H_x = H_0, T = T_\infty \text{ as } y \to \infty$$

They observed that for increasing values of magnetic field the viscous, magnetic, and thermal boundary-layer thicknesses are increased. Further, they concluded that for increasing values of magnetic force parameter the Nusselt number is decreased as shown in the below table.

From Table 5, they observed that for increasing values of magnetic force parameter the Nusselt number is decreased.

The hydromagnetic movement of a viscous, unsteady flow via a wedge with a permeable surface was researched by Mahmood et al. (2009). Further, they used the model as proposed by Davies (1963) as described in Eqs. (4), (6), and (7) subject to the boundary conditions,

$$u(x,0) = 0, v(x,0) = \mp V_0, H_x(x,0) = 0, H_y(x,0) = 0$$

$$u(x,\infty) = U_\infty(x), H_x(x,\infty) = H_\infty$$

Further, they assumed a constant transpiration along the wedge surface. They claimed that with increase of withdrawal of fluid both the momentum and magnetic boundary layers are decreased with the increase of transpiration

TABLE 4: For increasing values of magnetic Prandtl number the skin friction is slightly increased

Magnetic Prandtl number (ϵ)	**Skin friction**
1	0.3204
10	0.3210
100	0.3244

TABLE 5: For increasing values of magnetic force parameter the Nusselt number is decreased

Magnetic force parameter (β)	**Nusselt number Nu_x**
0.1	0.65048
0.3	0.61958
0.5	0.57693

parameter. In this review we report some results obtained by Mahmood et al. (2009) for different values of magnetic force parameter on skin friction against transpiration parameter ξ and are given in below Table 6.

From Table 6, they concluded that skin friction is reduced for increasing values of magnetic force parameter against transpiration parameter ξ.

Ashraf et al. (2012) simulated the illustration of the statistical assessment of the cumulative influence of radiant heat and hydromagnetics on convection flow along a convective surface. On the velocity distribution, temperature field, and transversal magnetization, the consequences of the magneto-Prandtl number and strong magnetic component are depicted graphically. The geometry of the flow domain was designated by choosing the following boundary conditions for the model as proposed by Davies (1963):

$$u = 0, v = V_0, T = T_w, H_x = H_w\left(x\right), H_y = 0 \text{ at } y = 0$$

$$u = 0, H_x = 0, T \to T_\infty$$

They reported that during the suction process the for enhancing quantities of magnetic force parameter the velocity, temperature, and magnetic field are dropped as shown in Figs. 1 and 2.

The importance of periodical MHD flow and heat transmission characteristics along the surface of a magnet cone was emphasized by Ilyas et al. (2020). To highlight the above-mentioned characteristics along the surface of magnetized cone, they used the model as proposed by Glauert (1964) and Davies (1963) subject to the following boundary conditions:

$$u = v = 0, H_x = H_0 = 0, H_y = 0 \text{ and } T = T_w \text{ at } y = 0$$

TABLE 6: Different values of magnetic force parameter on skin friction against transpiration parameter ξ

Transpiration parameter (ξ)	**Magnetic force parameter (β = 0.2)**	**Magnetic force parameter (β = 0.8)**
0.0	0.78085	0.33813
0.1	0.83914	0.38728
0.3	0.96025	0.49660
0.5	1.09039	0.62273
0.7	1.22898	0.76332
1.0	1.45085	0.99570
2.0	2.27988	1.88298
3.0	3.19107	2.85241
4.0	4.14182	3.85174
5.0	5.11212	4.86101
6.0	6.09271	5.87256
7.0	7.07917	6.88361
8.0	8.06920	7.89334
9.0	9.06155	8.90162
9.1	9.16088	9.00237
9.3	9.35959	9.40525
9.5	9.55836	9.40329
9.7	9.75718	9.60493
10.0	10.03560	9.88844

$$u \to U(\tau), T \to T_\infty, H_y \to H_\infty.$$

They reported that for increasing values of magnetic force parameter the velocity field is increased while the temperature and magnetic field are decreased slightly.

Ullah et al. reported the numerical results with distinct quantities of magnetic Prandtl number and magnetic force parameter in terms of phase angle and amplitude for mixed-convection flow along the non-conducting horizontal circular cylinder. They considered two-dimensional, unsteady, viscous incompressible fluid-flow phenomena. Further, they included oscillation by assuming the external fluid velocity as $u = U(\tau)$ far from the surface of a cylinder. They used the mathematical model as proposed by Glauert (1964) and Davies (1963) as given in Eqs. (4), (6), and (7), and extended this model by coupling the energy equations. They considered the following boundary conditions:

$$u = v = 0,\ H_x = H_y = 0 \text{ and } T = T_w \text{ at } y = 0$$

$$u \to U(\tau), T \to T_\infty, H_y \to H_0.$$

They reported that the phase angle of skin friction is decreased and there is no prominent change in the case of heat transfer and current density noted for the increasing values of magnetic force parameter. They also reported that the same mechanism is observed for the different values of magnetic Prandtl number.

4. CONCLUSION

We summarize the above study into the following points:

- The magnetic field behavior in momentum and thermal boundary layers, skin friction, heat transfer, and magnetic flux depends on magnetic force parameter and magnetic number.
- In different studies from classical to recent it is concluded that the phase angle of skin friction is decreased and no prominent change in the case of heat transfer and current density is noted for the increasing values of magnetic force parameter.
- For increasing values of magnetic Prandtl number the coefficient of skin friction is increased and magnetic intensity at the surface is decreased.
- Further, during the suction process for enhancing values of magnetic force parameter the velocity, temperature, and magnetic field quantities are dropped.
- It is also observed that the skin friction is reduced for the increasing values of magnetic force parameter versus transpiration along a non-magnetized surface.
- Further, it is also concluded that for the increasing values of magnetic force parameter the Nusselt number is decreased.

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