

Influence of space conjugate temperature varying non-uniform heat sink/source on hydromagnetic slip water-EG (50:50) nanofluid

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ABSTRACT

Significant cooling is an essential requirement in the field of aeronautical and bio-medical engineering, nuclear reactor system, solar collectors, and in development of electronic chips etc. Involving rotating conical geometries. In view of this, flow and heat transfer over conical geometries subject to different constraints of motion are highly needed. Implementation of magnetic field subject to flow pattern controls the fluid motion thereby imparting better cooling. Consideration of nanofluid instead of regular fluid yields prominent cooling of the associated surface. Such relevance has motivated the authors to work on magnetohydrodynamics and heat transfer investigation of water-EG (50:50) mixture based Al_2O_3 and Fe_3O_4 past heated and rotating down-pointing upright cone subject to impact of space and temperature varying non-uniform heat source or sink. Numerical solution of dimensionless governing equations is accomplished by implementing Runge-Kutta method. The findings indicate that swirl and axial velocities peter out with rise in magnetic parameter while both exhibit opposite impact in response to slip parameter. Temperature profiles upgrade due to amplification of space and temperature dependent parameters. Skin friction and heat transportation upsurge with growth of solid volume fraction of nanoparticle.

1. Introduction

Customizing the execution of industrial tools in the field of heat transfer (HT) is very challenging one. Early stage of research is mainly aimed using base fluids for HT phenomenon, but later on, to overcome this, Choi and Eastman [1] came up with an innovative type of fluids called nanofluids (NFs), which is a boon in enhancing the thermal conductivity. In recent years significant methods are developed to modify thermal performance of fluids by putting nano sized solid particles like Al_2O_3 , Cu, Ag etc. in fluids, which are poor in HT. Hakeem et al. [2] conducted a comparative study with magnetized/non-magnetized nanoparticles. Soret-Dufour impacts

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on magnetized Casson NF flow across an inclined plane were perused by Lorenzini et al. [3]. They found that the solutal and thermal distributions improve and velocity slows down with Casson parameter. The impact of Brownian movement and thermophoresis on chemically-reacting magnetized Hiemenz stream across a nonlinear expandable plate was investigated by Ibrahim et al. [4]. Impacts of slip on magnetic Casson nanofluid flow over a permeable stretched sheet were perused by Kumar et al. [5]. 3D NF stirring with a non-uniform heat sink/source was studied by Thumma et al. [6]. They discovered that an upsurge in the stretching ratio parameter had an impact on the axial velocity profile. Sheri and Thumma [7] investigated the thermal and solutal effects on natural convection flow when various volume fractions of copper-water nanofluid were present. They discovered that concentration profiles get more emaciated as Schmidt number increase. MHD convective NF flow over a rotating inclined thin layer of iron oxide based on sodium alginate was discussed by Dawar et al. [8]. The effects of magnetohydrodynamics, heat source/sink, and gravity modulation on the movement of a micropolar hybrid NF fluid across an inclined surface were studied by Ali et al. [9]. They found that fluid velocity amplifies with rising Hartmann number but emaciates with growth of micropolar material amount. The works on the related areas were seen in Refs. [10–34].

Fluid flow due to rotating conical geometries has gained importance in the field of aeronautical engineering, cooling nuclear reactor system, solar collectors, and medical field and in development of electronic chips etc. The investigation on free convection (Hering and Grosh [35]), mixed convection (Yih [36]), variable viscosity (Hossain et al. [37]), MHD (Chamkha and Al-Mudhaf [38]), and Soret/Dufour effect (Huang [39]) of nanofluids over rotating cone. The flow of MHD Ellis nanofluids around a spinning cone was perused by Ahmed et al. [40]. They stated that as bioconvection Rayleigh number ups, the temperature and velocity gradients decrease. The flow of water-copper NF and enhancement of HT between vertical concentric cones were studied by Bairi [41]. Their findings demonstrate that heat transmission improves with increasing thermal conductivity ratios, concentrations of nanoparticles, and Rayleigh numbers. The double-diffusive magnetized Jeffrey fluid flow via the disc and cone region was analysed by Khan et al. [42]. They discovered that when the thermic relaxation factor and Prandtl number increase, the thermal distribution slows down. The magnetic Casson liquid considering the impacts of Dufour and Soret because of a revolving cone was analysed by Mustafa et al. [43]. The magnetic parameter reduces the tangential and azimuthal velocities. Natural convective dissipative NF flow via a vertical cone was observed by Ragulkumar et al. [44]. They discovered that the fluid velocity and temperature of NF rise in tandem with increment in viscous dissipation parameter. The works on the related areas were carried out in Refs. [45–48].

The studied literature revealed that many works have carried out regarding the flow and HT of regular fluids or NFs over rotating cone with magnetic field. However, none of them investigated the influences of space and temperature varying heat source/sink on the stream and HT of NFs over rotating cone. The above gap in the existing literature and the relevance of NF flow over conical geometries of industrial importance were the main motivation behind the present investigation. Therefore, the objective of the existing work is to investigate the free convection flow about a vertical spinning cone under the effect of magnetic field, space and temperature varying heat source or sink using water-EG (50:50) mixture as base fluid and Al_2O_3 and Fe_3O_4 as nanoparticles. The novelties of the present study are:

- Introduction of space and temperature varying heat sink or source.
- Consideration of Al_2O_3/Fe_3O_4 + water-EG (50:50) NF as the flow medium over a vertical spinning cone.

