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Study of double-diffusive gravity modulated biothermal convection in porous media under internal heating effect

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Abstract Double diffusion combined with thermo-bioconvection in porous media under two prominent effects such as gravity modulation and internal heating are considered in the present work. The impact of considering double diffusion and porous medium in the current problem is studied graphically. We deal with linear and weakly nonlinear theory of the system. Linear theory helps in the analysis of onset of convection. The stability of the system is also discussed in this section. Onset of convection is governed by critical(threshold) Rayleigh number Ra_c. The marginal stability curves are plotted between critical Rayleigh number and wavenumber for all the parameters that exist in the study which helps in analyzing the stability of the system. Weakly nonlinear stability analysis is carried out to study heat and mass transfer in the system. Due to the gravity modulation, there arise amplitude and frequency for the corresponding fluid under convection. From such nonlinear study, we arrive at an amplitude equation called Ginzburg–Landau (GL) equation. Further, on solving GL equation, we discuss heat and mass transfer in terms of Nusselt number *Nu* and Sherwood number *Sh*. The graphical study of heat and mass transfer is performed by plotting *Nu* versus time scale τ and *Sh* against τ, respectively, for various parameters existing in the study. The convection cells arise due to temperature difference between the horizontal plates are shown in the form of streamlines and isotherms.

List of symbols

h	Thickness of the porous layer
T_{d}	Temperature at lower boundaries
T_{u}	Temperature at upper boundaries
g_0	Mean gravity
$\vec{g}$	Acceleration due to gravity
e	Unit vector in the direction of z-axis
δ	Amplitude of gravity modulation
ω_{g}	Frequency of gravity modulation
Ω	Non-dimensional frequency of gravity modulation
ε	Small dimensionless parameter
$\mathbf{V}_{\mathrm{D}}$	Darcy velocity
V	Velocity along y-axis
ϵ	Porosity of the porous medium
ν	Kinematic viscosity $\left(\frac{\mu}{\rho_0}\right)$
T	Temperature
T_0	Reference temperature
ρ_0	Density of the fluid at reference temperature
K	Permeability of the porous medium
P	Pressure

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β Thermal expansion coefficient
$\tilde{\mu}$ Brinkman effective viscosity
μ Fluid viscosity
$(\rho c)_\mathrm{f}$ Heat capacity of the fluid
$(\rho c)_\mathrm{m}$ Effective heat capacity
k_m Effective thermal conductivity
ϕ Concentration of microorganisms
$\delta\rho$ Density difference
Υ Average microorganism volume
D_m Diffusivity of microorganisms

1 Introduction

Numerous practical applications in different areas of study can be noticed on analyzing the fluid flow in porous media. Scientists and academicians are becoming interested in a new field of study called bioconvection in porous media that has recently arisen. The term bioconvection describes a collective dynamical convective event that takes place in a homogeneous suspension in which a fluid medium that is slightly less dense than the microorganisms is mixed with a non-heterogeneous mixture that comprises a high number of small, self-propelled microorganisms. The movement of bacteria and microorganisms is the subject of bioconvection, particularly in oil production technology. Thus, theoretical research on the interplay between bioconvection and natural convection is necessary. In this work, we examine the issue of porous media and talk about how it affects the migration of microbes under internal heating effect.

The term bioconvection was introduced by Platt [1], who also examined the moving polygonal patterns in Tetrahymena dense cultures. The bioconvection has been addressed in terms of Rayleigh–Taylor instability by Plesset and Winet [2]. The theoretical bioconvective model for gyrotactic bacteria was presented by Pedley et al. [3]. Later, in a finite depth layer, Hill et al. [4] investigated the growing number of microbial biothermal convection patterns.

Understanding the motion and importance of bioconvection in its interaction is crucial for the research of this process. Up till now, a large number of researchers have created computational simulations on thermal convection in porous media. Chandrashekhar [5], Drazin and Reid [6], Vafai [7] and Childress et al. [8] are the first who have modeled a thorough theory for gravitactic microorganism bioconvection. Also numerous scholars have created mathematical models on the concept of heat convection in fluid and concept of porous media. Ingham and Pop [9] conducted a thorough investigation on fluid layer instability caused by gravity within a porous material. Regarding porous media, Nield and Bejan [10] wrote about internal free convection. Vadasz [11] offered a thorough analysis of heat transport resulting from fluid movement in rotating porous media. The works described above primarily focus on the system's thermal instability. Comparable studies were carried out regarding natural convection in fluid-filled porous media, considering several parameters such as magnetic field, gravitational modulation and rotational modulation, among others. Zhao et al. [12] examined the same idea with changing gravity. The fluid's chaotic character is emphasized here. Bhadauria et al. [13] examined temperature modulation and gravity by taking into account a rotating porous material. Additionally, Bhadauria et al. [14] examined nonlinear thermal instability with the same temperature and gravity modulation in a rotating viscous fluid layer. This leads to a great deal of issues with the research of bioconvection in highly porous media, which is examined on the viscous fluid layer under the same temperature and gravity modulation similar to Darcy–Brinkman model. Regarding this, a great deal of issues with the Darcy–Brinkman model-based investigation of bioconvection in high porosity media came up.

The porous material can generate its own heat in a variety of real-world scenarios. This offers a different technique for creating a convective flow by generating heat locally inside the porous material.

The phenomenon known as bioconvection occurs when self-propelled microorganism species with densities higher than those of the surrounding fluid medium generate convective patterns. Taxis is the adaptable movement of living things in response to a variety of environmental indications, such as light, gravity, the availability of food, chemicals, etc. Based on the type of stimulus and whether the organism moves toward or away from it, taxis can be categorized. The term positive taxis describes the attraction that arises when a cell or organism moves in the direction of the stimulus. Conversely, moving away from the stimulant is described by negative taxis, often known as repulsion. The term gyrotactic microorganisms refers to the direction of an organism's movement in reaction to viscous and gravitational forces. In this research, we concentrate on this phenomena. Childress et al. [8] created the hypothesis and mathematical model of gravitactic microorganism bioconvection. The pattern created by the suspended microbes is explained by this notion.

Hill et al. [4] presented a theoretical bioconvective model designed specially for gravitactic microorganisms. The gyrotactic response of the microbes to break at the inflexible layer boundaries results in the existence of oscillatory or overstable modes. The theory for linear stability was created by Pedley et al. [15] to analyze gyrotactic microorganism stability-based bioconvection in a thin film of a simple fluid which acts as prerequisites for the essential start of bioconvection. The name for the spontaneous macroscopic convective fluid motion is bioconvection. The bioconvection phenomenon is caused by the interplay between microbes on different physical scales. Various types of microorganisms must be present for directed movement to occur in different bioconvection systems.

Microorganisms that move in water against gravity are known as gyrotactic microorganisms. These gyrotactic bacteria swim in a specific direction, which increases the base fluid density. Biotechnology and biosensors depend on bioconvection. Unstable stratification of density, which happens when biological entities have a density greater than the fluids around them, gives birth to bioconvection. These organisms build up and create instability, which causes distinct flow patterns to advance and makes the top layer denser than the bottom layer [16, 17]. Kuznetsov and Avramenko [18] reported on the bioconvection stability investigation of bacteria with gyrotactic motion in a porous medium saturated by fluid. Nield et al. [19] are a pioneer to examine bioconvection in a horizontal Darcy porous medium. The critical values of the Rayleigh number and the corresponding critical Rayleigh number for bioconvection are found for a variety of values of the bioconvection Peclet, gyrotaxis and cell eccentricity numbers. Kuznetsov [20] examined bio-nanoconvection, which adds suspended microorganisms to the nanofluid to increase system stability which is an extension of Nield et al. [19] problem. It is found that the inclusion of nanoparticles can either boost or decrease stability based on the top-heavy or bottom-heavy nature of the underlying nanoparticle distribution. Sharma and Kumar [21] employed analytical and quantitative methods to study the effects of high-frequency vertical oscillations on bioconvection in a dilute gyrotactic acid. Their conclusions point that the bioconvection Peclet number and low-amplitude high-frequency vertical vibrations stabilize the system. The main characteristics of bioconvection in porous media and nanofluids were thoroughly analyzed by Dmitrenko [22], who also presented a mathematical model based on the application of Darcy's law. In the past few decades, porous medium research has made extensive use of the Darcy–Brinkman model. The model's use was expanded by Zhao et al. [23], who examined thermo-bioconvection in a very porous media while taking a suspension of gyrotactic microorganisms into consideration. In order to investigate how thermo-bioconvection behaves when heated from below, they performed a stability study. A more thorough examination of the stability of shallow layers containing gyrotactic microorganisms swimming at random was carried out by Kushwaha et al. [24]. Recently, Arpan et al. [25] investigated the stability of thermo-bioconvection into an anisotropic porous fluid layer saturated with Jeffrey liquid, which is generated by gravitactic bacteria. In terms of thermophoresis and Brownian motion properties, Imran et al. [26] provided the rheological correlations between bioconvection and Buongiorno's nanofluid.

Further, Belabid and Allali [27] have examined how microorganisms affect thermal bioconvection. The results of the numerical simulations indicate that the microorganisms' presence increases heat transfer and destabilizes natural convection. Khan et al. [28] examined the flow of thermally generated magnetized couple-stress nanoparticles. They include flow characteristics including activation energy, chemical reaction, and radiation issues using a theoretical bioconvection model. The improved thermal property of a stretched surface bidirectional oscillation couple-stress nanofluid containing mobile microbes was examined by Aziz et al. [29]. They examined heat generation and thermal radiation in their analysis of thermal transportation. Kopp et al. [30] studied thermal convection to improve thermal characteristics as a result of heat transfer in porous media saturated with a blend of nanofluids and microbes. Azam [31] reported on the bioconvection flow of chemically reactive Sutterby nanofluid in the presence of nonlinear radiation and gyrotactic microbes.

The effects of gravity modulation on microorganisms that are oxytactic in a shallow fluid layer heated from below enable thermal bioconvection were investigated by Kuznetsov [32–34]. It has been shown that heating from below increases system instability and promotes the growth of bioconvection. Bioconvection with double diffusion in a porous media that is saturated with nanofluid was studied analytically and numerically by Saini and Sharma [35]. It is discovered that the Rayleigh number Ra decreases when gyrotactic microorganisms are present, indicating that convection occurs earlier than in nanofluid free of microbes. Their results demonstrate that when the concentration of bacteria and the bioconvection Peclet number increase, a system becomes unstable. Increasing the thermal anisotropy strength for low microbe concentrations causes the system to stabilize and the convective cell size to grow. The impact of g-jitter on weakly nonlinear biothermal convection in a porous medium has been documented by Kopp and Yanovsky [36]. They looked into the effects of gravity fluctuations on bio-porous convection, where the bio-convective cell is exclusively modeled as being controlled by vertical vibrations. Akhila et al. [37] recently reported on the influence of internal heating on bio-Darcy convection. It is discovered that in the presence of gravity modulations, bioconvection is even regulated by internally heated layers. According to their research, microbes can swim toward downwelling fluid when viscous and gravitational torques combine [38]. However, there is no documentation on what occurs when they trip over the limits. Whether or if the boundary's uneven heating could affect the convection of the swimming microorganisms. Recently, Kiran and Manjula [39, 40] made an investigation on the same gravity modulation in oscillatory convection for nanofluids and temperature modulation on bioconvection in porous media. The effect of gravity modulation in nanofluids and viscoelastic nanofluids in rotating porous medium with internal heating and throughflow was investigated in [41–47]. The goal of the current work is to explain bioconvection when plate modulation is present. It is clear from the literature that very limited research has been done on the addition of gravitactic microorganisms to a rotating porous layer that is being modulated by gravity.