2. Modeling

Consider an incompressible, steady, laminar and 2D magnetohydrodynamic flow of Al_2O_3/Fe_3O_4 + $H_2O-C_2H_6O_2$ (50:50) NF along a rotating down-pointing upright hot cone subjected to magnetic field and space conjugate temperature varying heat source or sink. Cone has angular velocity shown by Ω and the flow's local radius is taken as $r = x \sin \gamma$. Consider u, v, w are respectively velocity elements in x, y and θ directions. Problem geometry is shown in Fig. 1. Thermophysical properties of $H_2O-C_2H_6O_2$ (50:50) and Al_2O_3/Fe_3O_4 are mentioned in Table 1.

Using the assumptions mentioned earlier, governing equations would be ([35,38,44,47]):

$$\frac{\partial(ru)}{\partial x} + \frac{\partial(rv)}{\partial y} = 0, \quad (1)$$

$$\rho_{nf} \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} - \frac{w^2}{x} \right) = \mu_{nf} \frac{\partial^2 u}{\partial y^2} + (\rho\beta)_{nf} g \cos \gamma (T - T_\infty) - \sigma_{nf} B_o^2 u, \quad (2)$$

$$\rho_{nf} \left(u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + \frac{uw}{x} \right) = \mu_{nf} \frac{\partial^2 w}{\partial y^2} - \sigma_{nf} B_o^2 w, \quad (3)$$

$$(\rho C_p)_{nf} \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k_{nf} \frac{\partial^2 T}{\partial y^2} + \underbrace{\frac{k_{nf} u}{x v_{nf}} \left[A^* (T_w - T_\infty) \exp\left(-\frac{y}{L^2}\right) + B^* (T - T_\infty) \right]}_{\text{space and temperature dependent non-uniform heat source/sink}}, \quad (4)$$

Boundary conditions ([38,47]):

$$u = L_1 \frac{\partial u}{\partial y} + L_2 \frac{\partial^2 u}{\partial y^2}, v = 0, w = r\Omega + L_3 \frac{\partial w}{\partial y} + L_4 \frac{\partial^2 w}{\partial y^2}, T = T_w \text{ at } y = 0, \quad (5)$$

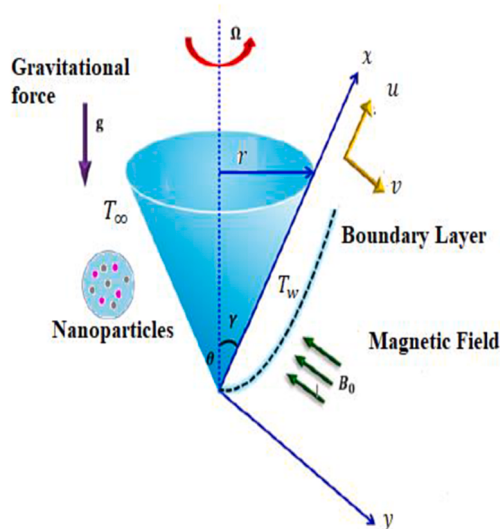


Fig. 1. Flow diagram.

Table 1

Thermal and physical properties of $H_2O-C_2H_6O_2$ (50:50) base fluid and Al_2O_3/Fe_3O_4 nanoparticles at 300K

	C_p	ρ	k	$\beta \times 10^{-5} \left(\frac{1}{K} \right)$	σ	Pr
EG and water (50:50) mixture (H_2O - $C_2H_6O_2$) (50:50)	3288	1056	0.425	0.00341	0.00509	29.86
Aluminium oxide. (Al_2O_3)	765	3970	40	0.85	35×10^6	–
Magnetite (Fe_3O_4)	670	5180	9.7	1.3	2.5×10^4	–

$$u = 0, v = 0, T = T_\infty \text{ as } y \rightarrow \infty, \quad (6)$$

Thermophysical models of nanofluid are described in Table 2.

Considering the transformation ([38,47]):

$$u = \frac{1}{2}\Omega x \sin \gamma F'(\eta), v = -(v\Omega \sin \gamma)^{\frac{1}{2}}F(\eta), w = \Omega x \sin \gamma G(\eta), = \left(\frac{\Omega \sin \gamma}{\nu}\right)^{1/2}y, \quad (7)$$

$$\Theta(y) = \frac{T-T_\infty}{T_w-T_\infty} \text{ where } T_w-T_\infty = (T_0-T_\infty) \frac{x}{L}, \quad (8)$$

We have

$$F''' - \left(\frac{F'^2}{2} - FF'' - 2G^2 \right) (1-\varphi)^{2.5} (1-\varphi + \varphi (\rho_s/\rho_f)) - \frac{\sigma_{nf}}{\sigma_f} (1-\varphi)^{2.5} MF' + 2(1-\varphi)^{2.5} (1-\varphi + \varphi ((\rho\beta)_s/(\rho\beta)_f)) \lambda \Theta = 0, \quad (9)$$

$$G''-(1-\varphi)^{2.5}(1-\varphi+\varphi(\rho_s/\rho_f))(F'G-FG')-\frac{\sigma_{nf}}{\sigma_f}(1-\varphi)^{2.5}MG=0, \quad (10)$$

Table 2

Thermophysical models of nanofluid.

Properties	Nanofluid
Kinematic viscosity	$\nu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}[(1-\phi)\rho_f + \phi\rho_s]}$
Density	$\rho_{nf} = (1-\phi)\rho_f + \phi\rho_s$
Thermal diffusivity	$\alpha_{nf} = \frac{k_{nf}}{(\rho C_p)_{nf}}$
Heat capacitance	$(\rho C_p)_{nf} = (\rho C_p)_f(1-\phi) + \phi(\rho C_p)_s$
Electrical conductivity	$\frac{\sigma_{nf}}{\sigma_f} = 1 + \frac{3(\sigma-1)\phi}{(\sigma+2) - (\sigma-1)\phi}$, where $\sigma = \frac{\sigma_s}{\sigma_f}$
Thermal conductivity	$\frac{k_{nf}}{k_f} = \frac{(2k_f + k_s) - 2\phi(k_f - k_s)}{(2k_f + k_s) + \phi(k_f - k_s)}$
Thermal expansion coefficient	$(\rho\beta)_{nf} = (\rho\beta)_f(1-\phi) + \phi(\rho\beta)_s$

$$H'' - \frac{k_f}{k_{nf}} \Pr (1 - \varphi + \varphi ((\rho C_p)_s / (\rho C_p)_f)) \left(\frac{1}{2} F' \Theta + \Theta' F \right) + \frac{1}{2} (1 - \varphi)^{2.5} (1 - \varphi + \varphi (\rho_s / \rho_f)) F' (A^* \exp(-\zeta) + B^* \Theta) = 0, \quad (11)$$

With boundary conditions

$$F' = \gamma_1 F'' + \gamma_2 F''', F = 0, G = 1 + \gamma_3 G' + \gamma_4 G'', \Theta = 1 \text{ at } \eta = 0, \quad (12)$$

$$F' \rightarrow 0, G \rightarrow 0, \Theta \rightarrow 0 \text{ as } \eta \rightarrow \infty, \quad (13)$$

In which $\lambda = \frac{Gr}{Re^2}$ is mixed convection parameter, $Re = \frac{L^2 \Omega \sin \gamma}{\nu}$ denotes Reynolds number, $Gr = \frac{g \beta_f \cos \gamma (T_0 - T_\infty) L^3}{\nu^2}$ is Grashof number, $M = \frac{\sigma B_0^2}{\rho \Omega \sin \gamma}$ is magnetic parameter, $Pr = \frac{\nu}{\alpha}$ is Prandtl number, $\gamma_1 = L_1 \left(\frac{\Omega \sin \gamma}{\nu} \right)^{\frac{1}{2}}$ and $\gamma_2 = L_2 \left(\frac{\Omega \sin \gamma}{\nu} \right)$ are first and second order slip parameter along axial direction, $\gamma_3 = L_3 \left(\frac{\Omega \sin \gamma}{\nu} \right)^{\frac{1}{2}}$ and $\gamma_4 = L_4 \left(\frac{\Omega \sin \gamma}{\nu} \right)$ are first and order slip parameter along swirl direction.

3. Local nusselt number (Nu) and skin friction coefficient(C_f)

Skin friction coefficients C_f and local Nusselt number Nu are defined as

$$C_f = \frac{2\tau_w}{\rho_f (\Omega x \sin \gamma)^2}, \quad (14)$$

$$\text{and } Nu = \frac{Lq_w}{k_f (T_w - T_\infty)}, \quad (15)$$

Where $q_w = -k_{nf} \left(\frac{\partial T}{\partial y} \right)_{y=0}$ is surface heat flux and $\tau_w = \mu_{nf} \left(\frac{\partial u}{\partial y} \right)_{y=0}$ indicates wall shear stress.

The dimensionless forms of C_f & Nu are

$$C_f Re^{1/2} = \left(\frac{\mu_{nf}}{\mu_f} \right) F''(0), \quad (16)$$

$$Nu Re^{-1/2} = - \left(\frac{k_{nf}}{k_f} \right) \Theta'(0), \quad (17)$$

4. Numerical procedure

The BVP may well be solved via fourth order Runge Kutta scheme with shooting techniques. We set $F = y_1$, $G = y_4$, $\Theta = y_6$, and hence,

$$y_1' = y_2,$$

$$y_2' = y_3,$$

$$y_3' = \left(\frac{F'^2}{2} - FF'' - 2G^2 \right) (1 - \varphi)^{2.5} (1 - \varphi + \varphi (\rho_s / \rho_f)) + \frac{\sigma_{nf}}{\sigma_f} (1 - \varphi)^{2.5} M F' - 2(1 - \varphi)^{2.5} (1 - \varphi + \varphi (\rho_s / \rho_f)) \lambda \Theta,$$

$$y_4' = y_5,$$

$$y_5' = (1 - \varphi)^{2.5} (1 - \varphi + \varphi (\rho_s / \rho_f)) (F'G - FG') + \frac{\sigma_{nf}}{\sigma_f} (1 - \varphi)^{2.5} M G,$$

$$y_6' = y_7,$$

$$y_7' = \frac{k_f}{k_{nf}} \Pr (1 - \varphi + \varphi ((\rho C_p)_s / (\rho C_p)_f)) \left(\frac{1}{2} F' \Theta + \Theta' F \right) - \frac{1}{2} (1 - \varphi)^{2.5} (1 - \varphi + \varphi (\rho_s / \rho_f)) F' (A^* \exp(-\zeta) + B^* \Theta).$$

The flow chart for numerical solutions of the developed problem is provided below.



Flow chart for Numerical solutions.