Consequently, our motivation emerged from the aim of comprehending the system dynamics through a more profound understanding of the interaction of gravitaxis, gyrotaxis and thermal convection in a rotating porous medium under gravity modulation. This work aims to investigate the behavior of weakly nonlinear thermo-bioconvection in Newtonian fluid containing gyrotactic organism within a porous medium. Using the Ginzburg–Landau (GL) model, the study focuses on the consequences of modulation in the gravitational field with internal heating effect. This multidisciplinary area of research has a wide range of applications in the current scenario. This aims to connect mathematical model for physical and biological situations.

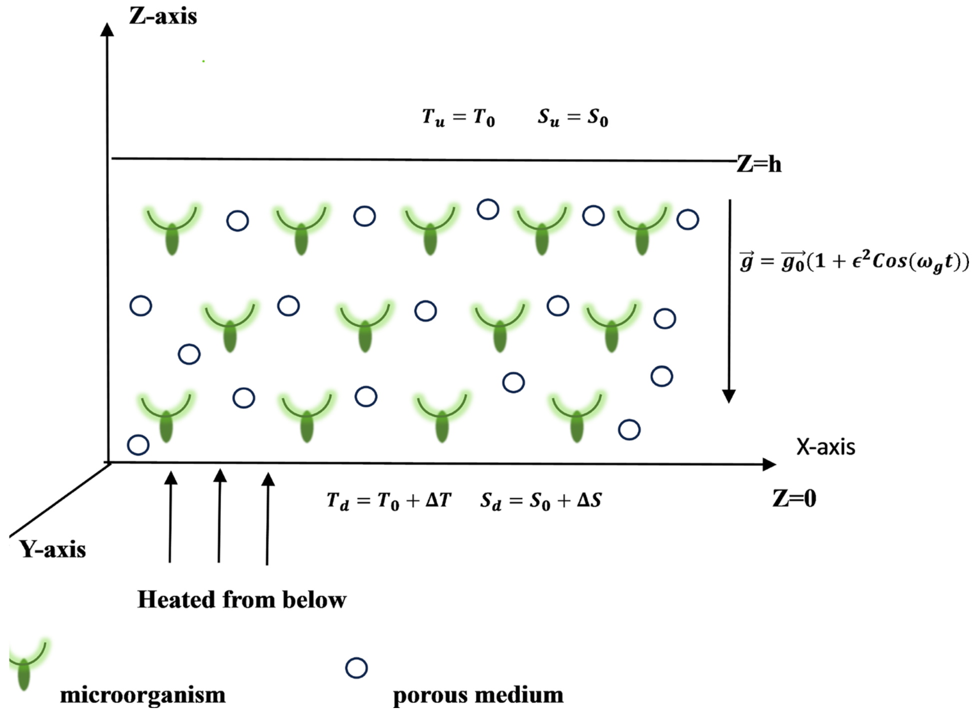

Fig. 1 Geometry of the problem

2 Problem statement and mathematical formulation

An incompressible viscous fluid containing gyrotactic microorganisms flowing in a porous medium whose layer is of thickness h between infinite horizontal plates is considered in the current investigation. We examine the horizontal laminar flow between thermally conductive boundaries. The boundaries are impermeable in nature. The bottom of the horizontal plate is heated. The temperaure and solutal concentration at the lower and upper boundaries are $(T_0 + \Delta T,\ S_0 + \Delta S)$ and $(T_0,\ S_0)$, respectively, as shown in Fig. 1. The gyrotactic microorganisms can move easily inside a Newtonian fluid saturated porous medium. The $z-$axis is pointing in the vertically upward direction whereas the gravity field is pointing downward in the direction opposite to $z-$axis. The gravity modulation is represented by $\mathbf{e}g_0(1 + \varepsilon^2\delta \text{ Cos } (\omega_g t))$ where $\mathbf{e}$ is a unit vector in the z direction, g_0 is the magnitude of the average gravitational field vector, ω_g is the gravity modulation frequency and δ is its amplitude. Boussinesq approximation is assumed to be true throughout the system, and Darcy–Brinkman model is employed in governing equations. The governing equations is as shown below [30, 36] and [37]

$$\nabla \cdot \mathbf{V}_D = 0 \tag{1}$$

$$\frac{\rho_0}{\epsilon}\frac{\partial \mathbf{V}_D}{\partial t} = -\nabla P + \tilde{\mu}\nabla^2\mathbf{V}_D - \frac{\mu}{K}\mathbf{V}_D - \mathbf{e}g(t)\rho_0(1 - \beta_1(T - T_0) + \beta_2(S - S_0)) - \mathbf{e}g(t)(\delta\rho)\Upsilon\phi \tag{2}$$

$$(\rho c)_m \frac{\partial T}{\partial t} + (\rho c)_f \mathbf{V}_D \cdot \nabla T = k_m \nabla^2 T + Q(T - T_0) \tag{3}$$

$$\frac{\partial \phi}{\partial t} = -\text{div}(\phi \mathbf{V}_D + \phi W_c \hat{\mathrm{I}} - D_m \nabla \phi) \tag{4}$$

$$\frac{\partial S}{\partial t} + \mathbf{V}_D \cdot \nabla S = k_s \nabla^2 S \tag{5}$$

$$g(t) = g_0(1 + \varepsilon^2\delta \text{ Cos } (\omega_g t)) \tag{6}$$

Here $\mathbf{V}_D = (u,\ v,\ w)$ is Darcy velocity in relation with fluid velocity as $\mathbf{V}_D = \epsilon\mathbf{V}$, where ϵ is the porosity of the porous medium. μ and $\tilde{\mu}$ in Eq. 2 represent fluid viscosity and Brinkman effective viscosity. Whereas, K is permeability of porous medium. The acceleration due to gravity is given by $g(t)$, with density ρ_0 at reference temperature T_0 and reference solutal concentration S_0. Thermal expansion and solutal expansion coefficients are given by β_1 and β_2, while the pressure is represented by P. The heat capacity of the fluid is $(\rho c)_f$ and the effective heat capacity is $(\rho c)_m$, respectively. Additionally, microorganism concentration is ϕ, and then, the difference in density of microorganisms and a base fluid is denoted by $\delta\rho$ which is $\rho_m - \rho_f$ while the mean microbial volume is Υ. Furthermore, the thermal diffusivity is k_m, and solutal diffusivity is k_s in the system. The diffusive nature of microorganisms is defined by D_m, and average velocity of the microorganism $W_c\hat{\mathrm{I}}(0)$ is a constant, and Q is internal heating coefficient.

The following are the boundary conditions [36, 37]

$$w = 0, \quad T = T_d, \quad S = S_d, \quad \mathbf{J} \cdot \mathbf{e} = 0, \quad \text{at} \quad z = h, \tag{7}$$

$$w = 0, \quad T = T_u, \quad S = S_u, \quad \mathbf{J} \cdot \mathbf{e} = 0, \quad \text{at} \quad z = 0, \tag{8}$$

where $\mathbf{J} = \phi \frac{\mathbf{V}_D}{\epsilon} + \phi W_c \hat{I} - D_m \nabla \phi$ is the microorganism flux. We introduce the non-dimensional parameters to analyze the problem, like [36]

$$(x^*, y^*, z^*)h = (x, y, z), \quad \alpha_m \mathbf{V}_D^* = \mathbf{V}_D h, \quad h^2 \tilde{\sigma} t^* = t \alpha_m, \quad (T_d - T_u)T^* = T - T_u,$$
$$(S_d - S_u)S^* = S - S_u, \quad \mu \alpha_m P^* = PK, \quad \tilde{\sigma} = \frac{(\rho c)_m}{(\rho c)_f}, \quad \phi^* = \phi \Upsilon, \quad \omega_g^* = \omega_g \frac{h^2 \tilde{\sigma}}{\alpha_m} \tag{9}$$

$\alpha_m = \frac{k_m}{(\rho c)_f}$ is the corresponding thermal diffusivity coefficient. We obtain the following system of dimensionless equations by using expression (9) and leaving off the asterisks.

$$\nabla \cdot \mathbf{V}_D = 0 \tag{10}$$

$$\frac{1}{\Upsilon_a} \frac{\partial \mathbf{V}_D}{\partial t} = -\nabla P + D_a \nabla^2 \mathbf{V}_D - \mathbf{V}_D - \mathbf{e} f_m \frac{\mathrm{R}_b}{L_b} \phi + \mathbf{e} f_m \mathrm{Ra} T - \mathbf{e} f_m \mathrm{Ra}_s S \tag{11}$$

$$\frac{\partial T}{\partial t} + (\mathbf{V}_D \cdot \nabla) T = \nabla^2 T + \mathrm{R}_i T \tag{12}$$

$$\frac{1}{\tilde{\sigma}} \frac{\partial \phi}{\partial t} = -\nabla \left(\phi \mathbf{V}_D + \frac{\mathrm{Pe}}{L_b} \phi \hat{\mathrm{I}}(t) - \frac{1}{L_b} \nabla \phi \right) \tag{13}$$

$$\frac{1}{\tilde{\sigma}} \frac{\partial S}{\partial t} + (\mathbf{V}_D \cdot \nabla) S = \frac{1}{Le} \nabla^2 S \tag{14}$$

where $f_m = 1 + \varepsilon^2 \delta \, \mathrm{Cos}\, (\omega_g t)$. In Eqs. (10)–(14), we have the subsequent dimensionless parameters such as $\Upsilon_a = \frac{\epsilon (\rho c)_m \tilde{\mu}}{\rho_0 k_m D_a} = \frac{\epsilon \tilde{\sigma} \mathrm{Pr}}{D_a}$, $\mathrm{Pr} = \frac{\tilde{\mu}}{\alpha_m \rho_0}$, $D_a = \frac{\tilde{\mu} K}{\mu h^2}$, $\mathrm{R}_b = \frac{g(\delta\rho) h K}{\mu D_m}$, $L_b = \frac{\alpha_m}{D_m}$, $\mathrm{Ra} = \frac{\rho_0 g h K \beta \Delta T}{\mu \alpha_m}$, $\mathrm{Ra}_s = \frac{\rho_0 g h K \beta_2 \Delta S}{\mu \alpha_m}$, $\mathrm{Pe} = \frac{W_c h}{D_m}$, $\mathrm{R}_i = \frac{Q h^2}{k_m}$ and $Le = \frac{\alpha_m}{k_s}$. These are named as modified Vadasz number, Prandtl number, Darcy number, bioconvection Rayleigh–Darcy number, bioconvection Lewis number, Rayleigh–Darcy number, solutal Rayleigh number, bioconvection Peclet number and internal Rayleigh number, respectively.