5. Validation

The existing outcomes are compared with that of Chamkha and Al-Mudhaf [12] and Khan et al. [26] and showed excellent agreement (See Tables 3 and 4). Ascending trend of HT rate is visualized with rise in Pr and λ .

6. Results and discussion

This part conveys the efficacies of space dependent parameter A^* , spin parameter ϵ , magnetic parameter M , nanoparticle volume fraction ϕ , and temperature dependent parameter B^* on the temperature profile $H(y)$, swirl $G(y)$ and tangential $F(y)$ velocity profiles in the case of LST is explored. Further, variations of dimensionless C_f and Nu in response to sundry physical parameters are examined. The values used for parameters in the simulation are: $\phi = 0.01, M = 1, \lambda = 0.3, A^* = B^* = 1, \gamma_1 = \gamma_2 = \gamma_3 = \gamma_4 = 0.01$.

Table 3
Comparison of Skin friction coefficient (Chamkha and Al-Mudhaf [12] and Khan et al. [26]).

Pr	λ	Chamkha et al. (2005)	Khan et al. (2020)	Present result
0.7	0	1.0255	1.0256	1.020466
	1	2.2012	2.2017	2.078857
	10	8.5041	8.5044	8.524585
10	0	1.0256	1.0254	1.020465
	1	1.5636	1.5636	1.596575
	10	5.0821	5.0825	5.335744

Table 4
Comparison of Nusselt number (Chamkha and Al-Mudhaf [12] and Khan et al. [26]).

Pr	λ	Chamkha et al. (2005)	Khan et al. (2022)	Present result
0.7	0	0.4299	0.4299	0.428518
	1	0.6120	0.6120	0.612017
	10	1.0097	1.3992	1.017308
10	0	1.4110	1.4113	1.408322
	1	1.5662	1.5663	1.607681
	10	2.3580	2.3582	2.35355

6.1. Influence of M on $F'(y)$, $H(y)$ and $G(y)$

Fig. 2a reveals the effect of M on $F'(y)$ and $G(y)$ at $\phi = 0.1$, $\varepsilon = 1$, $\Delta = 0.1$ and $Pr = 29.86$. When M ameliorates $F'(y)$ and $G(y)$ drop. This is because of the retarding nature of the Lorentz force which is due to the interaction of applied magnetic field and conducting NF thereby reduces the velocity boundary layer thickness. As visualized, Al_2O_3 has higher tangential velocity compared to Fe_3O_4 in $M = 0$ case. As M increases, magnetic nanoparticle Fe_3O_4 has high tangential velocity than non-magnetic nanoparticle Al_2O_3 . In case of $G(y)$, Al_2O_3 has higher spin velocity than Fe_3O_4 . The influence of M on $H(y)$ for both Al_2O_3 & Fe_3O_4 is portrayed in Fig. 2b. Further, $H(y)$ ameliorates due to rise in M . However, there is subtle variation of $H(y)$ between Al_2O_3 & Fe_3O_4 in appearance/non-appearance of magnetic field.

6.2. Influence of ε on $F'(y)$, $G(y)$ and $H(y)$

Fig. 3a depicts that increase in spin parameter ε accounts for increase in $F'(y)$ and decrease in $G(y)$. In fact, increase in spin parameter intensifies the movement of the nanoparticles within the NF thereby enhances the fluid motion. It is seen that Fe_3O_4 has higher tangential velocity than Al_2O_3 . In Fig. 3b it is evident that as spin parameter rises $H(y)$ peters out and Al_2O_3 has high temperature than Fe_3O_4 . This implicates that heat transfer rate ameliorates for enhancement of spin parameter. In fact, rise in spin parameter enables the nanoparticle move faster thereby spend less time near the solid boundary of the cone. As a result, temperature of the NF fluid within BL emaciates.

6.3. Influence of A^* and B^* on $H(y)$

The effect of A^* on $H(y)$ is revealed in Fig. 4a. Applying $A^* > 0$ (heat source) to BL produces energy which helps to increase $H(y)$ while considering heat sink ($\hat{A}^* < 0$) in BL may absorb energy that ensures $H(y)$ to drop. The nanomaterial Al_2O_3 has high temperature compared to Fe_3O_4 . The influence of temperature dependent heat generation or absorption on temperature profile is analysed in Fig. 4b. As B^* ameliorates, temperature profile uplifts. Further, Al_2O_3 has high temperature compared to Fe_3O_4 .

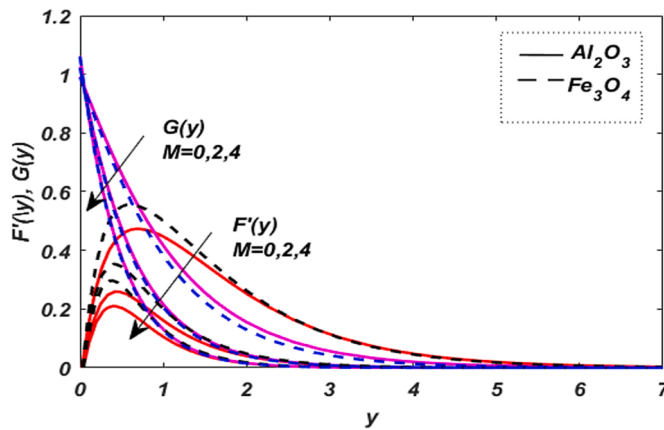


Fig. 2a. Role of M on $F'(y)$ and $G(y)$.