Non-dimensional boundary conditions are added to Eqs. (10)–(14) as follows: [36, 37]

$$w = 0, \quad T = 1, \quad S = 1, \quad \phi \mathrm{Pe} = \frac{d\phi}{dz}, \quad \text{at} \quad z = 0, \tag{15}$$

$$w = 0, \quad T = 0, \quad S = 0, \quad \phi \mathrm{Pe} = \frac{d\phi}{dz}, \quad \text{at} \quad z = 1, \tag{16}$$

2.1 Conduction (Basic) state

The basic or conduction state is considered to be time-independent and is as follows:

$$\mathbf{V}_D = \mathbf{V}_b = 0, \quad P = P_b(z), \quad \rho = \rho_b(z), \quad \phi = \phi_b(z), \quad T = T_b(z), \quad S = S_b(z) \tag{17}$$

Thus, the following equations must be computed in order to get the steady temperature profiles $T_b(z)$, microbial concentration $\phi_b(z)$ and pressure distribution $P_b(z)$ in the basic state:

$$\frac{d^2 T_b}{dz^2} + \mathrm{R}_i T_b(z) = 0 \tag{18}$$

$$\frac{d^2 S_b}{dz^2} = 0 \tag{19}$$

$$\frac{d\phi_b}{dz} = \phi_b(z) \mathrm{Pe} \tag{20}$$

$$\frac{dP_b}{dz} = -\frac{\mathrm{R}_b}{L_b} \phi_b(z) + \mathrm{Ra} T_b(z) - \mathrm{Ra}_s S_b(z) \tag{21}$$

The temperate distribution $T_b(z)$ and concentration distribution $S_b(z)$ are obtained on integrating Eqs. (18) and (19), respectively, and applying the boundary conditions in Eqs. (15 and 16).

$$T_b(z) = \frac{\mathrm{Sin}\, (\sqrt{\mathrm{R}_i}(1 - z))}{\mathrm{Sin}\, \sqrt{\mathrm{R}_i}} \tag{22}$$

$$S_b(z) = 1 - z \tag{23}$$

Moreover, $\phi_b(z)$ has a solution that we obtain as

$$\phi_b(z) = \phi_b(0)\exp(z\text{Pe}) \tag{24}$$

where the density at the layer's bottom is expressed as $\phi_b(0)$. The value of $\phi_b(0)$ is determined as follows:

$$\phi_b(0) = \frac{\langle\phi\rangle\text{Pe}}{\exp(\text{Pe}) - 1} \quad , \quad \langle\phi\rangle = \int_0^1 \phi_b(z)dz \tag{25}$$

Equation (20) shows that $\phi_b(z)$ is roughly constant in the layer in the situation of small Peclet number Pe ($\phi_b(z) \approx \phi_0$). We consider this instance in order to simplify the analysis.

Presuming that $P = P_0$ at $z = 1$, we determine the pressure distribution in the fundamental state as

$$P_b(z) = P_0 + \frac{\text{R}_b}{L_b\text{Pe}}\phi_b(0)(e^{\text{Pe}} - e^{z\text{Pe}}) + \frac{\text{Ra}}{\sqrt{\text{R}_i}\ \text{Sin}\,(\sqrt{\text{R}_i})}\Big(\text{Cos}\,(\sqrt{\text{R}_i}(1-z)) - 1\Big) - \text{Ra}_s\Big(z - \frac{z^2+1}{2}\Big) \tag{26}$$

2.2 Perturbed state

Small disruptions in the primary flow are caused by the heating from below to the fluid layer, and these can be stated as follows:

$$\mathbf{V}_D = \mathbf{V}', \quad T = T_b(z) + T', \quad S = S_b(z) + S', \quad P = P_b(z) + P', \quad \phi = \phi_b(z) + \phi', \quad \hat{\text{I}}(t) = \mathbf{e} + \hat{\mathbf{m}}'(t) \tag{27}$$

The perturbation of the unit vector, which represents the direction in which the microbes are swimming, can be described using the following equation by taking gravity modulation into account

$$\hat{\mathbf{m}}'(t) = \mathcal{B}_0(1 + \varepsilon^2\delta\ \text{Cos}\,(\omega_g t))\zeta\mathbf{i} - \mathcal{B}_0(1 + \varepsilon^2\delta\ \text{Cos}\,(\omega_g t))\xi\mathbf{j} + 0\cdot\mathbf{e} \tag{28}$$

Here, the corresponding unit vectors in x and y directions are denoted by $\mathbf{i}$ and $\mathbf{j}$. In the absence of modulation, the dimensionless parameter $\mathcal{B}_0$ describes the reorientation of microorganisms in relation to viscous resistance as a result of a gravitational moment is given by $\mathcal{B}_0 = (\frac{\mu\alpha_\perp}{\rho_0 g_0 d})(\frac{\alpha_m}{h^2})$. The ζ and ξ in the x and y elements of vector $\hat{\mathbf{m}}'$ are found in Eq. (28) which are given as follows: [30, 36, 37]

$$\zeta = -(1-\alpha_0)\frac{\partial w'}{\partial x} + (1+\alpha_0)\frac{\partial u'}{\partial z} \tag{29}$$

$$\xi = (1-\alpha_0)\frac{\partial w'}{\partial y} - (1+\alpha_0)\frac{\partial v'}{\partial z} \tag{30}$$

The cell eccentricity (α_0) is determined as $\alpha_0 = \frac{r_1^2 - r_2^2}{r_1^2 + r_2^2}$ in which the semi-major and semi-minor axes of the spherical cell are denoted by r_1 and r_2, respectively. The expressions for variables $\mathbf{V}'$, T', ϕ' and S' are as follows if we replace expressions from (24) into Eqs. (10)–(14).

$$\nabla\cdot\mathbf{V}' = 0 \tag{31}$$

$$\frac{1}{\Upsilon_a}\frac{\partial\mathbf{V}'}{\partial t} = -\nabla P' + D_a\nabla^2\mathbf{V}' - \mathbf{V}' - \mathbf{e}f_m\frac{\text{R}_b}{L_b}\phi' + \mathbf{e}f_m\text{Ra}T' - \mathbf{e}f_m\text{Ra}_sS' \tag{32}$$

$$\frac{\partial T'}{\partial t} + w'\frac{dT_b}{dz} + (\mathbf{V}'\cdot\nabla)T' = \nabla^2T' + \text{R}_iT' \tag{33}$$

$$\frac{1}{\tilde{\sigma}}\frac{\partial\phi'}{\partial t} = -\nabla\big(\phi'\mathbf{V}'\big) - w'\frac{d\phi_b}{dz} - \frac{\text{Pe}}{L_b}\frac{\partial\phi'}{\partial z} - \frac{1}{L_b}\nabla^2\phi' + \text{PeG}_0\phi_b(1 - \varepsilon^2\delta\ \text{Cos}\,(\omega_g t))\Lambda \tag{34}$$

$$\frac{1}{\tilde{\sigma}}\frac{\partial S'}{\partial t} + w'\frac{dS_b}{dz} + (\mathbf{V}'\cdot\nabla)S' = \frac{1}{Le}\nabla^2S' \tag{35}$$

where,

$$\Lambda = (1+\alpha_0)\frac{d^2w'}{dz^2} + (1-\alpha_0)\left(\frac{\partial^2w'}{\partial x^2} + \frac{\partial^2w'}{\partial y^2}\right)$$

$\text{G}_0 = D_m\mathcal{B}_0/h^2$ is a dimensionless orientation parameter when modulation is not present.

We define velocity in terms of stream function ψ for the 2-D flow model.

$$u' = \frac{\partial\psi}{\partial z} \quad \text{and} \quad w' = -\frac{\partial\psi}{\partial x} \tag{36}$$

We derive the following non-dimensionsal governing equations (on removal of asterisks) after substituting Eq. (36) into Eqs. (31)–(35) then applying the obtained outcomes into the conduction state by omitting the pressure:

$$\left(\frac{1}{\Upsilon_a}\frac{\partial}{\partial t} + 1 - D_a\nabla^2\right)\nabla^2\psi = f_m\frac{\text{R}_b}{L_b}\frac{\partial\phi'}{\partial x} - f_m\text{Ra}\frac{\partial T'}{\partial x} + f_m\text{Ra}_s\frac{\partial S'}{\partial x} \tag{37}$$

$$\frac{\partial \psi}{\partial x} - (\nabla^2 + \mathrm{R}_i)T' = -\frac{\partial T'}{\partial t} + \frac{\partial(\psi, T')}{\partial(x,z)} \tag{38}$$

$$\mathrm{PeG}_0(2 - f_m)\phi_0\hat{\alpha}\frac{\partial \psi}{\partial x} + \frac{\mathrm{Pe}}{L_b}\frac{\partial \phi'}{\partial z} - \frac{1}{L_b}\nabla^2\phi' = -\frac{1}{\tilde{\sigma}}\frac{\partial \phi'}{\partial t} + \frac{\partial(\psi, \phi')}{\partial(x,z)} \tag{39}$$

$$\frac{\partial \psi}{\partial x} - \frac{1}{Le}\nabla^2 S' = -\frac{1}{\tilde{\sigma}}\frac{\partial S'}{\partial t} + \frac{\partial(\psi, S')}{\partial(x,z)} \tag{40}$$

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}, \qquad \hat{\alpha} = \nabla^2 + \alpha_0\left(\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial z^2}\right)$$

Focusing on the stationary biothermal convective mode in the system, we rescale it by setting $\tau = \varepsilon^2 t$ using a small time variation. It is possible to express the nonlinear systems of Eqs. (37)–(40) in matrix form as below:

$$\begin{bmatrix} \nabla^2 - D_a\nabla^4 & f_m\mathrm{Ra}\frac{\partial}{\partial x} & -f_m\frac{\mathrm{R}_b}{L_b}\frac{\partial}{\partial x} & -f_m\mathrm{Ra}_s\frac{\partial}{\partial x} \\ \frac{\partial}{\partial x} & -(\nabla^2 + \mathrm{R}_i) & 0 & 0 \\ \mathrm{PeG}_0(2 - f_m)\phi_0\hat{\alpha}\frac{\partial}{\partial x} & 0 & \frac{\mathrm{Pe}}{L_b}\frac{\partial}{\partial z} - \frac{1}{L_b}\nabla^2 & 0 \\ \frac{\partial}{\partial x} & 0 & 0 & -\frac{1}{Le}\nabla^2 \end{bmatrix} \begin{bmatrix} \psi \\ T \\ \phi \\ S \end{bmatrix} = \begin{bmatrix} -\frac{\varepsilon^2}{\Upsilon_a}\frac{\partial}{\partial \tau}\nabla^2\psi \\ -\varepsilon^2\frac{\partial T}{\partial \tau} + \frac{\partial(\psi,T)}{\partial(x,z)} \\ -\frac{\varepsilon^2}{\tilde{\sigma}}\frac{\partial \phi}{\partial \tau} + \frac{\partial(\psi,\phi)}{\partial(x,z)} \\ -\frac{\varepsilon^2}{\tilde{\sigma}}\frac{\partial S}{\partial \tau} + \frac{\partial(\psi,S)}{\partial(x,z)} \end{bmatrix} \tag{41}$$