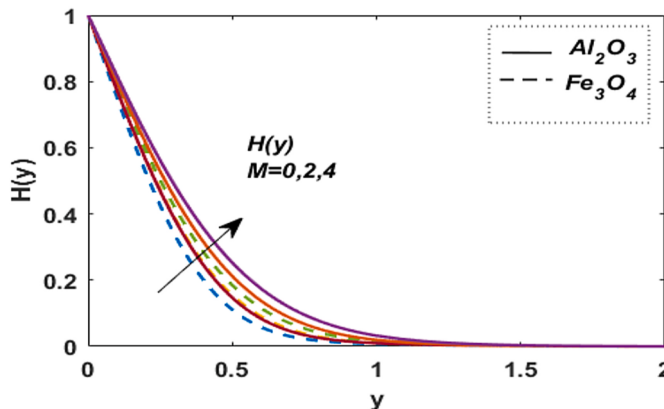
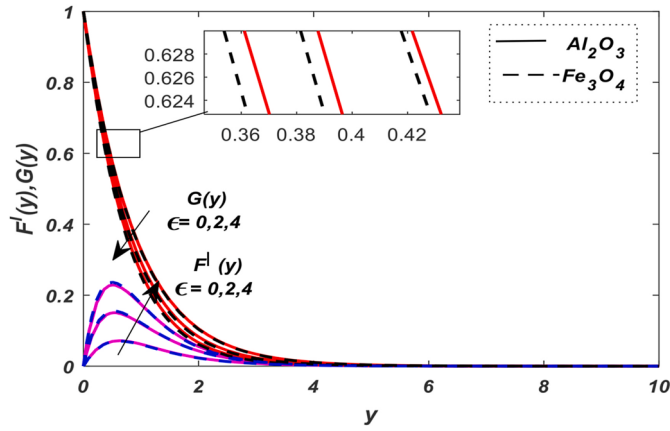
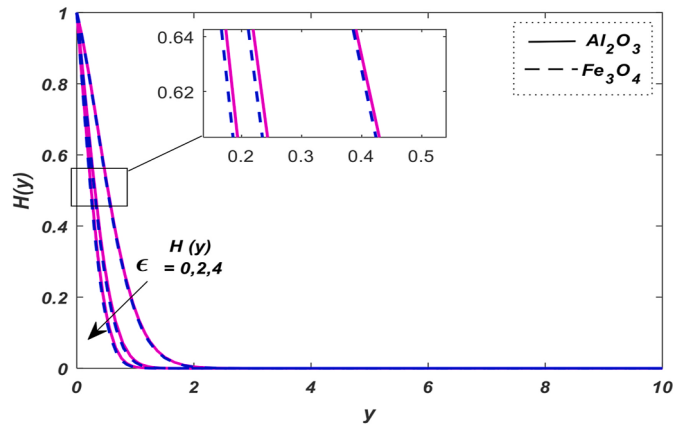
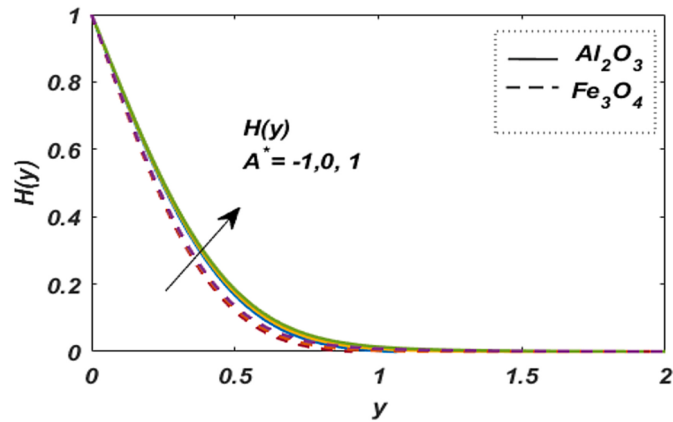
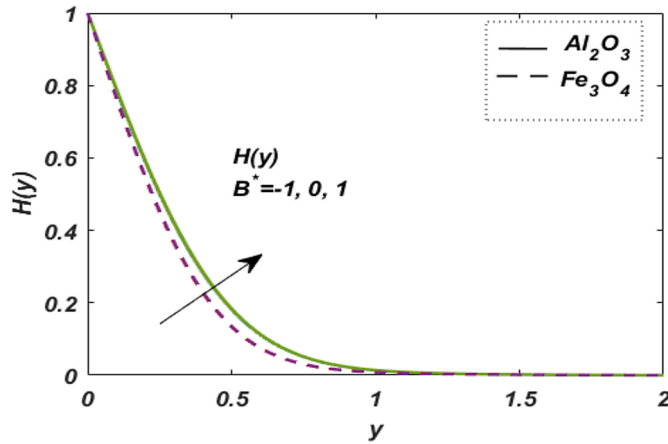
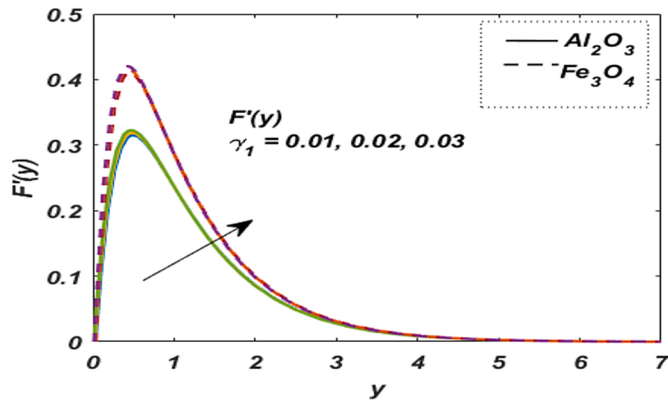
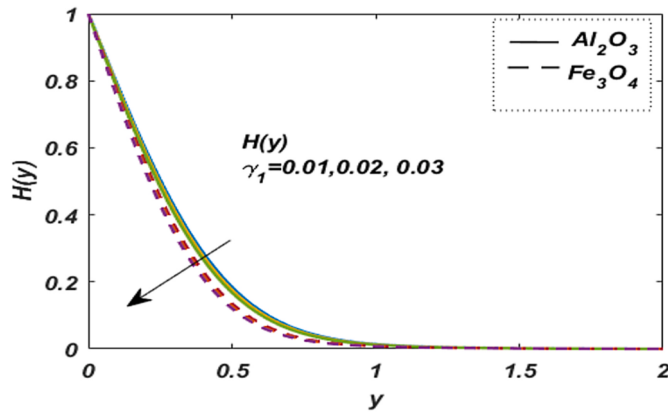


Fig. 2b. Role of M on $H(y)$.

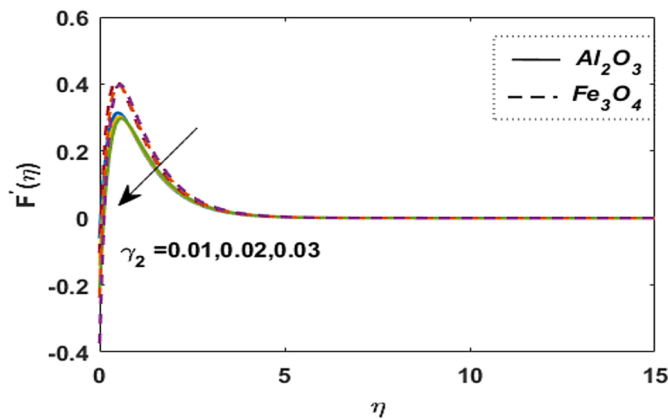
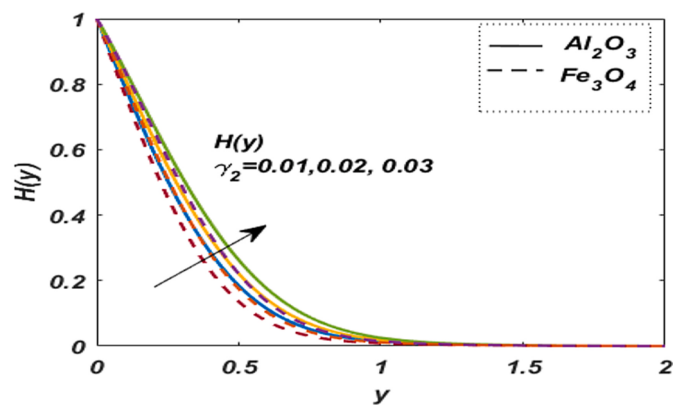
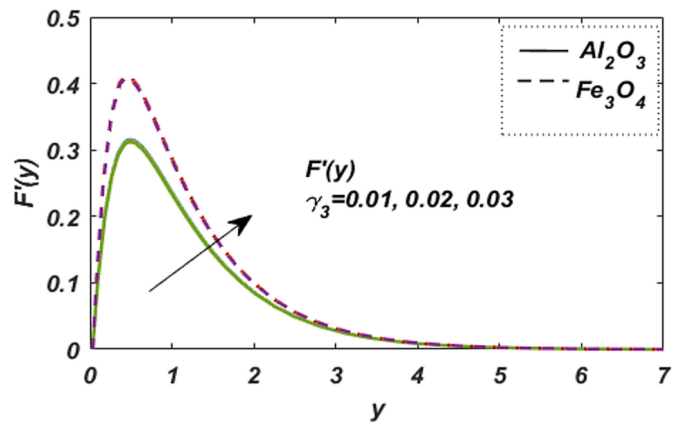
Fig. 3a. Role of ϵ on $F'(y)$ and $G(y)$.Fig. 3b. Role of ϵ on $H(y)$.Fig. 4a. Role of A^* on $H(y)$.

6.4. Influence of $\gamma_1, \gamma_2, \gamma_3, \gamma_4$ on $F'(y)$ and $H(y)$

Fig. 5a, displays the impact of first order slip factor along tangential direction γ_1 on $F'(\eta)$. As γ_1 increases there is enhancement in tangential velocity. The $F'(y)$ is higher in the case of Fe_3O_4 than Al_2O_3 . The $F'(y)$ profiles are characterized by overshoots near tangential surface of the cone. This is because $F'(\eta)$ is higher adjacent to solid surface than that of ambient NF. Fig. 5b indicates the effect of γ_1 on $H(y)$. As γ_1 increases there is drop in temperature. Intensifying slip on the cone's surface reduces the fluid stuck to it. Consequently, fluid temperature peters out. Further, Al_2O_3 has high temperature compared to Fe_3O_4 . As a consequence, the NF with Al_2O_3 as nanoparticle exhibits thicker thermal boundary layer. Fig. 5c displays the role of first order slip parameter along axial direction γ_2