In order to solve the system represented by Eq. (41), impermeable boundary conditions must be taken into account.

$$\psi = \nabla^2\psi = \phi = T = S = 0 \quad \text{on} \quad z = 0 \quad z = 1 \tag{42}$$

3 Weakly nonlinear stability

In order to examine the stationary convective instability, we use the following asymptotic expressions to Eq. (41).

$$\mathrm{Ra} = \mathrm{Ra}_c + \varepsilon^2\mathrm{Ra}_2 + \varepsilon^4\mathrm{Ra}_4 + \ldots, \tag{43}$$

$$\psi = \varepsilon\psi_1 + \varepsilon^2\psi_2 + \varepsilon^3\psi_3 + \ldots, \tag{44}$$

$$T = \varepsilon T_1 + \varepsilon^2 T_2 + \varepsilon^3 T_3 + \ldots, \tag{45}$$

$$\phi = \varepsilon\phi_1 + \varepsilon^2\phi_2 + \varepsilon^3\phi_3 + \ldots, \tag{46}$$

$$S = \varepsilon S_1 + \varepsilon^2 S_2 + \varepsilon^3 S_3 + \ldots \tag{47}$$

The critical Rayleigh number, Ra_c, denotes the point at which convection gets innitiated when gravity modulation is absent. Then, the system is solved for various orders of ε.

4 First-order system analysis

Here, we reduce the system to a linear model with minimal nonlinear effects at the smallest order of ε. The following is an expression for the system in matrix form:

$$\begin{bmatrix} \nabla^2 - D_a\nabla^4 & \mathrm{Ra}_c\frac{\partial}{\partial x} & -\frac{\mathrm{R}_b}{L_b}\frac{\partial}{\partial x} & -\mathrm{Ra}_s\frac{\partial}{\partial z} \\ \frac{\partial}{\partial x} & -(\nabla^2 + \mathrm{R}_i) & 0 & 0 \\ \mathrm{PeG}_0\phi_0\hat{\alpha}\frac{\partial}{\partial x} & 0 & \frac{\mathrm{Pe}}{L_b}\frac{\partial}{\partial z} - \frac{1}{L_b}\nabla^2 & 0 \\ \frac{\partial}{\partial x} & 0 & 0 & -\frac{1}{Le}\nabla^2 \end{bmatrix} \begin{bmatrix} \psi_1 \\ T_1 \\ \phi_1 \\ S_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{48}$$

For the boundary conditions considered in Eq. (42), the solution to the first-order system is as follows: [30, 36, 37]

$$\begin{aligned} \psi_1 &= A(\tau)\ \mathrm{Sin}\ (k_c x)\ \mathrm{Sin}\ (\pi z) \\ T_1 &= -\frac{A(\tau)k_c}{a^2 - \mathrm{R}_i}\ \mathrm{Cos}\ (k_c x)\ \mathrm{Sin}\ (\pi z) \quad , \quad a^2 = k_c^2 + \pi^2 \\ \phi_1 &= -\frac{k_c}{a^2}\mathrm{PeG}_0\phi_0 L_b((1 - \alpha_0)k_c^2 + (1 + \alpha_0)\pi^2)A(\tau)\ \mathrm{Cos}\ (k_c x)\ \mathrm{Sin}\ (\pi z) \\ S_1 &= -\frac{A(\tau)k_c Le}{a^2}\ \mathrm{Cos}\ (k_c x)\ \mathrm{Sin}\ (\pi z) \end{aligned} \tag{49}$$

The expression for stationary Rayleigh number Ra_c is as given below:

$$\mathrm{Ra}_c = \frac{a^2(1 + D_a a^2)(a^2 - \mathrm{R}_i)}{k_c^2} + \mathrm{PeG}_0\phi_0\mathrm{R}_b((1-\alpha_0)k_c^2 + (1+\alpha_0)\pi^2)\left(1 - \frac{\mathrm{R}_i}{a^2}\right) + Le\mathrm{Ra}_s \tag{50}$$

In order to identify the beginning of convection, we must minimize Ra_c in relation to k_c^2 in order to get the wavenumber k_c. Ra_c can be differentiated with regard to k_c^2, and the derivative can then be set to zero. We then determine the wavenumber for the onset of convection by solving this equation. The threshold stationary Rayleigh number for the Darcy–Brinkman model of a porous medium can be obtained thoroughly from Eq. (50) if the fluid is free of microorganisms.

But when there is no internal heating in the system, we have regular bioconvection, which is brought on by the movement of microbes. Here R_b is the bioconvection governing parameter.

5 Second-order system analysis

The Jacobian expressions, which are nonlinear in nature in the right-hand side of Eq. (37), explain the interaction among fluid velocity, temperature and microbiological diffusivity that introduce the nonlinear effects at this order. The following is an another form for the system of equations in this second order:

$$\begin{bmatrix} \nabla^2 - D_a\nabla^4 & \mathrm{Ra}_c\frac{\partial}{\partial x} & -\frac{\mathrm{R}_b}{L_b}\frac{\partial}{\partial x} & -\mathrm{Ra}_s\frac{\partial}{\partial z} \\ \frac{\partial}{\partial x} & -(\nabla^2 + \mathrm{R}_i & 0 & 0 \\ \mathrm{PeG}_0\phi_0\hat{\alpha}\frac{\partial}{\partial x} & 0 & \frac{\mathrm{Pe}}{L_b}\frac{\partial}{\partial z} - \frac{1}{L_b}\nabla^2 & 0 \\ \frac{\partial}{\partial x} & 0 & 0 & -\frac{1}{Le}\nabla^2 \end{bmatrix} \begin{bmatrix} \psi_2 \\ T_2 \\ \phi_2 \\ S_2 \end{bmatrix} = \begin{bmatrix} N_{21} \\ N_{22} \\ N_{23} \\ N_{24} \end{bmatrix} \tag{51}$$

where,

$$N_{21} = 0, \quad N_{22} = \frac{\partial(\psi_1, T_1)}{\partial(x,z)}, \quad N_{23} = \frac{\partial(\psi_1, \phi_1)}{\partial(x,z)}, \quad N_{24} = \frac{\partial(\psi_1, S_1)}{\partial(x,z)} \tag{52}$$

To obtain the second-order solutions, the first-order solutions are taken from Eq. (49). The second-order solutions considering boundary conditions (42) are as follows:

$$\psi_2 = 0 \tag{53}$$

$$T_2 = \frac{-k_c^2\pi}{2(a^2 - \mathrm{R}_i)(4\pi^2 - \mathrm{R}_i)}A^2(\tau)\ \mathrm{Sin}\ (2\pi z) \tag{54}$$

$$\phi_2 = \frac{-k_c^2 L_b \Pi}{8a^2\pi}A^2(\tau)\ \mathrm{Sin}\ (2\pi z) \tag{55}$$

$$S_2 = \frac{-k_c^2 Le^2 \Pi}{8a^2\pi}A^2(\tau)\ \mathrm{Sin}\ (2\pi z) \tag{56}$$

where, $\Pi = \mathrm{PeG}_0\phi_0 L_b((1-\alpha_0)k_c^2 + (1+\alpha_0)\pi^2)$.

The following expression can be used to assess the heat transfer using Nusselt number $\mathrm{Nu}(\tau)$ for the stationary convective mode:

$$\mathrm{Nu}(\tau) = \frac{\left[\frac{k_c}{2\pi}\int_0^{\frac{2\pi}{k_c}}(\frac{\partial T_2}{\partial z})dx\right]_{z=0}}{\left[\frac{k_c}{2\pi}\int_0^{\frac{2\pi}{k_c}}(\frac{\partial T_b}{\partial z})dx\right]_{z=0}} + 1 \quad \text{and} \quad \mathrm{Sh}(\tau) = \frac{\left[\frac{k_c}{2\pi}\int_0^{\frac{2\pi}{k_c}}(\frac{\partial S_2}{\partial z})dx\right]_{z=0}}{\left[\frac{k_c}{2\pi}\int_0^{\frac{2\pi}{k_c}}(\frac{\partial S_b}{\partial z})dx\right]_{z=0}} + 1$$

$$\mathrm{Nu}(\tau) = \frac{k_c^2\pi^2\ \mathrm{Sin}\ \sqrt{\mathrm{R}_i}}{(a^2 - \mathrm{R}_i)(4\pi^2 - \mathrm{R}_i)\sqrt{\mathrm{R}_i}\ \mathrm{Cos}\ \sqrt{\mathrm{R}_i}}A^2(\tau) + 1 \quad \text{and} \quad \mathrm{Sh}(\tau) = \frac{k_c^2 Le^2}{4a^2}A^2(\tau) + 1 \tag{57}$$

Once the amplitude $A(\tau)$ expression is derived, the heat transfer in the system is analyzed using $\mathrm{Nu}(\tau)$. The asymptotic expansion in Eqs. (41)–(47) makes obvious that the gravity modulation effect becomes important only when ε is taken at its third order. The mean Nusselt number $\tilde{Nu}$ is defined over an appropriate interval $(0, 2\pi)$ for gravity modulation on heat transport.

$$\tilde{Nu} = \frac{1}{2\pi}\int_0^{2\pi}\mathrm{Nu}(\tau)d\tau \tag{58}$$

Mean Nusselt number $\tilde{Nu}$ is evaluated numerically against several parameters, and corresponding graphical analysis is discussed in Fig. 7.

6 Third-order system analysis

In this order, we obtain

$$\begin{bmatrix} \nabla^2 - D_a\nabla^4 & \mathrm{Ra}_c\frac{\partial}{\partial x} & -\frac{\mathrm{R}_b}{L_b}\frac{\partial}{\partial x} & -\mathrm{Ra}_s\frac{\partial}{\partial z} \\ \frac{\partial}{\partial x} & -(\nabla^2+\mathrm{R}_i) & 0 & 0 \\ \mathrm{PeG}_0\phi_0\hat{\alpha}\frac{\partial}{\partial x} & 0 & \frac{\mathrm{Pe}}{L_b}\frac{\partial}{\partial z} - \frac{1}{L_b}\nabla^2 & 0 \\ \frac{\partial}{\partial x} & 0 & 0 & -\frac{1}{Le}\nabla^2 \end{bmatrix} \begin{bmatrix} \psi_3 \\ T_3 \\ \phi_3 \\ S_3 \end{bmatrix} = \begin{bmatrix} N_{31} \\ N_{32} \\ N_{33} \\ N_{34} \end{bmatrix} \tag{59}$$

where,

$$\begin{aligned} N_{31} &= \left[\frac{a^2}{\Upsilon_a}\frac{\partial A(\tau)}{\partial \tau} - \mathrm{Ra}_c\frac{k_c^2 A(\tau)}{a^2-\mathrm{R}_i}\delta\,\mathrm{Cos}\,(\Omega\tau) + \mathrm{Ra}_2\frac{k_c^2 A(\tau)}{a^2-\mathrm{R}_i} + \frac{\mathrm{R}_b}{L_b}\frac{k_c^2 A(\tau)}{a^2}\Pi\delta\,\mathrm{Cos}\,(\Omega\tau)\right. \\ &\quad \left. +\mathrm{Ra}_s\frac{k_c^2 Le A(\tau)}{a^2}\delta\,\mathrm{Cos}\,(\Omega\tau)\right]\mathrm{Sin}\,(k_c x)\,\mathrm{Sin}\,(\pi z) \\ N_{32} &= \left(\frac{k_c}{a^2}\frac{\partial A(\tau)}{\partial \tau} - \frac{k_c^3 A^3(\tau)\pi^2}{(4\pi^2-\mathrm{R}_i)(a^2-\mathrm{R}_i)}\mathrm{Cos}\,(2\pi z)\right)\mathrm{Cos}\,(k_c x)\,\mathrm{Sin}\,(\pi z) \\ N_{33} &= \left(\frac{k_c\Pi}{\tilde{\sigma}a^2}\frac{\partial A(\tau)}{\partial \tau} + \frac{k_c\Pi}{L_b}\delta\,\mathrm{Cos}\,(\Omega\tau)A(\tau) - \frac{k_c^3 A^3(\tau)}{4a^2}\Pi L_b\,\mathrm{Cos}\,(2\pi z)\right)\mathrm{Cos}\,(k_c x)\,\mathrm{Sin}\,(\pi z) \\ N_{34} &= \left(\frac{k_c Le}{\tilde{\sigma}a^2}\frac{\partial A(\tau)}{\partial \tau} - \frac{k_c^3 A^3(\tau)Le^2}{4a^2}\mathrm{Cos}\,(2\pi z)\right)\mathrm{Cos}\,(k_c x)\,\mathrm{Sin}\,(\pi z) \end{aligned}$$

By applying the solvability condition, we can extract the Ginzburg–Landau (GL) equation for the stationary convective mode with time-periodic coefficients from the third-ordered solution.

$$\int_{z=0}^{z=1}\int_{x=0}^{x=\frac{2\pi}{k_c}}\left[\psi_1^* N_{31} + T_1^* N_{32} + \phi_1^* N_{33} + S_1^* N_{34}\right]dxdz = 0 \tag{60}$$

where ψ_1^*, T_1^*, ϕ_1^* and $S_!^*$ are obtained by taking the adjoint of Eq.(48)

$$B_1\frac{\partial A}{\partial \tau} - B_2(\tau)A + B_3 A^3 = 0 \tag{61}$$

where,

$$B_1 = \frac{a^2}{\Upsilon_a} - \mathrm{Ra}_c\frac{k_c^2}{(a^2-\mathrm{R}_i)^2} + \frac{\mathrm{R}_b}{\tilde{\sigma}}\frac{k_c^2}{a^4}\Pi - \frac{\mathrm{Ra}_s}{\tilde{\sigma}}\frac{k_c^2 Le}{a^4} \tag{62}$$

$$B_2(\tau) = \frac{k_c^2}{a^2-\mathrm{R}_i}\mathrm{Ra}_c\left(\frac{\mathrm{Ra}_2}{\mathrm{Ra}_c} - \delta\,\mathrm{Cos}\,(\Omega\tau)\right) + 2\frac{k_c^2}{a^2}\frac{\mathrm{R}_b}{L_b}\Pi\delta\,\mathrm{Cos}\,(\Omega\tau) - \mathrm{Ra}_s\delta\,\mathrm{Cos}\,(\Omega\tau)\frac{k_c^2 Le}{a^2} \tag{63}$$

$$B_3 = k_c^4\left(\frac{\mathrm{Ra}_c\pi^2}{(a^2-\mathrm{R}_i)(4\pi^2-\mathrm{R}_i)} + \frac{\mathrm{R}_b L_b\Pi}{4a^4} - \frac{\mathrm{Ra}_s Le^2}{4a^4}\right) \tag{64}$$

The equation for amplitude given in Eq. (61) is non-autonomous. A built-in function NDSolve of Mathematica has been used to solve it numerically. The initial or the preliminary condition $A(0) = A_0$, where A_0 is some selected value for the amplitude, is used to solve the problem. Since we are interested in the nonlinearity close to the critical stage of convection, we make the assumption that $\mathrm{Ra}_2 \approx \mathrm{Ra}_c$. Therefore, a modest expansion parameter ε^2 in weak nonlinear convection is the relatively minor departure of critical Rayleigh number Ra_c from Ra (Rayleigh number): [36]

$$\varepsilon^2 = \frac{\mathrm{Ra}-\mathrm{Ra}_c}{\mathrm{Ra}_c} << 1$$

The unmodulated case of Eq.(61), which is analytically obtained, is as given below:

$$A_u(\tau) = \frac{A_0}{\sqrt{\frac{B_3}{B_2}A_0^2 + \left(1-\frac{B_3}{B_2}A_0^2\right)\exp\left(\frac{-2\tau B_2}{B_1}\right)}} \tag{65}$$

where $A_u(\tau)$ is the unmodulated amplitude, B_1 and B_3 are as given in Eqs.(62) and (64), while $B_2 = \frac{k_c^2}{a^2-\mathrm{R}_i}\mathrm{Ra}_2$

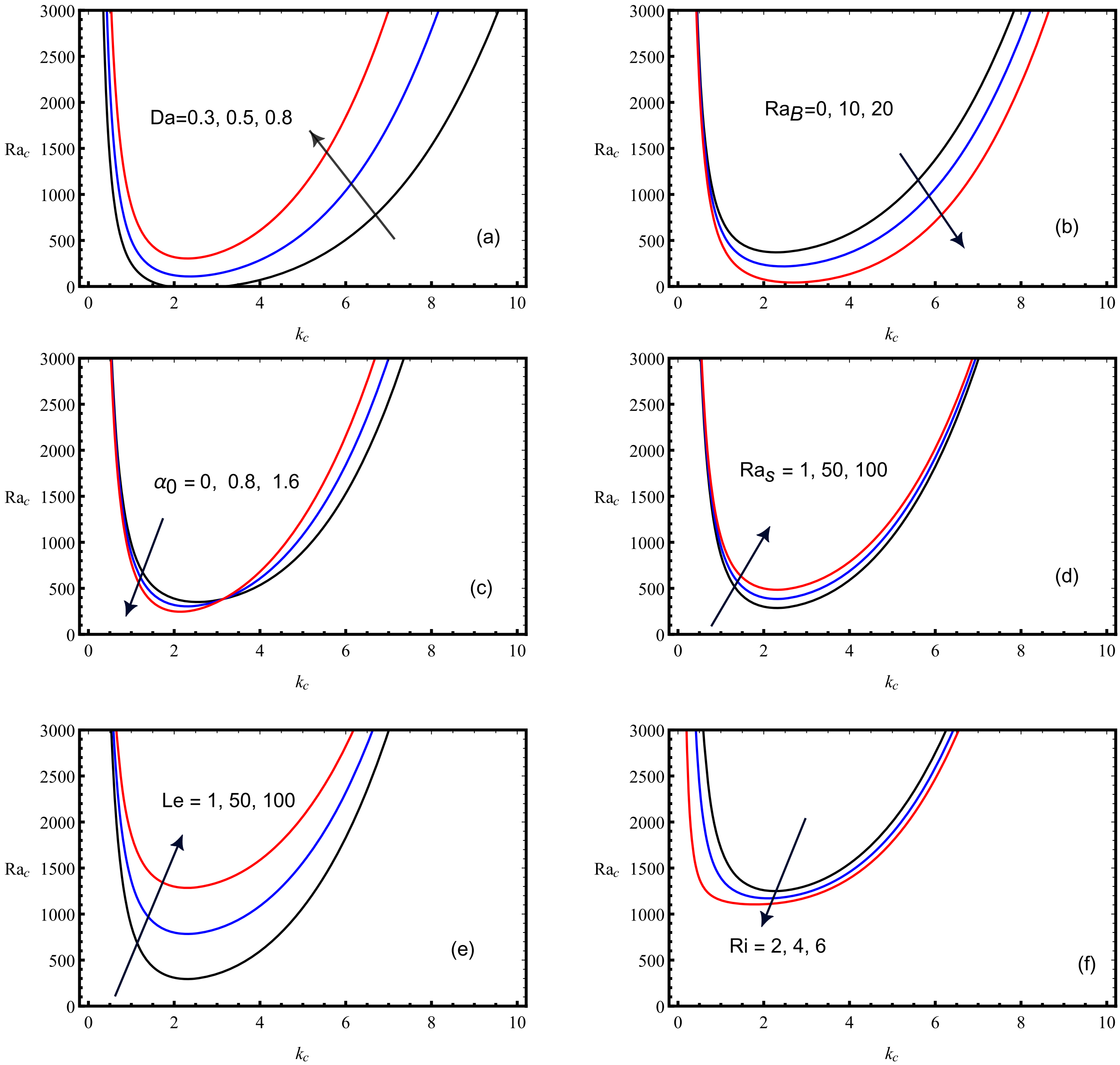

Fig. 2 The graphical analysis of the critical Rayleigh number Ra_c against wavenumber k_c

7 Results and discussion

Perturbation technique is employed in the investigation to derive Ginzberg–Landau amplitude equation for stationary mode of double-diffusive thermo-bioconvection. A numerical solution to the GL equation is determined for assumed initial values. The obtained amplitude is used to calculate Nusselt number and Sherwood number which are the rates of heat and mass transfer, respectively. The linear theory brings out the threshold or critical Rayleigh number Ra_c for the corresponding wavenumber k_c. This Ra_c gives the clear picture about the onset of convection. We come across various parameters in the current study which are having their impact on commencement of convection, heat and mass transfer in the system. The graphical representation of Ra_c versus k_c for various values of the existing parameters is plotted in Fig. 2. In Fig. 2a, the effect of Darcy number D_a on the onset of convection is shown. Increasing values of D_a leads to rise in threshold Rayleigh number. This implies that there is a delay in onset of convection. This means that the system is stabilized. The effect of modified bioconvection Rayleigh number ($\mathrm{Ra}_B = \mathrm{PeG}_0\phi_0\mathrm{R}_b$) on the onset of bioconvection is exhibited in Fig. 2b. The term Ra_B is directly proportional to gyrotactic number G_0. Increase in Ra_B leads to decrease in critical Rayleigh number. Naturally there is an advancement in onset of convection leaving the system in destabilized state. The critical Rayleigh number against dimensionless wavenumber is plotted for varying values of α_0 which is