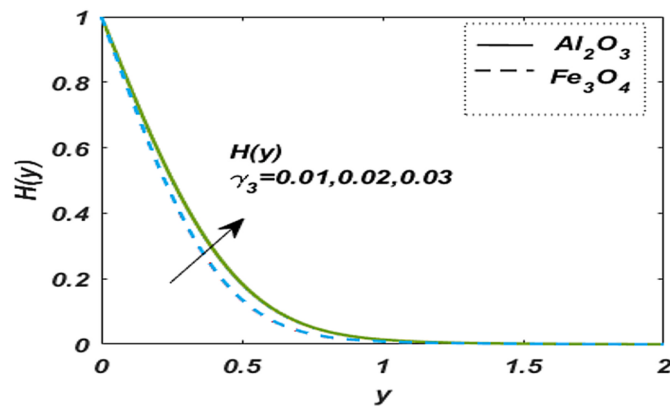
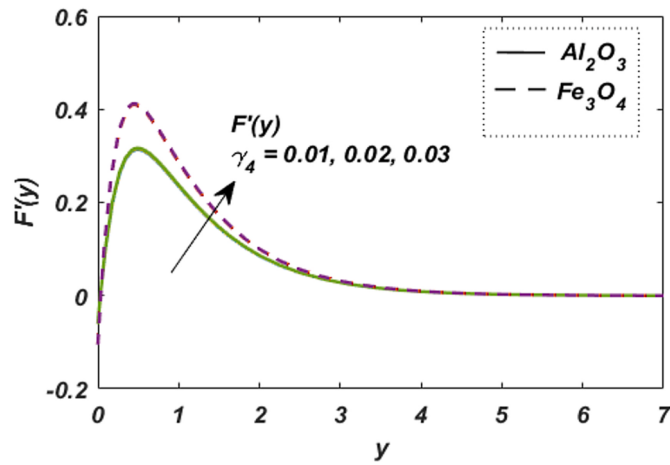
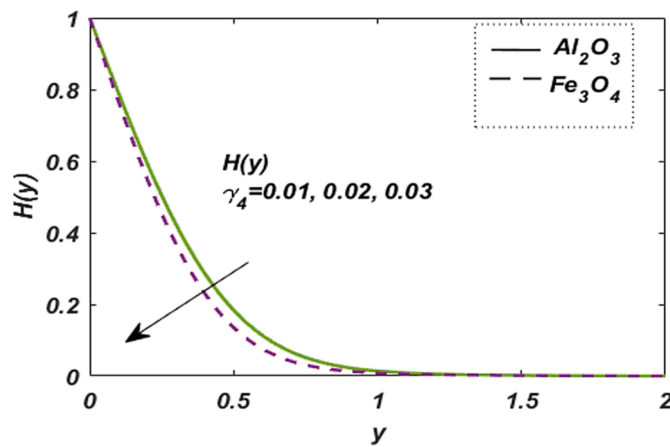
Fig. 4b. Role of B^* on $H(y)$.Fig. 5a. Role of γ_1 on $F'(\eta)$.Fig. 5b. Role of γ_1 on $H(\eta)$.

on velocity profile $F'(\eta)$. As γ_2 rises there is emaciation in $F'(\eta)$ and therefore, velocity boundary layer thickness decays. So rise in slip parameter γ_2 generates decelerated flow of NFs in BL region near the surface. Amplifying slip on the cone's surface can lessen the liquid stuck to it which may lead the shear stress to diminish and consequently can impact the movement of the fluid. In addition, Fe_3O_4 possesses higher velocity than that of Al_2O_3 . In case of temperature profile $H(y)$, increase in γ_2 leads to intensification of temperature $H(y)$. Further, Al_2O_3 has high temperature than Fe_3O_4 (Fig. 5d). Fig. 5e illustrates that increasing the value of γ_3 leads to augmentation of $F'(\eta)$. Further, $F'(\eta)$ of Al_2O_3 nanofluid is lower than that of Fe_3O_4 nanofluid. A similar trend η is visualized for temperature profile $H(\eta)$ in response to rise in γ_3 (Fig. 5f). Fig. 5g and Fig. 5h illustrate that the amplifying γ_4 leads to enhancement of $F'(\eta)$ and enervation in $H(\eta)$. In Fig. 5g, Al_2O_3 is dominated by Fe_3O_4 , while opposite trend is visualized in Fig. 5h.

Fig. 5c. Role of γ_2 on $F(\eta)$.Fig. 5d. Role of γ_2 on $H(\eta)$.Fig. 5e. Role of γ_3 on $F(\eta)$.

6.5. Influence of ϕ, M, A^*, B^* on $C_f Re^{1/2}$ and $Nu Re^{-1/2}$

The values of $C_f Re^{1/2}$ and $Nu Re^{-1/2}$ for sundry ϕ , M , A^* , B^* for Al_2O_3 and Fe_3O_4 with LST is listed in Table 5. It is noticed that $Nu Re^{-1/2}$ decreases whereas $C_f Re^{1/2}$ increases as A^* and B^* grow. By comparing nanoparticles Al_2O_3 and Fe_3O_4 , Fe_3O_4 possesses significantly higher $C_f Re^{1/2}$ and $Nu Re^{-1/2}$. Both $Nu Re^{-1/2}$ and $C_f Re^{1/2}$ peter out due to strengthening of magnetic field. However, they exhibit reverse trend with increment in volume fraction ϕ .