Table 1 Numerical values of critical Rayleigh number Ra_c and corresponding wavenumbers k_c for variable parameters:

Fixed parameters	Variable parameter	Ra_c	k_c
$\mathrm{Ra}_B = 15, \alpha_0 = 0.5,\ Le = 2,$	0.3	43.215	2.46279
$\mathrm{R}_s = 10, \mathrm{R}_i = 0.2$	$Da = 0.5$	108.827	2.36575
	0.8	304.416	2.30925
$Da = 0.8, \alpha_0 = 0.8,\ Le = 2,$	1.0	563.986	2.26089
$\mathrm{R}_s = 10, \mathrm{R}_i = 0.2$	$\mathrm{Ra}_B = 10$	397.023	2.29149
	20	211.181	2.32758
$Da = 0.8, \mathrm{Ra}_B = 15, Le = 2,$	0.0	351.452	2.55372
$\mathrm{R}_s = 10, \mathrm{R}_i = 0.2$	$\alpha_0 = 0.8$	304.146	2.30925
	1.6	245.703	2.32758
$Da = 0.8, \mathrm{Ra}_B = 15, Le = 2,$	1	286.146	2.30925
$\alpha_0 = 0.8, \mathrm{R}_i = 0.2$	$\mathrm{R}_s = 50$	384.146	2.30925
	100	484.146	2.30925
$Da = 0.8, \mathrm{Ra}_B = 15, \mathrm{R}_s = 10,$	1	294.146	2.30925
$\alpha_0 = 0.8, \mathrm{R}_i = 0.2$	$Le = 50$	784.146	2.30925
	100	1284.146	2.30925
$Da = 0.8, \mathrm{Ra}_B = 15, \mathrm{R}_s = 10,$	2	269.822	2.26613
$\alpha_0 = 0.8, Le = 2$	$\mathrm{R}_i = 4$	230.976	2.20296
	6	190.879	2.11136

observed in Fig. 2c keeping the values of other parameters fixed. From the graph, it can be observed that increasing values of α_0 leads to decreasing threshold value for the onset of biothermal convection. The spherical shape of the microorganism has the stabilizing effect on the onset of convection. As the irregularities in the shape of the microorganisms increase, enhancement in biothermal convection is observed. Hence, the morphological aspects of microorganisms play a prominent role in the stability of biothermal convection. The effect of solutal Rayeigh number Ra_s and Lewis number Le on the system is of similar kind. Increasing values of those two leads to delay in onset of convection. Hence, it stabilizes the system. In Fig. 2f, we observe the effect of internal Rayleigh number R_i on the onset of bioconvection. There is an obvious increase in temperature in the system with the increase in the value of R_i. Therefore, the process of convection takes place faster. Hence, the advancement in onset of convection leads to destabilization of the system.

A new observation is made in Table 1 which indicates the values of critical Rayleigh number Ra_c and their corresponding wavenumber k_c with respect to all existing parameters.

In Fig. 3, we can find the graphs of Nusselt number versus time scale τ with respect to various parameters existing in the system. Modified Vadasz number helps in enhanced heat tranfer in the system initially for small time scale τ. Later, there is no difference among the curves for larger values of τ as shown in Fig. 3a. Further, Fig. 3b illustrates the impact of the modified bioconvective Rayleigh–Darcy number Ra_B on the thermo-Nusselt number. Heat transmission with the gravity modulation decreases as the bioconvective Rayleigh–Darcy number rises. The growth in the concentration of microorganisms causes an increase in Ra_B. As a result of the rush in the system, the microorganism-induced convective flow intensity decreases. Thus, the amount of heat transfer also decreases throughout the time interval. The heat transport in the system can be significantly affected by the shape of the microorganisms, as demonstrated by the graph in Fig. 3c. Physically speaking, it makes sense since the way microorganisms are shaped influences how they travel through a fluid and interact with it, which, in turn, influences convective heat transmission. Because of their symmetrical structure, spherical bacteria can move through fluids with remarkable ease. This encourages effective heat transmission and mixing, which raises convective heat transfer rates and, in turn, Nusselt numbers. However, the migration of the microorganisms through the fluid gets more complex if they have non-spherical or irregular forms. Their shape's asymmetries and imperfections might cause altered flow patterns. In contrast with spherically-shaped microorganisms, this could hinder the convective heat transfer process, resulting in decreased heat transfer rates and Nusselt numbers. The results obtained are comparable with Kopp and Yanovsky [36] and Akhila et al. [37].

The effect of the modulation frequency Ω is presented in Fig. 3d. To be more precise, heat transfer is increased at lower frequency modulation [low frequency ($\Omega = 2$)] in comparison with higher vibrational rates ($\Omega = 5$) and ($\Omega = 25$). The influence of δ, the amplitude of modulation on heat transfer within the system is depicted in Fig. 3e. The δ values between 0.1 and 0.3 are taken into consideration in the study. These values were chosen with the intention of improving heat transfer. In Fig. 3f, we find the impact of Lewis number Le on heat transfer in the system. Lewis number shows its significance only when concentration of solute is involved in the system. Increase in Nusselt number is observed with increase in Lewis number. Another parameter which shows its existence during double diffusion is solutal Rayleigh number Ra_s. In the present investigation, the values for Ra_s are taken as 1100 and

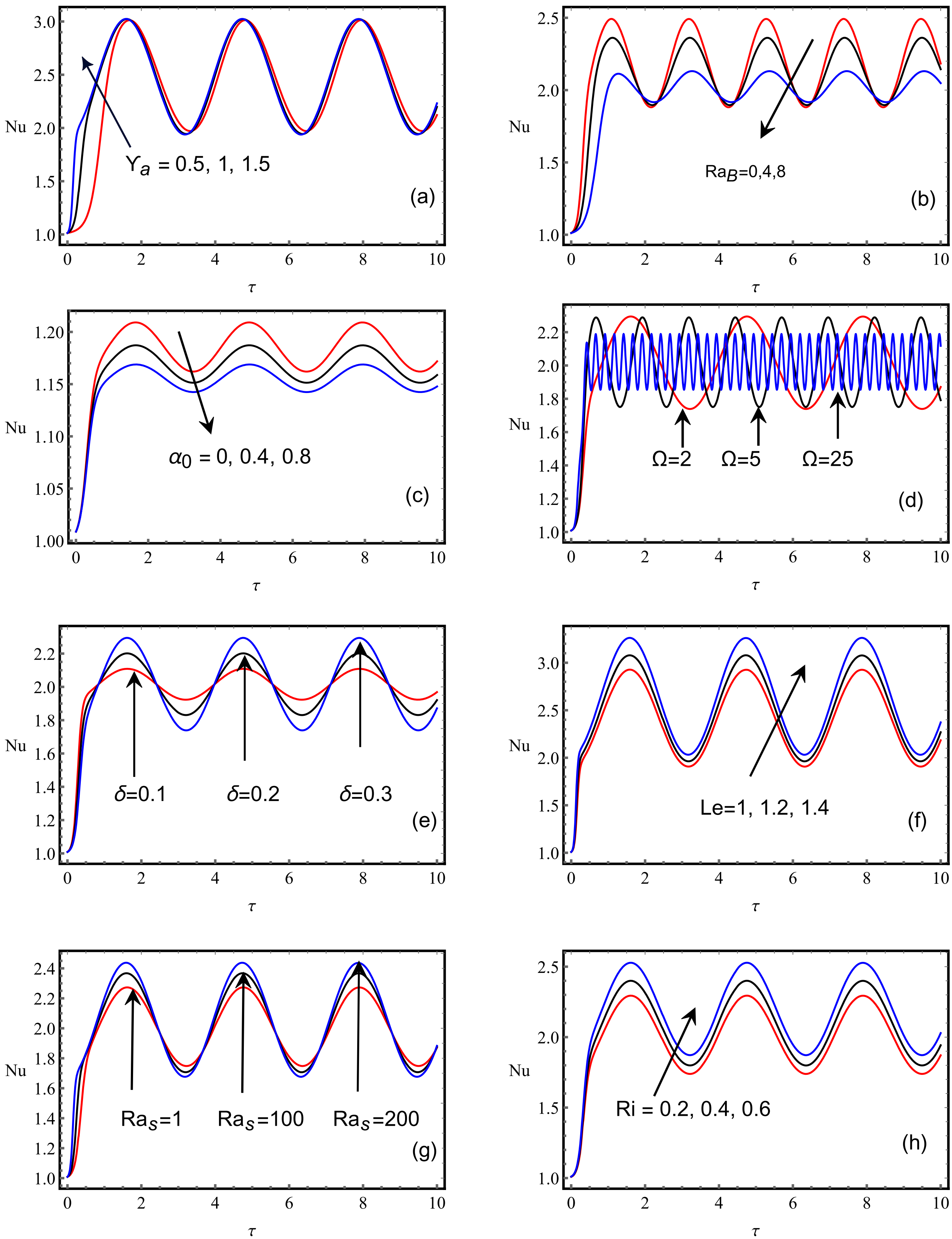

Fig. 3 The graphical representation of the Nusselt number *Nu* against time τ with respect to various parameters