Fig. 5f. Role of γ_3 on $H(\eta)$.Fig. 5g. Role of γ_4 on $F(\eta)$.Fig. 5h. Role of γ_4 on $H(\eta)$.

7. Conclusions

The existing study deals with the free convective flow and HT of Al_2O_3/Fe_3O_4 + water-EG (50:50) NF over a vertical spinning cone subject to magnetic field, space and temperature varying heat source or sink. The current problem may be utilized in the field of aeronautical engineering, cooling nuclear reactor system, solar collectors, and medical field and in development of electronic chips etc. Employing rotating conical geometries. It is worth to mention the following points from the investigation:

Table 5The values of $C_f Re^{1/2}$ and $Nu Re^{-1/2}$ for sundry ϕ , M , A^* , B^* for Al_2O_3 and Fe_3O_4 with linear surface temperature.

ϕ	M	A^*	B^*	$C_f Re^{1/2}$		$Nu Re^{-1/2}$	
				Al_2O_3	Fe_3O_4	Al_2O_3	Fe_3O_4
0.01	1	1	1	2.127103	3.237418	2.175797	2.398573
0.02				3.279087	5.316236	2.445315	2.686296
0.03				4.373248	7.292505	2.640275	2.87445
0.01	1	1	1	2.127103	3.237418	2.175797	2.398573
	2			1.986764	3.089628	2.03683	2.278629
	3			1.903397	2.997533	1.942622	2.19384
0.01	1	-2	1	2.048414	3.103143	2.235011	2.44377
		0		2.098257	3.186411	2.196482	2.413952
		2		2.159075	3.296437	2.153922	2.382585
0.01	1	1	-1	2.120062	3.225694	2.195829	2.41859
			0	2.12356	3.231517	2.185878	2.408656
			1	2.127103	3.237418	2.175797	2.398573
0.01	1	1	0.1	1.204577	1.641722	1.864215	2.030396
			0.5	2.956383	4.673896	2.354564	2.584571
			1	4.853887	7.987911	2.607726	2.808399

- Axial velocity $F(y)$ reduces with rise in M, ϵ, γ_2 while it shows reverse trend with growth of γ_1 .
- Swirl velocity $G(y)$ peters out due to amplification of M, ϵ, γ_3 while it reverses the trend with rise in γ_4 .
- Temperature profiles $H(y)$ upgrades with increase in $M, A^*, B^*, \gamma_2, \gamma_3$ while reverse effect is attained due to rise in $\epsilon, \gamma_1, \gamma_4$.
- As observed, Fe_3O_4 exhibits higher velocity than that of Al_2O_3 while Al_2O_3 possesses higher temperature than that of Fe_3O_4 .
- The surface viscous drag $(C_f Re^{\frac{1}{2}})$ and heat transfer rate $(Nu Re^{-\frac{1}{2}})$ enervate due to strengthening of applied magnetic field while the reverse effect is visualized with amplification of ϕ for both Al_2O_3/Fe_3O_4 + water nanofluids.

7.1. Research questions to understand the significance of the results of the current study

- What are the roles of the first and second order slip mechanisms on the behavior of velocities and temperature of magnetic nanofluid flow over rotating conical surfaces.
- Mention the influence of space and temperature varying heat generation/absorption on thermal behavior of magnetic NF flow over rotating conical surfaces.
- In what way the findings of the existing study serve scientifically to the society/industrial world?

CRediT authorship contribution statement

Yaqun Niu: Formal analysis, Writing. **M.K. Nayak:** Methodology, Writing, Simulation, Validation. **S. Yashodha:** Methodology, Writing, Simulation. **S. Nazari:** Conceptualization, Methodology, Formal analysis. **A.K Abdul Hakeem:** Supervision, Conceptualization. **Rifaqat Ali:** Writing, Formal analysis. **Syed Zaheer Abbas:** Formal analysis. **Ali J. Chamkha:** Supervision.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Data availability

Data will be made available on request.

Nomenclature

(u, v, w)	velocity elements in the x, y and θ directions (ms^{-1})
x	coordinate measured along the plane (m)
y	coordinate normal to the plane (m)
MHD	magnetohydrodynamic
NF	nanofluid
HT	heat transfer
BL	boundary layer

ρ	Density (Kg m^{-3})
(ρC_p)	specific heat capacity
μ	dynamic viscosity
σ	electrical conductivity
k	thermal conductivity ($\text{Wm}^{-1}\text{K}^{-1}$)
β	thermal expansion coefficient (K^{-1})
T	temperature (K)
ν	kinematic viscosity (m^2s^{-1})
γ	half of the vertex angle (rad)
Ω	angular speed of cone (rads^{-1})
g	acceleration due to gravity (ms^{-2})
B_0	magnetic field strength
L_1, L_2	first and second order slip factors along axial direction (m)
L_3, L_4	first and second order slip factors along swirl direction (m)
Gr	Grashof number
Re	Reynolds number
A^*	space dependent parameter
B^*	temperature dependent parameter
Pr	Prandtl number
λ	mixed convection parameter
ε	spin parameter
M	magnetic parameter
C_f	skin friction coefficient
γ_1, γ_2	first and second order slip parameters along axial direction
γ_3, γ_4	first and second order slip parameters along swirl direction
Nu	Nusselt number
ϕ	volume fraction of nanomaterial
Suffixes	
nf	nanofluid
f	base fluid
s	solid nanoparticle

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