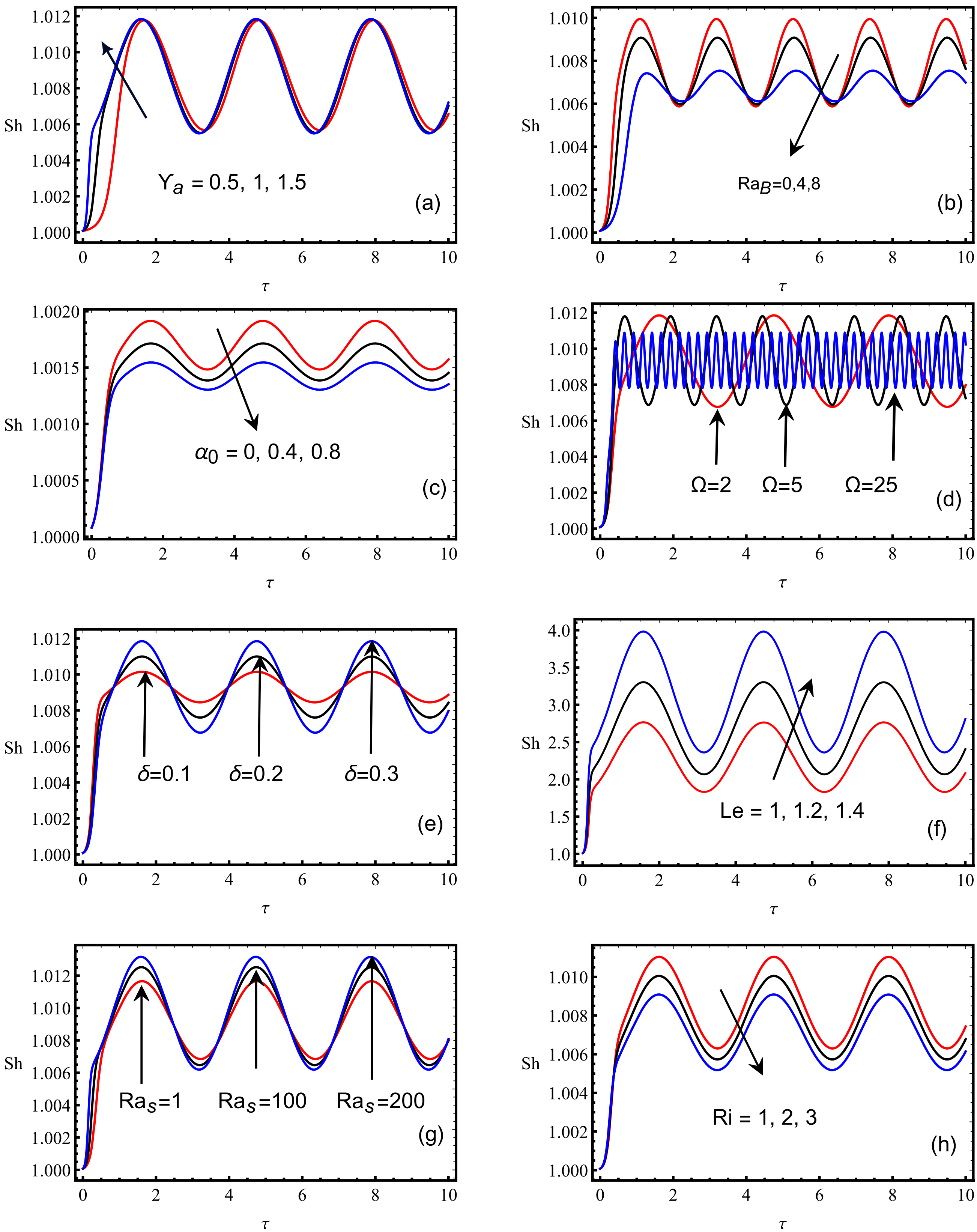

Fig. 4 The graphical representation of the Sherwood number *Sh* against time τ with respect to various parameters

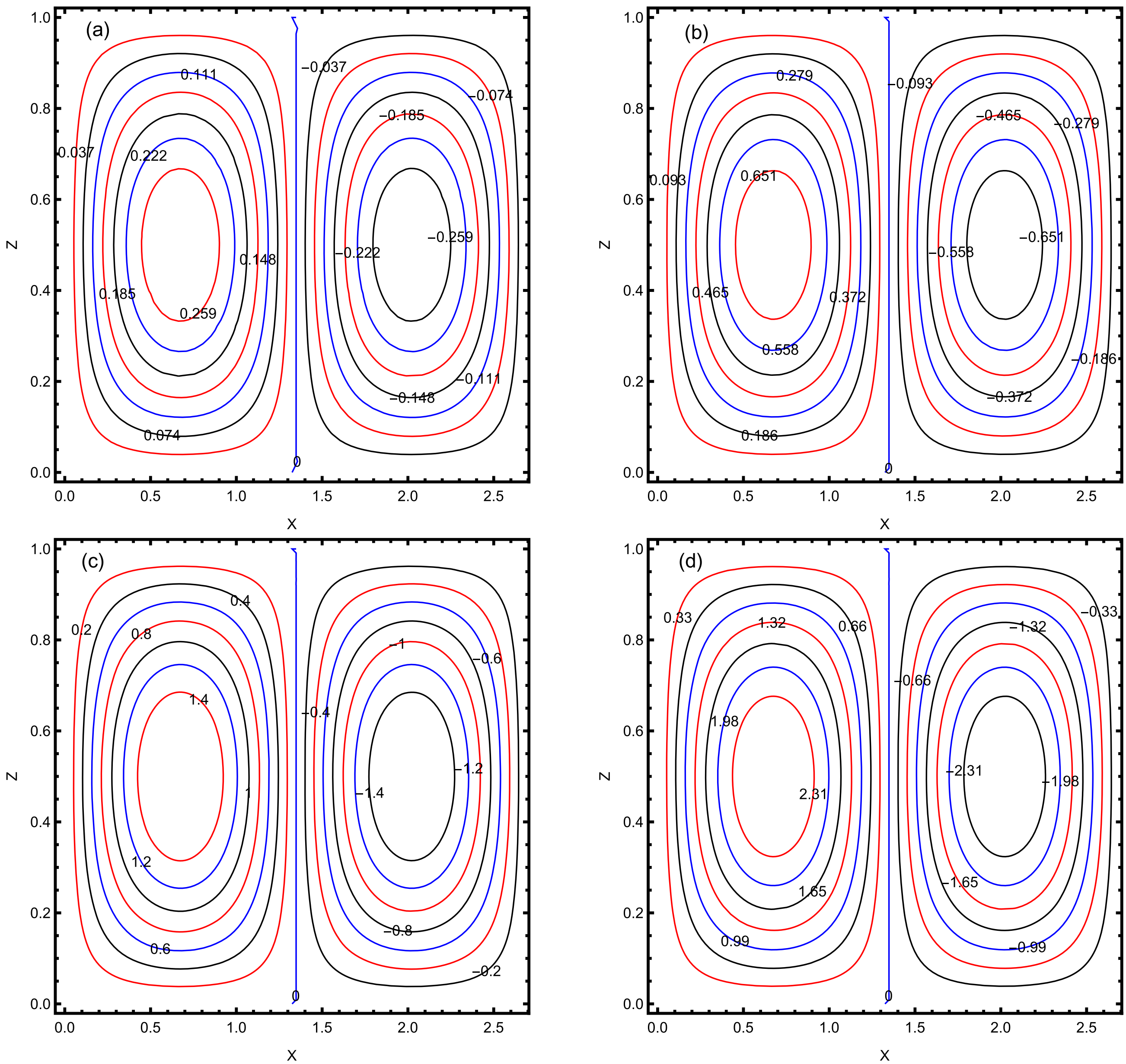

Fig. 5 Streamlines: **a** $\tau = 0.0$, **b** $\tau = 0.1$, **c** $\tau = 0.2$, **d** $\tau = 0.3$

200. Heat transfer increases accordingly. As we have included internal heating effect in the present problem, there is an obvious emergence of internal Rayleigh number which advances heat transfer with increase in the value of R_i.

The next set of graphs depicted in Fig. 4 explains the mass transfer rates against time scale τ. The effect of gravity modulation along with the various parameters on mass transfer is very much similar to heat transfer as discussed in Fig. 3. The only difference we observe is from internal Rayleigh number. As the values of R_i increase, there is a decrease in Sherwood number. Hence, internal Rayleigh number helps in increased heat transfer and decreased mass transfer in the system which we can observe in Figs. 3h and 4h. Internal heat source increases temperature gradient but reduces concentration gradient within the medium. Higher temperature leads to greater molecular kinetic energy which transfers from higher energy to lower energy molecules. This enhances heat conduction in the medium. The same higher temperature leads to faster diffusion, i.e., there exists a diminished concentration gradient. Therefore, internal heat source or internal Rayleigh number enhances heat transfer while decreases mass transfer. The results converge with Kopp and Yanovsky [36] and Akhila et al. [37].

A set of impressive graphs are observed in Figs. 5 and 6 which are streamlines and isotherms, respectively. In Rayleigh–Bénard experimental setup, as the bottom layer is heated, gradually convection cells are generated when the Rayleigh number exceeds the critical Rayleigh number. Those cells are pictorially represented by streamlines and isothermal lines. These lines for various

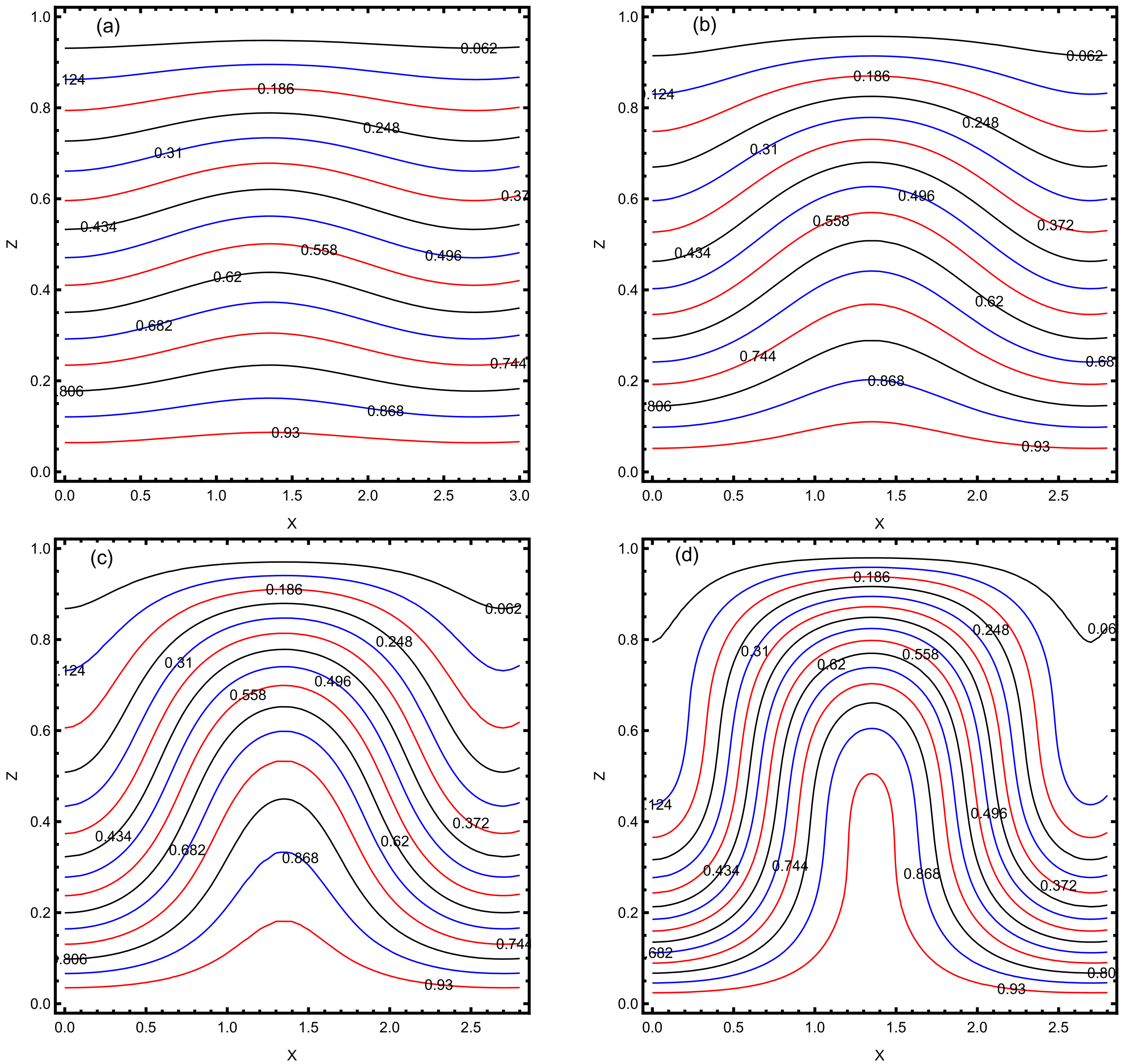

Fig. 6 Isotherms: **a** $\tau = 0.0$, **b** $\tau = 0.1$, **c** $\tau = 0.2$, **d** $\tau = 0.3$

time scales are discussed. Streamlines move with different velocities where one in clockwise and other in anticlockwise direction. Isotherms are generated due to varying temperature with respect to variable time scale τ. The effects of parameters, gravity modulation and internal heat source are discussed in all the above graphs.

The next set of graphs shown in Fig. 7 discuss about the mean Nusselt number against several parameters. In Fig. 7a, $\tilde{Nu}$ against Ω is plotted for varying modified Vadasz number. As the value of Υ_a increases, there is a decrease in mean Nusselt number. The same behavior is observed in Fig. 7b–e but with different nature. In these graphs, $\tilde{Nu}$ is plotted against α_0, R_b, R_s and R_i for varying Ra_B, Le and D_a, respectively. But in Fig. 7f, we observe an opposite nature when compared to above ones. Here, mean Nusselt number increases with increase in L_b when it is plotted against α_0.

Figure 8 indicates the comparison between analytical and numerical results obtained for an unmodulated case of Nusselt number and Sherwood number. Here, we observe the coincidence of both the results on heat and mass transfer which shows that numerical and analytical validations are in good agreement.

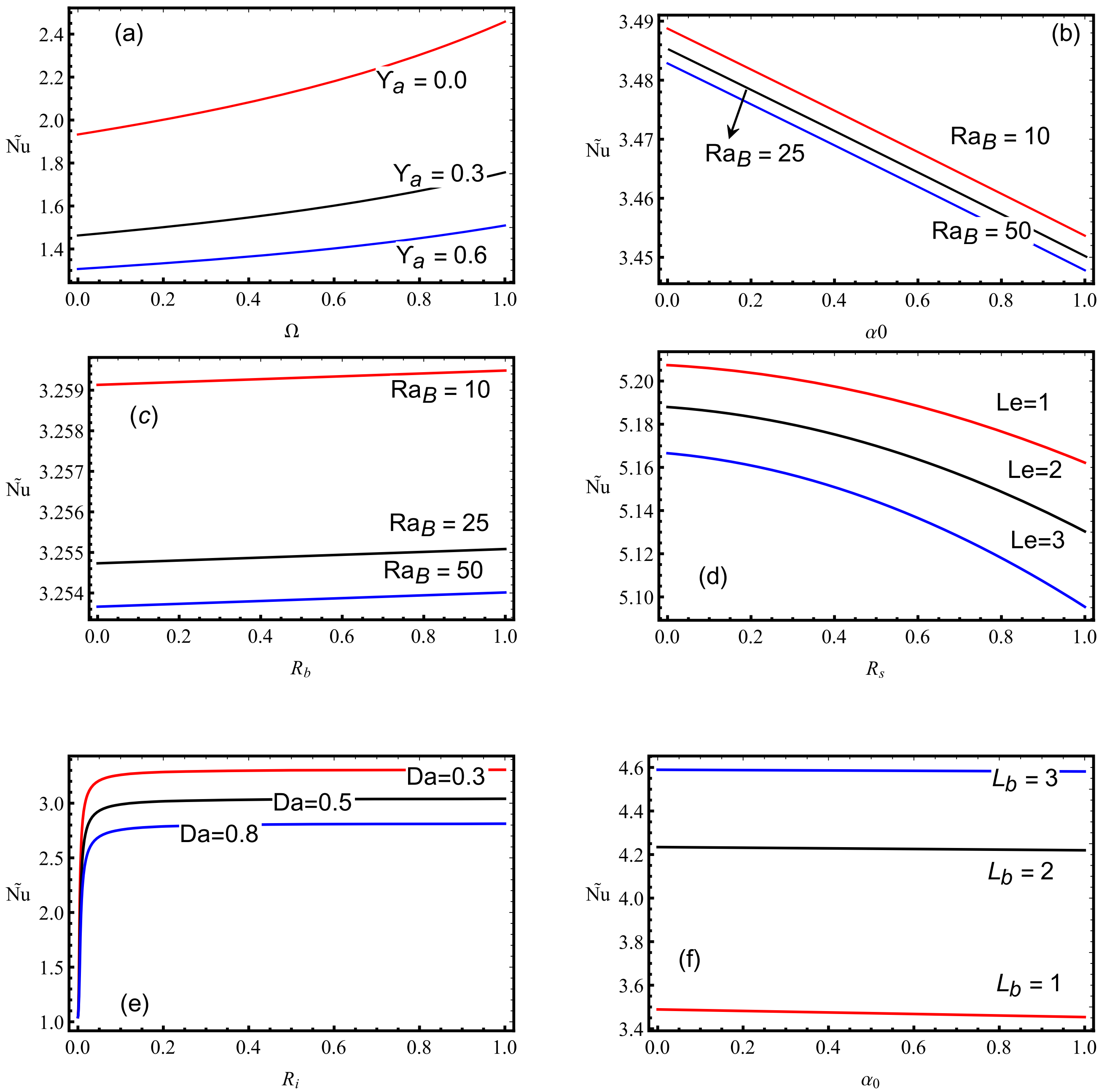

Fig. 7 The graphical analysis of the mean Nusselt number with repect to various parameters

8 Conclusions

In order to draw conclusion, we established a hypothesis on weak nonlinear stability for stationary biothermal convection due to gravity modulation in a porous media saturated with Newtonian fluid including gyrotactic microorganisms with internal heating effect. For our investigation, we use perturbation theory and focus on the minute parameter ε, which represents the departure from the critical Rayleigh number. We take into account the modulated gravity field's small amplitude up to the second order in ε in our study. Finding results compatible with linear theory, we discover that the parametric modulation has no substantial effect on the convection development in the first order of ε. But when we increase ε to the third order, we get Ginzburg–Landau amplitude equation which is nonlinear in nature that shows the existence of nonlinearity throughout the system. Based on the findings of our numerical results, we have deduced a number of conclusions. These findings clarify the effects of porous media and gravity modulation on biothermal convection. Based on the acquired results, we make the following conclusions:

1. The parameters D_a, Ra_s and Le help in stabilizing the system by delaying the onset of convection.

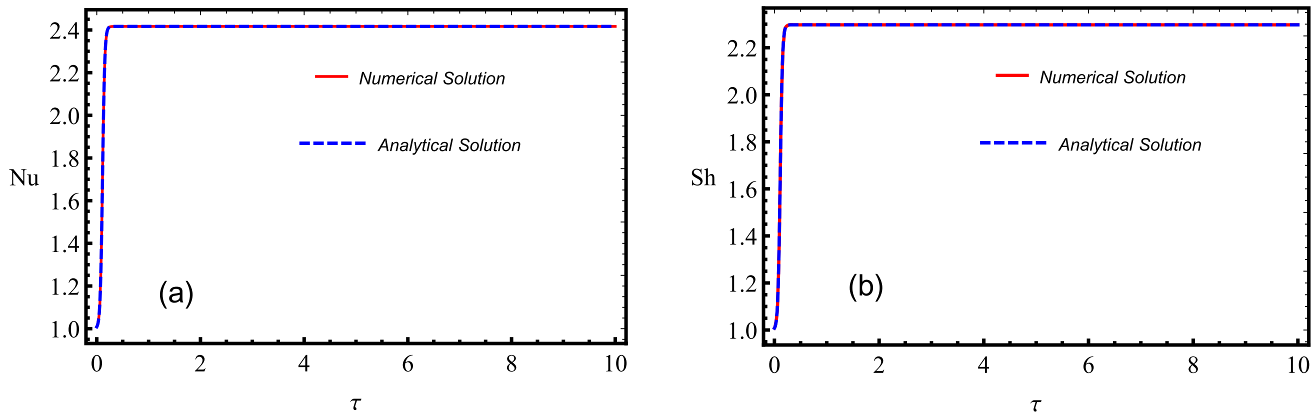

Fig. 8 Comparison between numerical and analytical solutions of Nusselt number and Sherwood number

2. Modified bioconvective Rayleigh number Ra_B advances the onset of bioconvection, and the system gets destabilized due to its increasing values.
3. The cell eccentricity α_0 destabilizes the system by enhancing the onset of convection. The irregularity in the shape of microorganisms leads to advancement in onset of convection.
4. Internal Rayleigh number R_i helps in advancement of onset of convection by destabilizing the system.
5. An increase in heat and mass transport is observed when the parameter Υ_a is raised.
6. Increase in Ra_B number results in decreasing heat and mass transmission because of gravity modulation and internal heating effect in porous media.
7. The microorganisms' spherical form helps to improve the efficiency of the heat and mass transmission mechanism. The system's heat transfer decreases as the shape becomes more irregular.
8. Heat and mass transmission is inhibited when the modulation frequency Ω is increased since this lowers the fluctuations of the Nusselt numbers $\mathrm{Nu}(\tau)$.
9. Heat and mass transport is improved by raising the modulation amplitude δ.
10. Lewis number Le and solutal Rayleigh number Ra_s help in increase in heat and mass transport with increase in their values.
11. Significant aspect of internal Rayleigh number R_i is it improves heat transport and diminishes mass transfer on increasing its values.
12. Streamlines show the convective cells in counterclockwise pattern.
13. Isotherms show the convection with increase in time scale.
14. Numerical and analytical results for unmodulated case for heat and mass transfer coincide with each other.

Our study provides an insight on how stationary thermo-bioconvection behaves in a porous media saturated by Newtonian fluid when subjected to gravitational modulation and internal heating effect. We may better comprehend the impact of various factors on the convection process by examining their impacts on Υ_a, D_a, Ra_B, α_0, Ω, δ, Le, Ra_s and R_i. Gaining this insight is essential to manage and control heat transmission and mass transmission in the system as effectively as possible. It is feasible to improve or control the heat and mass transfer in a system containing Newtonian fluid with gyrotactic microorganisms in a porous medium by adjusting the previously specified parameters. With this understanding, more effective thermo-solutal transfer systems may be created, convection control techniques can be developed and thermal management can be enhanced in a variety of applications. Thermal bioconvection is a realistic problem. But the addition of gravitation effect and internal heat source adds up to its application as gravitation has an effect on bioconvection, which is a phenomenon resulting from the negative gravitactic behavior of microorganisms. When the density gradient generated by gravitaxis becomes large enough, it leads to an overturning convection and the formation of characteristic patterns involving highly concentrated aggregation of cells into extended two-dimensional structures. Additionally, factors such as the Vadasz number, modulation amplitude, modified bioconvection Rayleigh–Darcy number, internal Rayleigh number and cell eccentricity also impact heat transfer in bioconvection.

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Declarations

Conflict of interest The authors declare that they have no conflict of interest.

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